1 Hypothesis Testing

- Simple Hypothesis Testing: H0: $\theta = \theta_0$, H1: $\theta = \theta_1$
  - Likelihood Ratio Test (LRT): Reject H0 if
  $$\Lambda(x) = \frac{f_{X,\theta_0}(x)}{f_{X,\theta_1}(x)} < c$$
  - Choose the threshold $c < 1$ so that type 1 error, i.e. $P(choose\ H1|H0) \leq \alpha$, e.g. $\alpha = 0.05$ (called significance level)
  - Or choose c in order minimize "Bayes risk", which is $P(choose\ H1|H0)P(H0) + P(choose\ H0|H1)P(H1)$
  - Here x can be a single observation or a set of independent and identically distributed (iid) observations. If it is a set of observations, then we use the joint PDF, i.e. we write
  $$\Lambda(\underline{x}) = \frac{f_{\underline{X},\theta_0}(\underline{x})}{f_{\underline{X},\theta_1}(\underline{x})} < c$$
  where $f_{\underline{X},\theta}(\underline{x})$ denotes the joint PDF of $X_1, X_2, \ldots X_n$
- Composite Hypothesis Testing (two sided): H0: $\theta = \theta_0$, H1: $\theta \neq \theta_0$
  - Likelihood Ratio Test (LRT): Reject H0 if
  $$\Lambda(\underline{x}) = \frac{f_{\underline{X},\theta_0}(\underline{x})}{\max_\theta f_{\underline{X},\theta}(\underline{x})} < c$$
  - Choose $c < 1$ so that type 1 error, i.e. $P(H1|H0) \leq \alpha$, e.g. $\alpha = 0.05$ (called significance level)
  - Or choose c in order minimize "Bayes risk"
  - To evaluate the denominator, we need to compute the Maximum Likelihood (ML) estimate of θ, denoted by $\hat{\theta}$
- Composite Hypothesis Testing (one sided): H0: $\theta \leq \theta_0$, H1: $\theta > \theta_0$
  - Likelihood Ratio Test (LRT): Reject H0 if
  $$\Lambda(\underline{x}) = \frac{\max_{\theta \leq \theta_0} f_{\underline{X},\theta}(\underline{x})}{\max_\theta f_{\underline{X},\theta}(\underline{x})} < c$$
  - Choose $c < 1$ so that the worst case type 1 error, i.e. $\max_{\theta < \theta_0} P(H1|\theta \leq \theta_0) = P(H1|\theta = \theta_0) \leq \alpha$, e.g. $\alpha = 0.05$ (called significance level)
  - The way to compute the above is as follows: compute the ML estimate of θ, call it $\hat{\theta}$.
  - If $\hat{\theta} < \theta_0$, then numerator = denominator, i.e. $\Lambda(x) = 1 > c$ and thus H0 is accepted,
  - If $\hat{\theta} > \theta_0$, then if $\hat{\theta} > \theta_0 + a$, then reject H0, else accept H0.
- We have evaluated the LRT for a Gaussian pdf with unknown mean in all the three cases above, in class.
- P values: is an important concept to know. See link.
- See the link to Glossary of Hypothesis Testing