MODIFIED-CS-RESIDUAL FOR RECURSIVE RECONSTRUCTION OF HIGHLY UNDERSAMPLED FUNCTIONAL MRI SEQUENCES

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ABSTRACT

In this work, we study the application of compressive sensing (CS) based approaches for blood oxygenation level dependent (BOLD) contrast functional MR imaging (fMRI). In particular, we show, via exhaustive experiments on actual MR scanner data for brain fMRI, that our recently proposed approach for recursive reconstruction of sparse signal sequences, modified-CS-residual, outperforms other existing CS based approaches. Modified-CS-residual exploits the fact that the sparsity pattern of brain fMRI sequences and their signal values change slowly over time. It provides a fast, yet accurate, reconstruction approach that is able to accurately track the changes of the active pixels, while using only about 30% measurements per frame. Significantly improved performance over existing work is shown in terms of practically relevant metrics such as active pixel time courses, activation maps and receiver operating characteristic (ROC) curves.

Index Terms— Compressive Sensing, Functional MRI

1. INTRODUCTION

The static sparse recovery problem has been well studied for a while, e.g. [1, 2]. The recent work on Compressed Sensing (CS) [3, 4] provides the missing theoretical guarantees for these approaches to work. The goal of sparse recovery (henceforth referred to as CS) is to recover a (approximately) sparse signal or image from undersampled measurements by using the fact of sparsity. In this work, we evaluate CS based approaches for blood oxygenation level dependent (BOLD) contrast functional MR imaging (fMRI). We show, via exhaustive experiments on actual MR scanner data for brain fMRI, that our recently proposed approach for recursive reconstruction of sparse signal sequences, modified-CS-residual, outperforms other existing CS based approaches.

In BOLD contrast fMRI, a time-series of T_2^*-weighted images are collected as the subject is presented a controlled stimulus. To achieve whole-brain coverage fMRI is typically performed at a low spatial (e.g., $3 \times 3 \times 3\ mm^3$ voxels) and temporal (e.g., volume repetition time of $2-3$ seconds) resolution. This provides a sufficient signal-to-noise ratio for robust detection of BOLD contrast by statistical testing. However, if CS based approaches can be applied to fMRI it may ultimately enable higher spatial and temporal resolution functional brain imaging, which potentially provides a new view of human brain function [5].

The application of CS to MRI was first developed in detail in [6]. The most straightforward application of CS to fMRI images reconstruction would be to perform CS on each slice of data independently (simple-CS). For time sequences, batch-CS [7] improves simple-CS by jointly reconstructing the entire sequence by treating it as a 3D sparse signal. Because it uses sparsity also along the time axis, it is able to achieve accurate reconstructions using much fewer measurements than simple-CS. But the reconstruction can only be performed on the entire *batch* of data after all sampling is completed. Also, for an N-frame acquisition, its computational complexity is roughly N^2 times that of simple-CS, while its memory requirement is N times that of simple-CS. In recent work, [8, 9] proposed Kt-FOCUSS, which uses the fact that a sequence of MR image data is sparse in the $y-f$ domain where f denotes temporal frequency. The key idea is to reconstruct $kY-t$ "frames" using FOCUSS[2] where kY denotes the phase encoding direction (y-axis of the 2D discrete Fourier transform (DFT) plane). Kt-FOCUSS is still a batch method, which means it is still (a) non-causal, i.e. it needs to wait to acquire the entire N frame sequence before doing the reconstruction (or one needs to re-run it in a batch fashion again at each time which is slow), and (b) its memory requirement is still N times that of simple-CS. But its reconstruction is fast because it is done on one $kY-t$ "frame" at a time and because often it only runs a a few iterations of FOCUSS starting from previous "frame" as initial guess. The same memory and non-causality issues also remain with Kt-FOCUSS with motion compensation (MC) [8]. Moreover, as we demonstrate in our experiments, for the fMRI based BOLD contrast detection application that we study here, its performance is, in fact, slightly worse than our proposed recursive approach (modified-CS-residual) because of its assumption of Fourier sparsity along the time axis – it tries to recover the sparsest sum of sinusoids to represent the time sequence for a given pixel.

In recent work, we studied the problem of recursively reconstructing a time sequence of (approximately) sparse signals from highly undersampled measurements and proposed two sets of approaches – LS-CS and KF-CS [10] and later modified-CS and

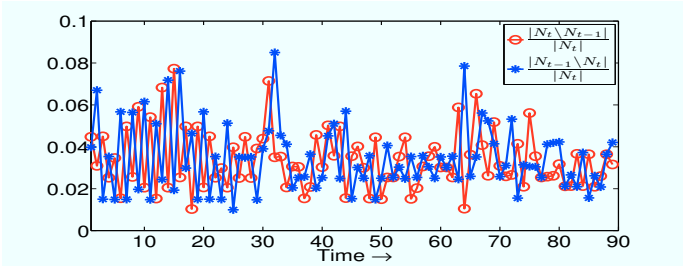


Fig. 1. **Slow support change plots for a simulated brain fMRI sequence (details are given in Sec 4)**. N_t refers to the 99% energy support of the two-level Daubechies-4 2D discrete wavelet transform (DWT) of the image at time t. $|N_t| \approx 0.05m$. We the plot support changes, additions and deletions, with respect to the previous frame

modified-CS-residual [11, 12]. By "recursive", we mean that we use only the previous reconstruction and the current measurements' vector to recover the current signal. As a result, these are (a) causal approaches, i.e. they can recover the current frame as soon as its MR data gets acquired; and (b) they have the same storage (memory) and computational complexity as that of simple-CS (and hence much lower than that of batch methods), but they can achieve significantly lower reconstruction errors than simple-CS when the available number of measurements is too few for simple-CS.

In all the above works, we have done experiments only on either fully simulated data or simulated MRI data, i.e. real medical image sequences, but random-sampled MRI is simulated by taking the 2D discrete Fourier transform (DFT) of the image and randomly sampling it. Moroever, only the mean squared error (MSE) has been used as the performance evaluation metric. But we know that when using actual MR scanner data, (a) there are multiple sources of noise and modeling error so that the resulting 2D-DFT of the image is no longer conjugate symmetric (its inverse DFT is not fully real); and (b) randomly sampling the 2D-DFT plane is not a practical scanning approach. In practice, one can only random sample in one direction e.g. one can only random sample rows or columns of the 2D-DFT plane. (c) Moreover, it is well known to the image processing and medical imaging communities that MSE over the entire image is not a useful performance metric since it does not capture errors in individual pixels very well. But often errors in even a few pixels can be quite problematic, e.g. they can indicate incorrect active regions.

In this work, we perform a detailed experimental evaluation of modified-CS-residual for

1. a real functional MRI application (that of detecting the active region in the brain as a stimulus is provided to the subject);
2. with actual MR scanner data that is acquired in a practically sensible fashion (randomly sample the ky axis); and
3. using practically relevant performance metrics – activation maps and receiver operating characteristic (ROC) curves.

Modified-CS relies on a key assumption that the sparsity pattern (support change in the sparsity basis) changes slowly with time for most practical image sequences. We demonstrate this for brain fMRI sequences in Fig. 1. Notice that the maximum support change is less than 7% of the support size in most cases and in the worst case it is less than 10%. Denote the support estimate from the previous time by T. The key idea of modified-CS is to find the solution that is sparsest outside of T while satisfying the data constraint.

Some other related approaches include Dynamic-LASSO [13] which is a causal but batch approach (with very high computational and storage cost) and it assumes that the sparsity pattern of the image sequence *does not* change with time; or [14] which recovers the difference image by doing CS on the measurement differences(CS-diff). Both CS-diff and our earlier work on LS-CS and KF-CS have already been demonstrated to have worse performance than modified-CS [11, 12]. Approaches related to modified-CS for a static problem but with partial support knowledge include [15, 16].

The paper is organized as follows. We formulate our problem in the next section. Then modified-CS-residual is developed in Sec 3. Experimental results are discussed in Sec 4. Conclusions and future work are given in Sec. 5.

2. PROBLEM DEFINITION

2.1. Notation

A_T denotes the sub-matrix containing the columns of A with indices belonging to T. For a vector, the notation $(\beta)_T$ (or β_T) refers to a sub-vector that contains the elements with indices in T. The set operations, $\cup, \cap$ stand for set union and intersection, respectively, and $T_1 \setminus T_2 := T_1 \cap T_2^c$ denotes set difference. We use T^c to denote the complement of T with respect to $[1, ..., m]$, i.e. $T^c := \{i \notin T, i \in [1, ..., m]\}$. For a set T, $|T|$ denotes its size (cardinality). But for a scalar, β, $|\beta|$ denotes its magnitude.

Let N_t denote the current set of nonzero coefficients (significantly nonzero coefficients in case of compressible sequences) of a signal x_t. N_t consists of three parts: $N_t \triangleq T \cup (\Delta)_t \setminus (\Delta_e)_t$ where $(\Delta)_t$ and T are disjoint and $(\Delta_e)_t \subseteq T$. T is the known part of support while $(\Delta_e)_t$ is the error in the known part of support and $(\Delta)_t$ is the unknown part. $\hat{x}_t$ denotes the estimate of x_t and $\hat{N}_t$ denotes the estimate of N_t.

2.2. Problem Formulation

We formulate the problem for a single slice of fMRI acquired over time. Let $(I_t)_{m_1 \times m_1}$ denote the image at time t and let $m := m_1^2$ be its dimension. The full sampling measurement model is

$$Y_{full,t} = S_t + Z_t \tag{1}$$

where $Y_{full,t}$ is the measured k-space data at time t. S_t is the ideal k-space data and Z_t is the measurement noise, which is modeled as a complex Gaussian noise. The image reconstructed from the full Fourier samples, I_t, can be rewritten as

$$I_t = F' Y_{full,t} F' = I_{true,t} + \eta_t \tag{2}$$

where F is the DFT matrix and $I_{true,t}$ is the ideal image reconstructed from noise-free k-space data. $\eta_t = F' Z_t F'$ is the degrading noise in image domain, which is complex and zero mean Gaussian with variance σ_η^2. We further model the complex image I_t as follows. Each pixel is made up of the baseline MR signal, the functional signal of interest, nuissance signals[17], and the degrading noise signal. Then we model a slice in an fMRI time-sequence as [18].

$$I_t(i,j) = I_b(i,j) + \nu_t(i,j) + \alpha(i,j) \cdot b_t(i,j) + \eta_t(i,j) \tag{3}$$

Here, i, j are the pixel indices with $i, j \in \{1, \ldots, m\}$. I_b is the baseline MR signal which does not change over time. $b_t(i,j)$ denotes the unit-amplitude BOLD signal shape in pixel (i,j), the exact form of which depends on the hemodynamic response function (HDR) corresponding to the pixel. $\alpha(i,j)$ is the non-negative amplitude of the BOLD signal in pixel (i,j) that will be equal to zero in inactive pixels. ν_t is the nuissance signal, which are modeled only for completeness since we aim to faithfully reconstruct I_t from highly undersampled data. From these definitions, the contrast-to-noise ratio (CNR) of the BOLD signal in each pixel can be expressed as $CNR(i,j) = \frac{\alpha(i,j)}{\sigma_\eta}$. MR images, especially MR brain images are known to be compressible in the wavelet transform domain[6]. Hence, we set up the measurement model of CS as follows. Let X_t denote the 2D discrete wavelet transform (DWT) of the image representation from ideal k-space, i.e. $X_t := W I_{true,t} W'$, where W is the DWT matrix. Then $Y_{full,t} = F W' X_t W F + Z_t$. We capture a smaller number, $n < m$, of Fourier coefficients of the images. Since we only sample in kY direction, this can be modeled by applying an $\frac{n}{m_1} \times m_1$ sampling mask, M (which contains a single 1 at a different location in each row and all other entries are zero) to $Y_{full,t}$ to obtain the measurements Y_t,i.e. $Y_t = M Y_{full,t} = M(FW' X_t W F + Z_t)$. The above can also be transformed to a 1D problem by using Kronecker product, denoted by $\bigotimes$. Let $y_{full,t} := vec(Y_{full,t})$, $x_t := vec(X_t)$ and $z_t := vec(Z_t)$. Here, $vec(X_t)$ denotes the vectorization of the matrix X_t formed by stacking the columns of X_t into a single column vector. Then $y_{full,t} = F_{1D} W'_{1D} x_t + z_t$ where $F_{1D} = F \bigotimes F$, $W'_{1D} = W' \bigotimes W'$. An $n \times m$ mask $M_{1D} = Id_{m_1} \bigotimes M$ is applied to $y_{full,t}$ to undersample the Fourier coefficients to obtain y_t where Id_{m_1} is an $m_1 \times m_1$ identity matrix. The above can be rewritten as

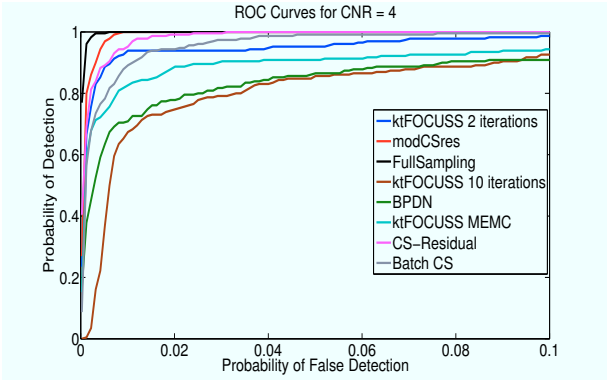


Fig. 2. Comparing modified-CS-residual, Kt-FOCUSS with different iterations and ME/MC, BPDN, CS-residual,and Batch CS with full sampling. At $t = 1$, $n = 100\%m$ measurements are used. For $t > 1$, $n = 0.3m$ measurements are used.

$$y_t = Ax_t + z_t, \; where \; A := H\Phi, \tag{4}$$

where $H := M_{1D}F_{1D}$ and $\Phi := W'_{1D}$. For our algorithm, we require A be satisfying $S = (|T| + 2|\Delta|)$ RIP property[3].

Our final goal is to detect the active pixels' region from the reconstructed sequence, i.e. detect the region where $b_t(i,j) > 0$.

3. MODIFIED-CS-RESIDUAL

BPDN[1] is the most commonly used method in noisy CS. Modified-BPDN[19] tries to find the signal sparsest outside of the set T while satisfying the data constraint. For signal sequences with slow changing support, we can use $T = \hat{N}_{t-1}$. When the measurements are few(smaller than what CS needs), modified-BPDN is known to have much smaller reconstruction error than that of CS(as long as $|\Delta|$ and $|\Delta_e|$ are small) [19].

Furthermore, by using this fact that signal/image also changes slowly over time, we can apply modified-BPDN on the observation residual computed using the previous signal estimate (or using the first signal estimate), i.e. we can solve

$$\arg\min_{\beta} \; \|y_t - Ax_{t,temp} - A\beta\|_2^2 + \gamma\|\beta_{T^c}\|_1 \tag{5}$$

with $\hat{x}_{t,temp} = \hat{x}_{t-1}$ or $\hat{x}_{t,temp} = \hat{x}_1$. The reconstructed signal $\hat{x}_t$ is then given by

$$\hat{x}_t = \hat{\beta} + \hat{x}_{t,temp} \tag{6}$$

We refer the above as modified-CS-residual. If n is small and γ is not large enough, modified-BPDN will not have a unique minimizer. Modified-CS-residual in (5) ensures that the chosen minimizer is the one closest to $\hat{x}_{t,temp}$. Assuming that $\hat{x}_{t,temp}$ is a good initial estimate of x_t, this would be the correct one. In our experiments, we used $\hat{x}_{t,temp} = \hat{x}_1$, the baseline signal at the first frame. The entire algorithm is summarized in Algorithm 1.

Algorithm 1 Modified-CS-residual

Initialization: Do inverse DFT for x_1 and set $\hat{N}_1 = \{k : |(\hat{x}_1)_k| \geq \tau\}$. For $t > 0$, do,

1. **Modified-CS-residual**
   (a) Set $\hat{x}_{t,temp} = \hat{x}_1$.
   (b) **Do Modified-CS-residual.** Compute $\hat{\beta} = \arg\min_{\beta} \; \|y_t - A\hat{x}_{t,temp} - A\beta\|_2^2 + \gamma\|(\beta)_{\hat{N}_{t-1}^c}\|_1$.
   (c) **Compute the support.** Set $\hat{x}_t = \hat{x}_{t,temp} + \hat{\beta}$ and compute $\hat{N}_t = \{k : |(\hat{x}_t)_k| \geq \tau\}$.
2. **Output** $\hat{N}_t$ and $\hat{x}_t$. Increment t and go to step 1.

4. EXPERIMENTAL RESULTS

In this section, we show experiments on real fMRI sequences. We evaluate the performance of detection using 'activation map', 'Receiver operating characteristic(ROC)' and 'time course'. Two-level Daubechies-4 2D discrete wavelet transform(DWT) is used as the sparsifying basis. N_t refers to the 99% energy support of the wavelet coefficients of each frame. Variable density undersampling scheme(which samples from a distribution that has more weight on the low frequencies) [6] is used in our experiments. The sampling mask, M, is varying for each t. In our experiments, the reconstruction of the whole sequences takes 4 seconds for all BPDN, modified-CS-residual, CS-residual, Kt-FOCUSS with 2 iterations.

4.1. Real Brain Sequence(Simulated Activation)

To quantify detection performance using ROC curves, we need to know the ground truth for active regions. Hence in the first experiment, we captured a rest brain sequence (brain fMRI when no stimulus was provided to the subject) using a real MR scanner, but we added the activation later in software. Rest fMRI (TR/TE = 2500/24.3 ms, 90 degree flip angle, 3 mm slick thickness, 22 cm FOV, 64×64 matrix, 90 volumes) was performed using a $3T$ whole-body MR scanner and a gradient-echo echo-planar imaging(EPI) acquisition sequence. We added synthetic BOLD contrast at an average CNR of 4 to pixels corresponding to motor activation on one slice. The 64×64 slice image has 23 active pixels. The BOLD signal was created by convolving a bi-Gamma HDR model (6-s onset delay, 4-s FWHM) with binary-valued function representing a block stimulus (30 s active, 30 s rest; start/end in rest condition). 10 separate observations were generated by resampling with the wavestrapping technique[20] the original rest fMRI data and adding activation to the appropriate pixels to compute descriptive statistics and compute meaningful performance curves.

We compare modified-CS-residual, Kt-FOCUSS, BPDN, batch-CS, CS-residual with IDFT using full sampling. CS-residual, an improved version of CS-diff, refers to doing BPDN on the observation residual computed using the first reconstructed frame. Fig. 2 shows the ROC curves of all methods. From the figure, it is clear that modified-CS-residual has the best performance since the its ROC curve is strictly higher than those of other methods and closest to full sampling. We do not show N-RMSE plot since it can not show the detection performance. But modified-CS-residual has similar N-RMSE as those of Kt-FOCUSS and CS-residual and they are much smaller than other methods. For Kt-FOCUSS, increasing the number of iterations will not help improve the detection performance even if it can reduce N-RMSE. With more iterations, the temporal DC component of Kt-FOCUSS reconstruction becomes better while many other nonzero frequency components are eliminated. Hence, the reconstructed signal is more 'flat' with more iterations which worsens the detection for active pixels but reduces N-RMSE. Similarly, Kt-FOCUSS with ME/MC also has smaller N-RMSE but worse detection performance. CS-residual does not use the slow support change, therefore it has worse detection than modified-CS-residual.

Time courses for one active pixel are shown in Fig. 3. It is also observed that modified-CS-residual does best to track the time course of true(fully sampled) signal, thus providing good reconstruction and detection.

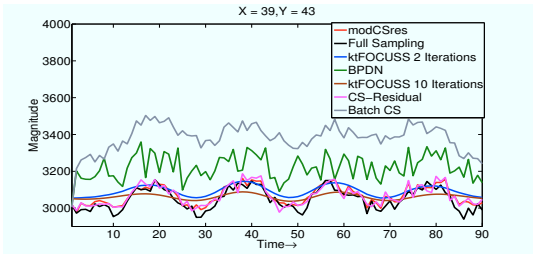
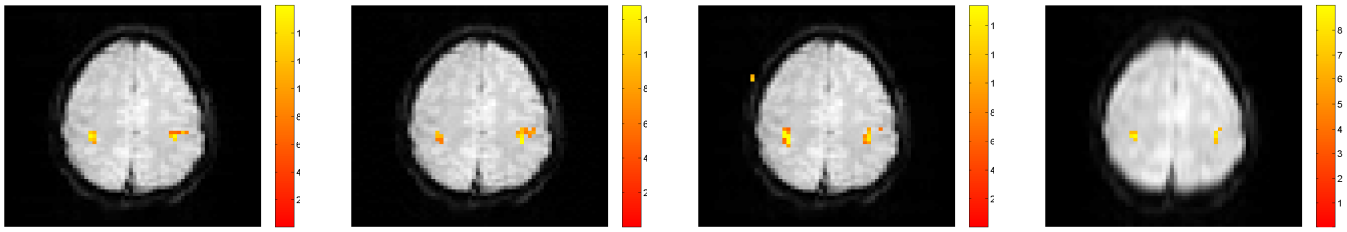


Fig. 3. Time courses of one active pixel.



(a) Full sampling (b) Modified-CS-residual (c) Kt-FOCUSS (d) BPDN

Fig. 4. Comparing activation maps for modified-CS-residual, Kt-FOCUSS, and BPDN with full sampling for each reconstruction. We can see modified-CS-residual has the closest detected regions to full sampling. Modified-CS-residual only has 1 missing active pixel and 5 false ones while Kt-FOCUSS has 4 missing and 11 false ones. BPDN has 7 missing active pixels and 2 false ones.

4.2. Real Brain Sequence(Real Activation)

For real data sequences, we cannot use ROC curves to compare the performances of different methods since no ground truth is available. Our comparison is based on how the detected activation can approximate the activation of IDFT using full Fourier samples. Activation maps for a given threshold in t-test are used to study the detected activation. Different from the simulated sequence, the activations of the real data are not so ideal. For active brain imaging, we used the same experimental setup as the one in Sec. 4.1 except using $n = 0.33m$ measurements for $t > 1$. The activation maps are shown in Fig. 4 for the reconstructions using modified-CS-residual, Kt-FOCUSS and BPDN compared with full sampling when threshold for t-test is set the same for all algorithms. The Bonferroni-corrected threshold is chosen as 5 computed from the dataset. We easily observe that modified-CS-residual has most active pixels detected and few false detection while both Kt-FOCUSS and BPDN has many missing detection.

5. CONCLUSIONS AND FUTURE WORK

We studied the problem of recursively and causally reconstructing a sequence of fMRI sequences from a reduced number of Fourier measurements. We demonstrated improved reconstruction and activation pattern detection performance of our proposed solution, modified-CS-residual on the real fMRI sequences, compared to existing work. In future, we want to do joint real-time detection and reconstruction to further improve performance. Also, higher spatial and temporal resolution sequences will be experimented.

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