

Evaluation of the MYNN PBL Scheme Closure Constants for Low-Level Jet Events in a Stable Boundary Layer

David E. Jahn, Eugene Takle, William Gallus

IGERT Wind Energy Science Engineering and Policy (WESEP)
Program

And Dept. of Geological and Atmospheric Sciences
Iowa State University

Outline

International Experience

- Basic WESEP info.
- Finding a research collaborator
- Description of experience (with pictures!)

Research Project and Results

- Background theory
- Use of observations from tall tower in Germany to identify low-level jet (LLJ) cases
- LES model to generate turbulence-scale data for LLJ cases
- Use of LES results to modify BL parameterization scheme of a numerical weather prediction model

International Experience: the Basics

- 2-3 months working at a research center/university/national lab in another country
- Basic expenses paid (accommodations, food, transportation)
- Working as a visiting researcher (i.e., not necessarily hired by the host institute)
- Need collectively to define a research project/goal commensurate with length of stay and mutually beneficial

ForWind Center for
Wind Energy Research
at
the University of
Oldenburg
in
Oldenburg, Germany
Energy Meteorology Group



For(schung)

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the University of
Oldenburg
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Energy Meteorology Group





Wind Resource

**Turbine and
Blade Design**

**Tower Design,
Production
Techniques**

WHERE is Oldenburg?





DER ANDERE MÜLLER VOM SIEL
25. Mai bis 24. August 2014
Kunstmuseum für Kunst und Kulturgeschichte, Prinzengalerie

Restaurant EGERT'S

Pizza
Bratwurst - Pommes

GIKO

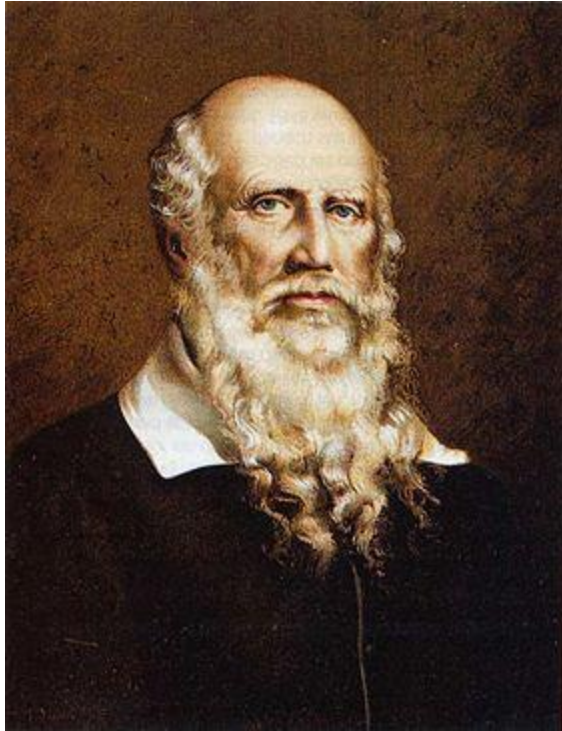
BURGER KING

GARTEN

2







Friedrich Ludwig Jahn





Shut up!
Ask more
questions!



Achtung!
Hier kein Räum-
und Streudienst.
Benutzung auf eigene Gefahr!

 **Hunde
bitte an
die Leine**



FREIE
CHRISTEN
GEMEINDE

Deelweg 14

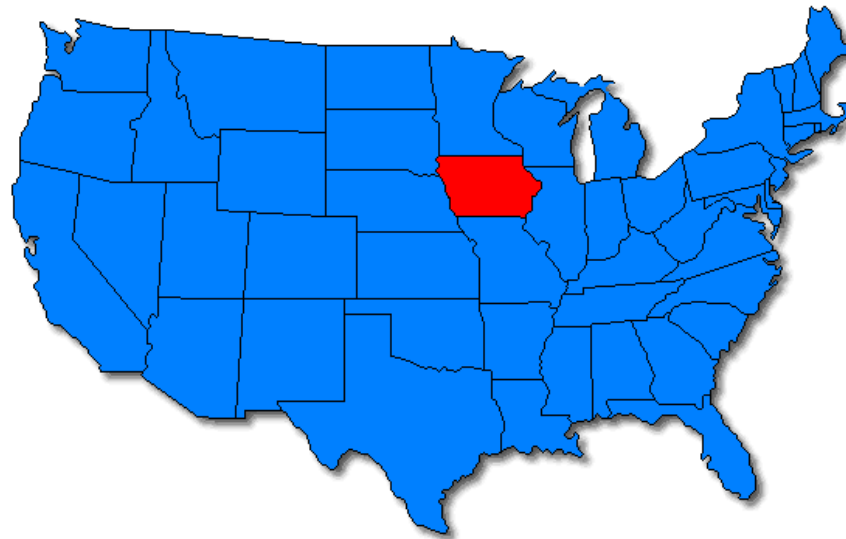


Begeisterte Kollegen!





WHERE is Iowa?!



WHAT is Iowa?

- Corn and soybeans
- Ethanol & other biofuels
- Wind energy
 - 5178 MW, 3216 turbines
 - 3rd in US for capacity
 - 1st in US for %energy, 27.5%
- Iowa State University
 - 33,241 students
 - Engineering & agricultural roots
 - Award-winning campus beauty
 - Wind energy research & education



Wind Energy Science Engineering & Policy

PhD Program that is *interdisciplinary*

Research and course work in:

- Meteorology
- Aerospace Engineering
- Electrical Engineering
- Industrial Engineering
- Mechanical Engineering
- Material Science
- Statistics
- Government Policy
- Economics
- Communications & Journalism



International Experience: Challenges

- Plan for several months to identify and set-up an opportunity (Dr. McCalley can help)
- Finding housing
- Language is generally not a problem
- Expect some level of culture-shock

International Experience: Benefits

- Work and interact with collaborators overseas on a project related to your research
- Expand your network of professionals
- Promote collaboration between ISU and international researchers and research centers
- Experience a new culture, learn another language
- Learn about yourself

Jahn Family Ancestral Home Großstechau, Germany







Thank you ...

WESEP/NSF IGERT grant,
Dr. McCalley, Dr. Takle, and Dr.
Gallus

Evaluation of the **MYNN** PBL Scheme Closure Constants for Low-Level Jet Events in a Stable Boundary Layer

MYNN = Mellor, Yamada, Nakanishi, Niino

Mellor (1973)

Mellor & Yamada (1974, 1982)

Nakanishi (2000, 2001)

Nakanishi & Niino (2004, 2006)

Impact of Wind Ramp on Wind Power

Wind Ramp: Change in power $> 50\%$ wind power capacity within 1-2 hours

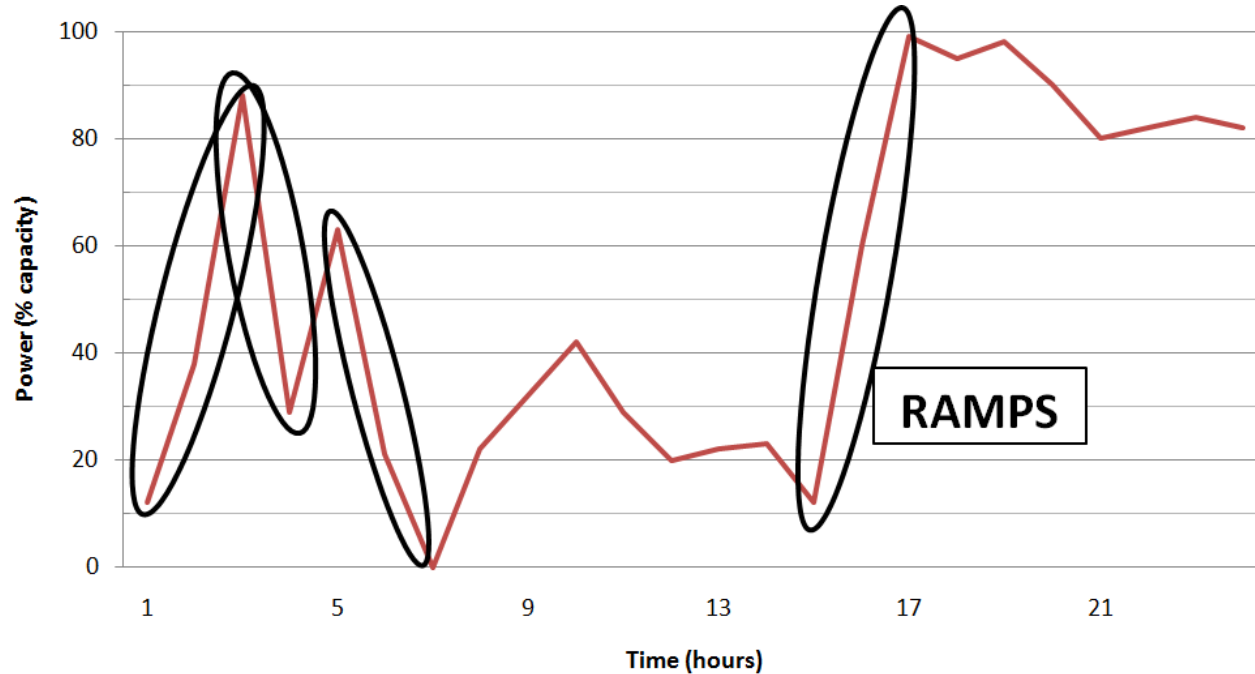
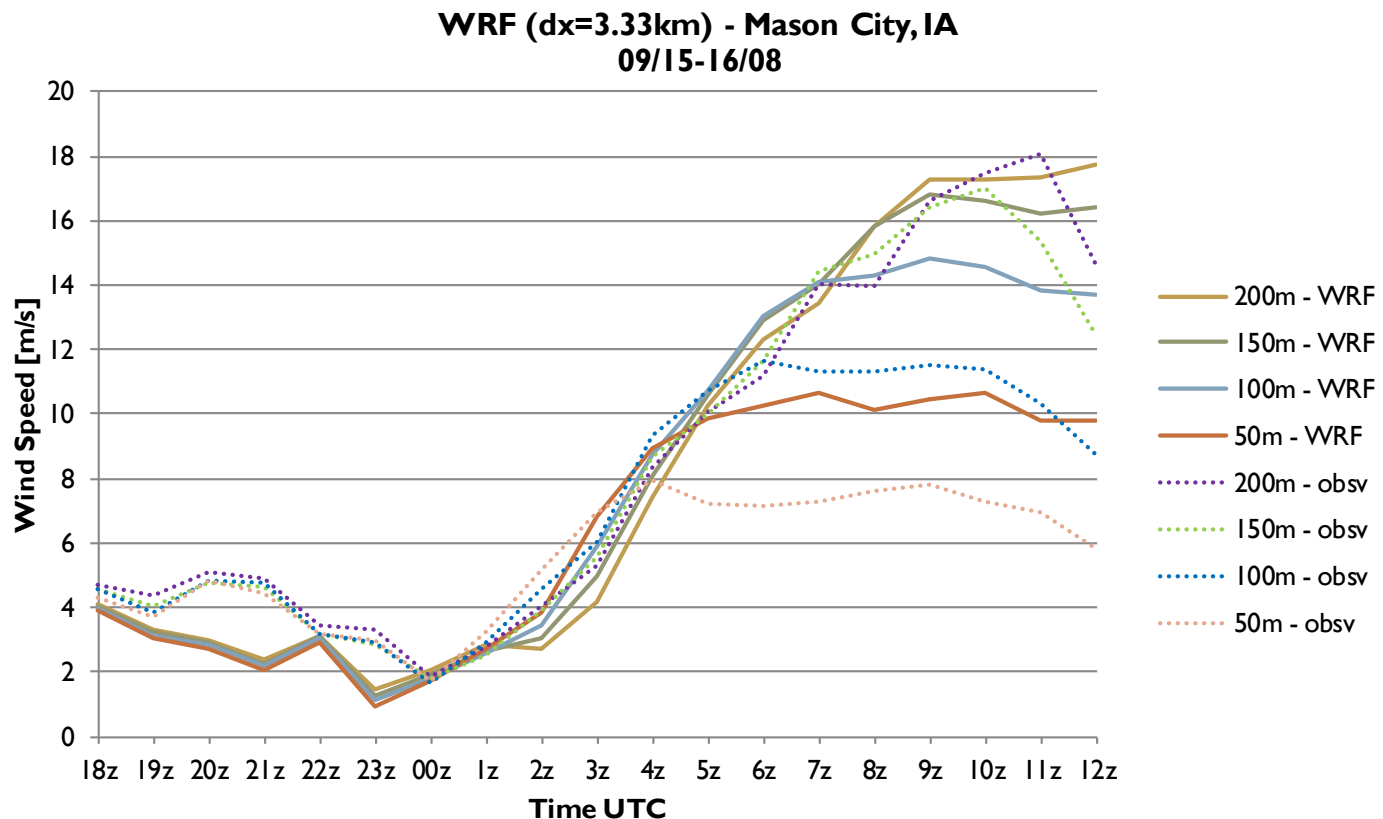


Figure taken from
Ferreira et al. (2010)

WRF Forecast of Wind Ramp Case



Accuracy of NWP Forecasts

- Probability of forecast of a ramp event one day ahead is 30-35% (Greaves 2009; Deppe et al. 2013)
- Operational NWS 6-hour wind forecasts RMS error 3-4 m/s (Benjamin et al. 2013)

Accuracy of NWP Forecasts

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NWP Community has called for focused research on:

- **PBL Schemes** (Schreck et al. 2008; Storm and Basu 2010; Deppe et al. 2013)
- **Especially related to the SBL** (Fernando and Weil 2010; Grisogono 2010; Hu et al. 2013)

Limitations of research to date

- Bulk of research has involved the **evaluation** of existing PBL schemes and **not modification** to the model itself
- PBL schemes have been developed as a “**one size fits all**” approach
- PBL schemes have, for the most part, been **tuned for neutral cases** (i.e., not directly for the SBL)

Limitations of research to date

- Bulk of research has involved the **evaluation** of existing PBL schemes and **not modification** to the model itself
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- PBL schemes have, for the most part, been **tuned for neutral cases** (i.e., not directly for the SBL)

→ **Leaves room for unique research in improving PBL schemes:**

- Digging into the scheme to seek means for improvement
- Specifically for the stable boundary layer (SBL) and wind ramp/LLJ events

MYNN Scheme Improvement for LLJ: Revisit Closure Constants

What are **closure constants**?

Need to reference MYNN basic theory ...

MYNN: Basic Theory

- Reynolds-Averaged Navier-Stokes Eqs.
- Neglected molecular viscosity
- First-order eq. with a second-order term

$$\frac{\partial \bar{U}_j}{\partial t} + \bar{U}_k \frac{\partial \bar{U}_j}{\partial x_k} = -\delta_{i3}g + f_c \epsilon_{ij3} \bar{U}_j - \frac{1}{\bar{\rho}} \frac{\partial \bar{P}}{\partial x_j} - \frac{\partial \overline{u_k u_j}}{\partial x_k}$$

Time-tendency

Advection by mean wind

Gravity force

Coriolis force

Pressure-gradient force

Turbulent momentum flux
divergence

MYNN: Basic Theory

- Reynolds-Averaged
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Time-tendency

Advection by mean wind

Gravity force

Coriolis force

Pressure-gradient force

Turbulent momentum flux divergence

MYNN Scheme: Solving for turbulent fluxes $\overline{u_i u_j}$

$$\begin{aligned} \text{Time-tendency} &= \text{Energy redistribution} \\ \frac{D\overline{u_i u_j}}{Dt} &= \overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} \\ &+ \text{Dissipation} \quad + \quad \text{Buoyancy} \\ &+ \nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} \quad + \quad \overline{g u_j \theta} \\ &+ \text{Diffusion} \quad + \quad \text{Shear production} \\ &+ \overline{u_i u_j u_k} \quad + \quad \overline{u_k u_i} \frac{\partial \overline{U_j}}{\partial x_k} \end{aligned}$$

MYNN Scheme: Solving for turbulent fluxes $\overline{u_i u_j}$

$$\begin{aligned} \text{Time-tendency} \quad \frac{D\overline{u_i u_j}}{Dt} &= \text{Energy redistribution} \quad \overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} \\ &+ \text{Dissipation} \quad \nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} \\ &+ \text{Buoyancy} \quad \overline{g u_j \theta} \\ &+ \text{Diffusion} \quad \overline{u_i u_j u_k} \\ &+ \text{Shear production} \quad \overline{u_k u_i} \frac{\partial \overline{U_j}}{\partial x_k} \end{aligned}$$

Appx.

Dissipation Approximation

$$\boxed{\text{Momentum Dissipation}} = \frac{1}{B_1} \boxed{\frac{(2 * TKE)^{3/2}}{\text{mixing length}}}$$

$$TKE = \frac{1}{2} (u^2 + v^2 + w^2)$$

*Mixing length = f(stability, height above the surface)**

*Mellor and Yamada (1984), Nakanishi (2001)

Dissipation Approximation

Momentum
Dissipation

$$= \frac{1}{B_1} \frac{(2 * TKE)^{3/2}}{\text{mixing length}}$$

The diagram illustrates the dissipation approximation equation. On the left, a box contains the text "Momentum Dissipation". This is followed by an equals sign. To the right of the equals sign, the fraction $\frac{1}{B_1}$ is circled in red. A red line extends from the bottom of this circle to the text "Closure Constant" below. To the right of the circled fraction is another box containing the expression $\frac{(2 * TKE)^{3/2}}{\text{mixing length}}$.

Closure Constant

Dissipation Approximation

$$\text{Momentum Dissipation} = \frac{1}{B_1} \frac{TKE^{3/2}}{\text{mixing length}}$$

$$\text{Heat Dissipation} = \frac{1}{B_2} \frac{TKE^{1/2} \theta^2}{\text{mixing length}}$$

Dissipation Approximation

$$\text{Momentum Dissipation} = \frac{1}{B_1} \frac{TKE^{3/2}}{\text{mixing length}}$$

$B_1 = 24.0$ in MYNN scheme (WRF version 3.5.1)

$$\text{Heat Dissipation} = \frac{1}{B_2} \frac{TKE^{1/2} \theta^2}{\text{mixing length}}$$

$B_2 = 15.0$ in MYNN scheme (WRF version 3.5.1)

(Based on study of near-neutral cases by Nakanishi 2001)

Dissipation Approximation

$$\text{Momentum Dissipation} = \frac{1}{B_1} \frac{TKE^{3/2}}{\text{mixing length}}$$

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$$\text{Heat Dissipation} = \frac{1}{B_2} \frac{TKE^{1/2} \theta^2}{\text{mixing length}}$$

$B_2 = 15.0$ in MYNN scheme (WRF version 3.5.1)

Should B_1 and B_2 remain the same for all cases?

Calculate closure constants for the stable BL in context of LLJ cases

Start with prognostic equation for TKE

Calculate closure constants for the stable BL in context of LLJ cases

Start with prognostic equation for TKE

$$\begin{aligned} & \boxed{\text{Time-tendency}} \quad \frac{\partial(TKE)}{\partial t} \quad \equiv \quad \boxed{\text{TKE vert. flux}} \quad -\frac{\partial}{\partial z} (\overline{w(TKE)^{1/2}}) \\ & + \boxed{\text{Shear}} \quad -\overline{uw} \frac{\partial U}{\partial z} \quad + \quad \boxed{\text{Buoyancy}} \quad \frac{g}{\Theta_o} \overline{w\theta} \quad + \quad \boxed{\text{Dissipation}} \quad -\frac{1}{B_1} \frac{(TKE)^{3/2}}{L} \end{aligned}$$

Determining Closure Constants

Neglect first two terms*

(Level 2.0 of 1.5 order TKE closure scheme)

$$\begin{aligned} & \boxed{\text{Time-tendency}} \quad = \quad \boxed{\text{TKE vert. flux}} \\ & \quad \frac{\partial (\text{TKE})}{\partial t} \quad = \quad - \frac{\partial}{\partial z} (w(\text{TKE})^{1/2}) \\ & + \boxed{\text{Shear}} \quad + \quad \boxed{\text{Buoyancy}} \quad + \quad \boxed{\text{Dissipation}} \\ & \quad - \overline{uw} \frac{\partial U}{\partial z} \quad + \quad \frac{g}{\Theta_o} \overline{w\theta} \quad + \quad - \frac{1}{B_1} \frac{(\text{TKE})^{3/2}}{L} \end{aligned}$$

*Nakanishi (2001)

Determining Closure Constants

$$B_1 = \frac{(2 * TKE)^{3/2} / L}{\frac{g}{\theta_0} \overline{w\theta} + \overline{uw} \frac{\partial U}{\partial z}}$$

Using explicit values of turbulence fluxes from large eddy simulations of LLJ cases

$$\overline{w\theta} \quad \overline{uw} \quad TKE$$

First Step: Select Wind LLJ Cases



- Tall tower near Hamburg, Germany
 - Wind speed and dir. (cup anemometer and wind vane) and
 - 3D wind measurements (sonic anemometer),
 - Temp., RH at
 - Obsv. heights: 10, 50, 110, 175 [m]
 - 1-min. avg. data
 - variances, covariances (since 2004)

Brummer, B., Lange, I., Konow, H., 2006: Atmospheric boundary layer measurements at the 280 m high Hamburg weather mast 1995–2011: mean annual and diurnal cycles. *Meteorologische Zeitschrift*, **21**, No. 4, 319-335

First step: Select LLJ Cases

Using Hamburg tower data 2010-2012

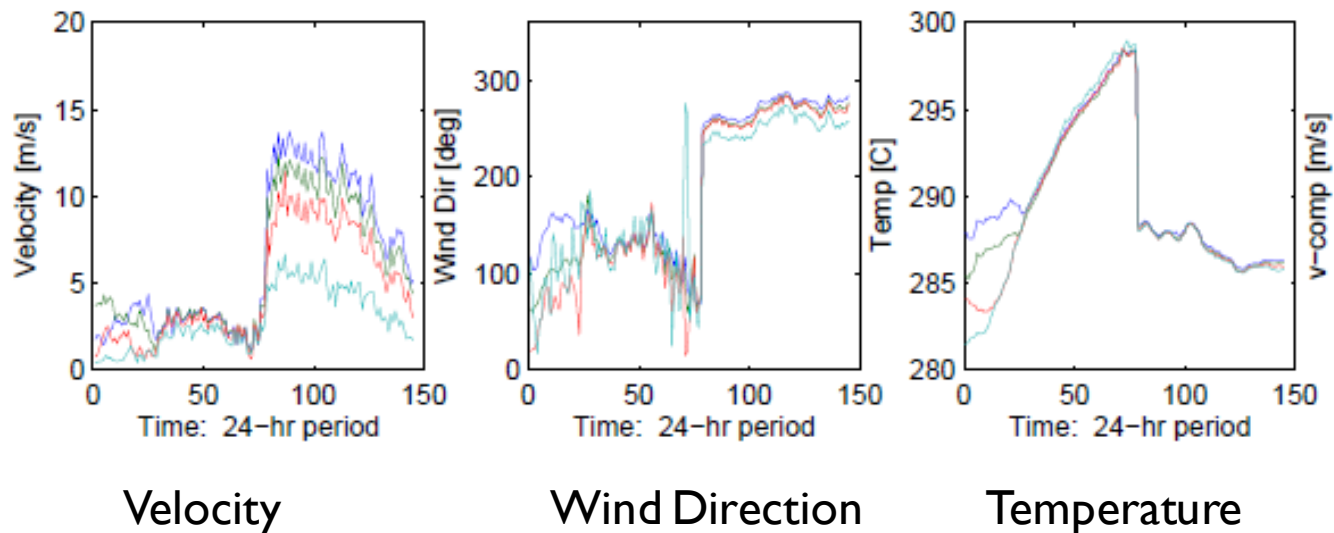
(roughly 1,000 days)

Looking for LLJ cases:

- Wind ramp > 4 m/s in 1-hr.
- Stable BL, preferably nocturnal
- No influence of front
- No influence of nearby convection
- No cases with BL wind out of west
(eliminate impact of city of Hamburg)

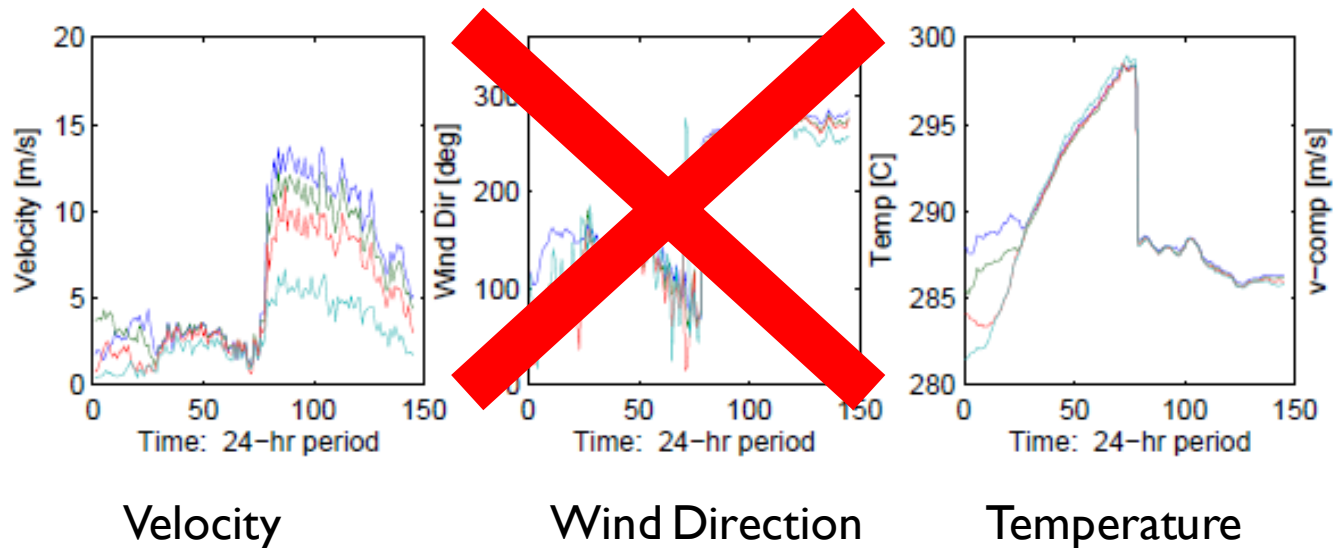


Select LLJ Cases from Hamburg data



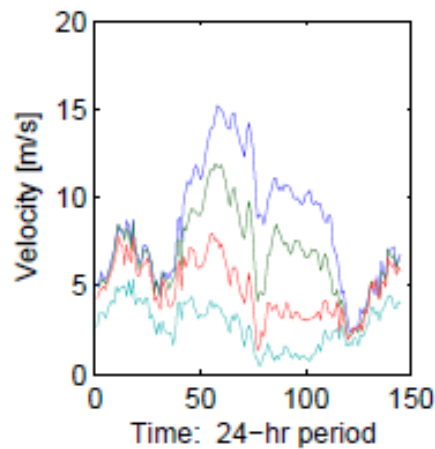
Blue – 175m
Green – 110m
Red – 50m
Aqua – 10m

Select LLJ Cases from Hamburg data

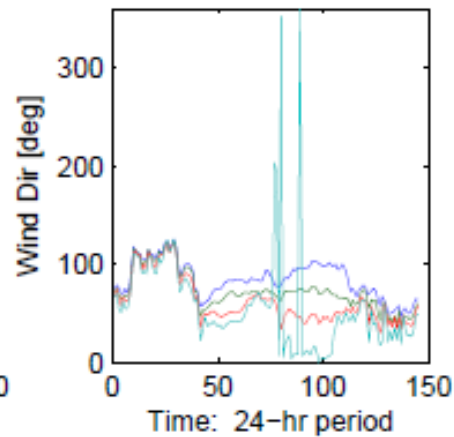


Blue – 175m
Green – 110m
Red – 50m
Aqua – 10m

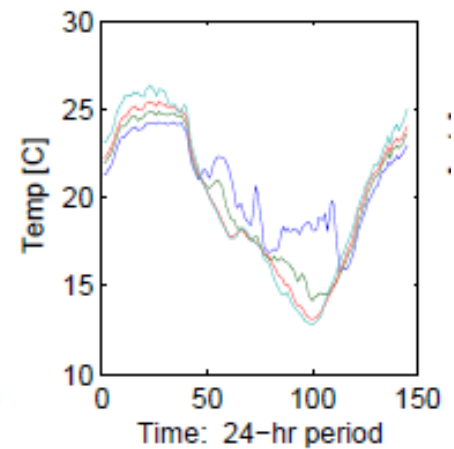
Select LLJ Cases from Hamburg data



Velocity



Wind Direction

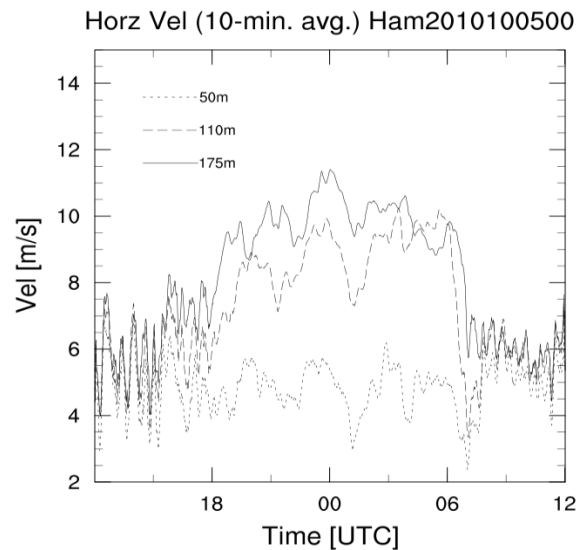


Temperature

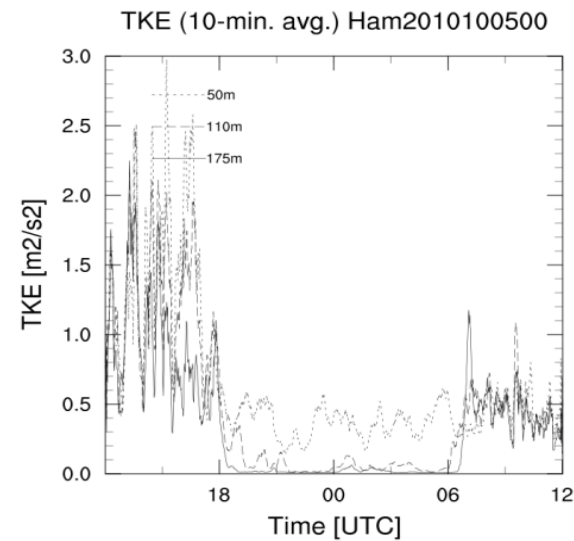
Blue – 175m
Green – 110m
Red – 50m
Aqua – 10m

Example Case: 09/05/2010

Hamburg Tower Observations



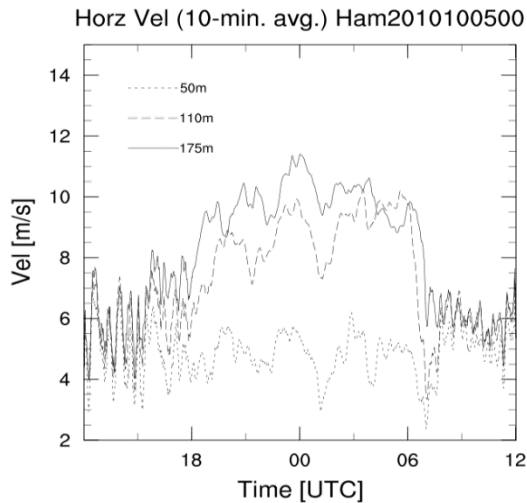
Velocity [m/s]



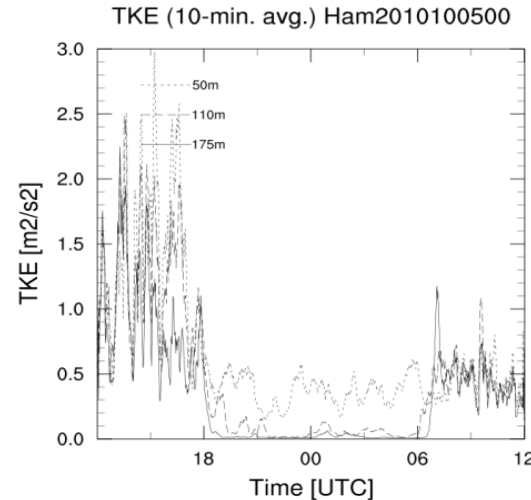
TKE [m2/s2]

Example Case: 09/05/2010

Velocity
[m/s]



TKE [m²/s²]



Time-tendency

$$\frac{\partial(TKE)}{\partial t}$$

=

TKE vert. flux

$$-\frac{\partial}{\partial z} \overline{w(TKE)^{1/2}}$$

+

Shear

$$-\overline{uw} \frac{\partial U}{\partial z}$$

+

Buoyancy

$$\frac{g}{\Theta_o} \overline{w\theta}$$

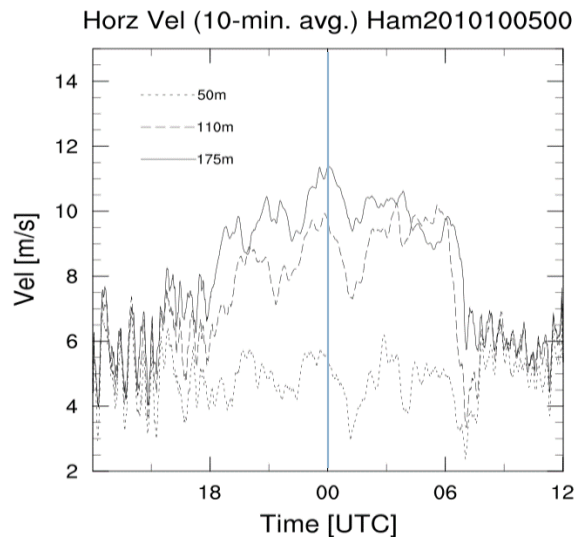
+

Dissipation

$$-\frac{1}{B_1} \frac{(TKE)^{3/2}}{L}$$

LES Simulation of a LLJ case

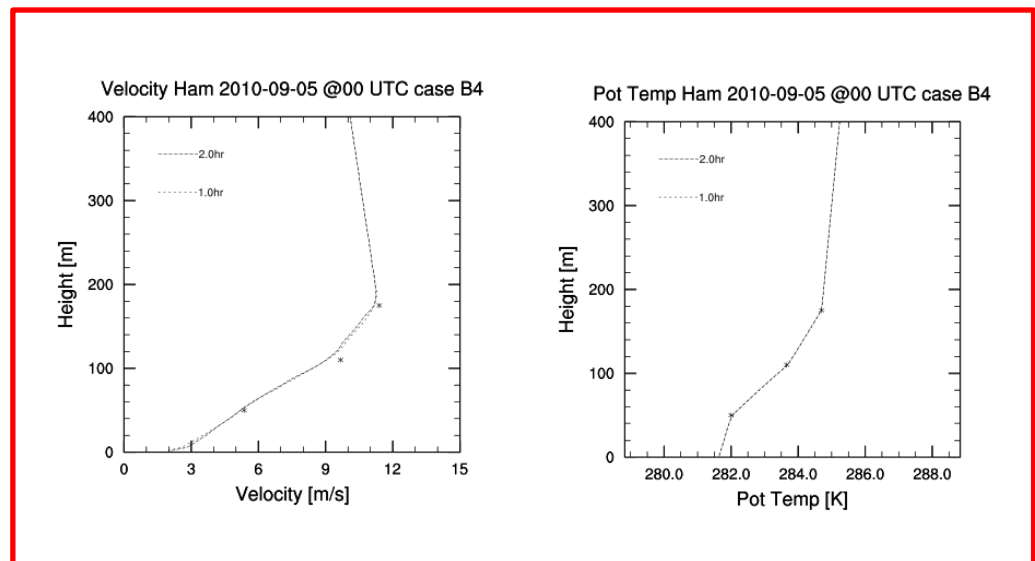
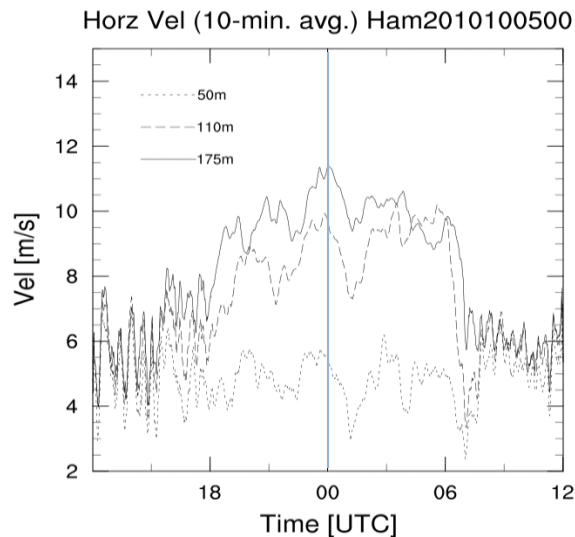
- WRF-LES model
 - Initialized using a vertical profile of wind velocity and pot. temp.



LES Simulation of a LLJ case

- WRF-LES model
 - Initialized using a vertical profile of wind velocity and pot. temp.

Initialize LES Model

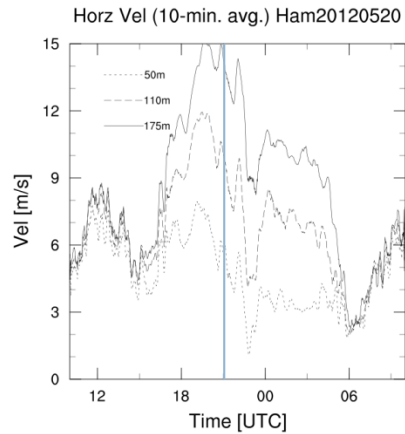


LES Simulation of a LLJ case

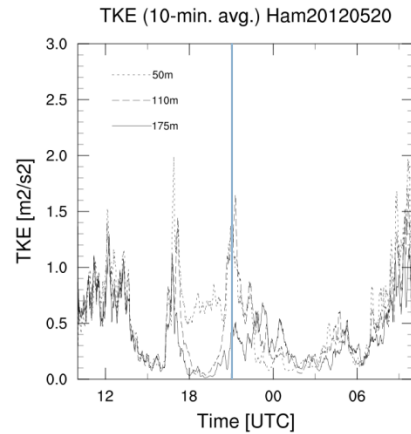
- WRF-LES model
 - Horizontally homogeneous
 - 4m grid resolution (dx, dy, dz)
 - Domain 400m x 400m x 1300m
 - Run for 2 hours to allow for stable solution

Observations

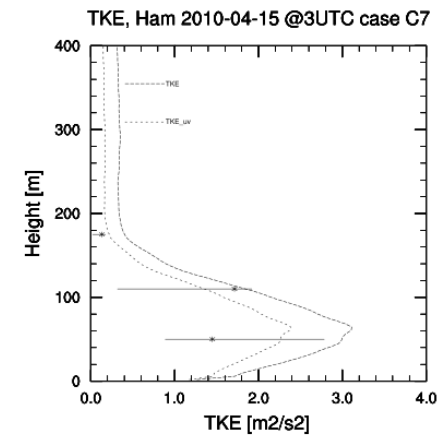
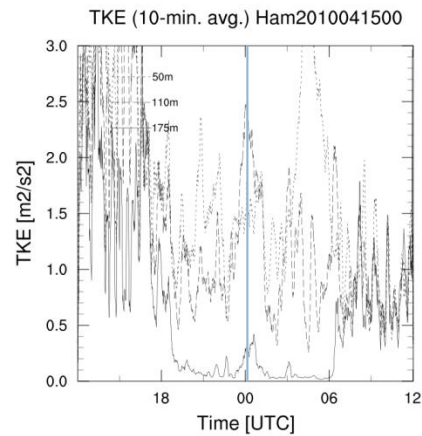
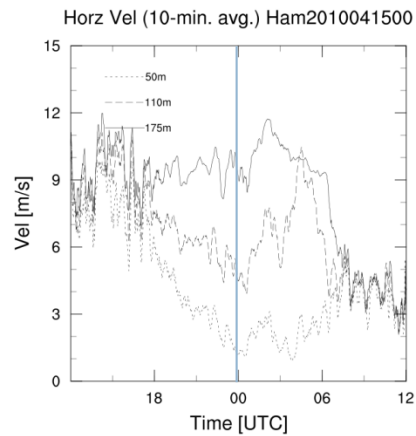
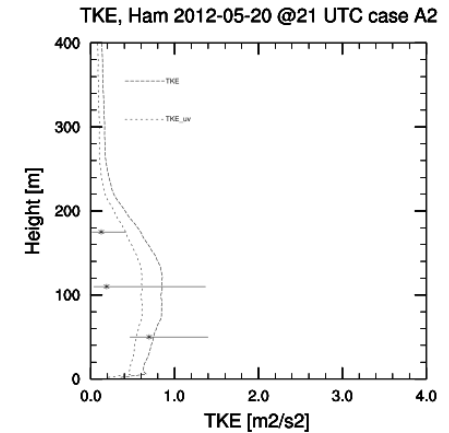
Velocity



TKE

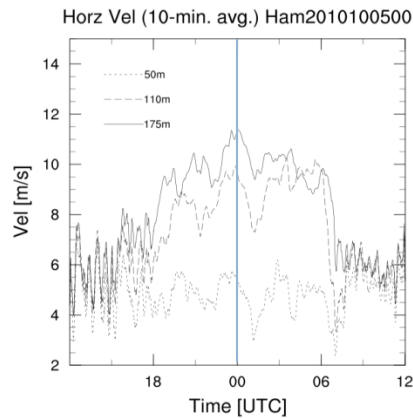


TKE

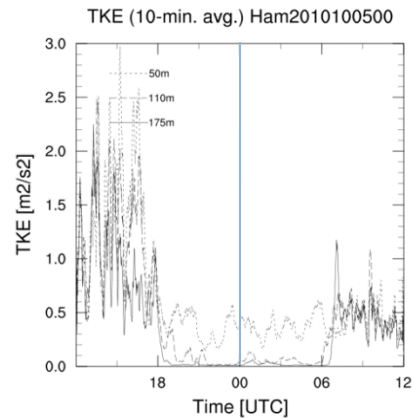


Observations

Velocity

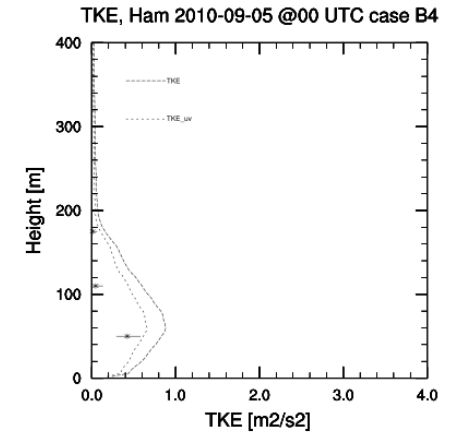


TKE



Results

TKE



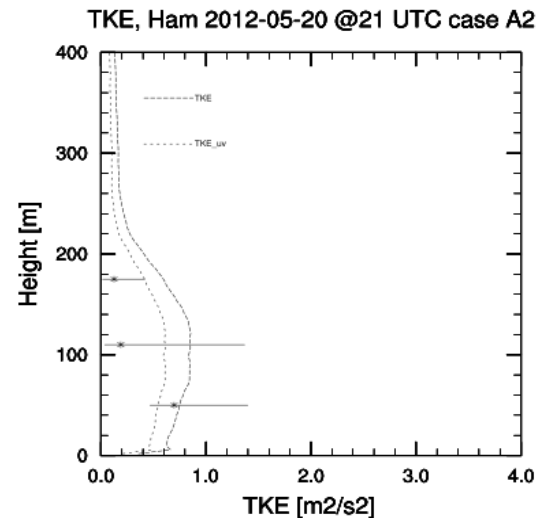
Calculate Closure Constants

LES results provide explicit values for variance and covariance values:

$$\overline{w\theta} \quad \overline{uw} \quad TKE$$

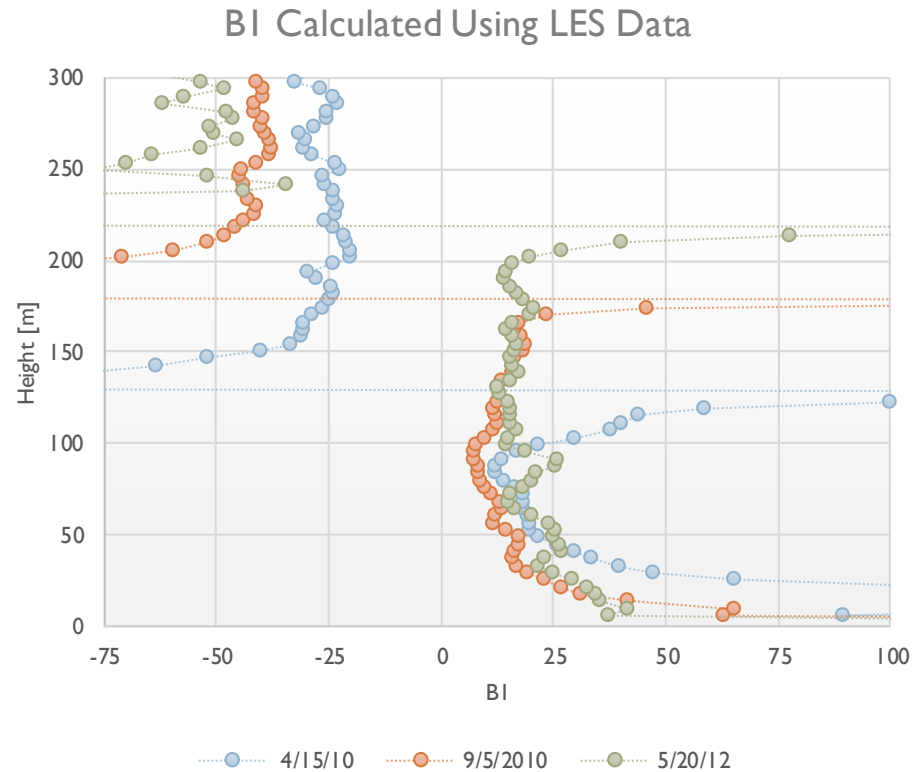
$$B_1 = \frac{(TKE)^{3/2}/L}{\frac{g}{\theta_0} \overline{w\theta} + \overline{uw} \frac{\partial U}{\partial z}}$$

$$B_2 = \frac{(TKE)^{1/2} \overline{\theta^2}/L}{\overline{w\theta} \frac{\partial \Theta}{\partial z}}$$



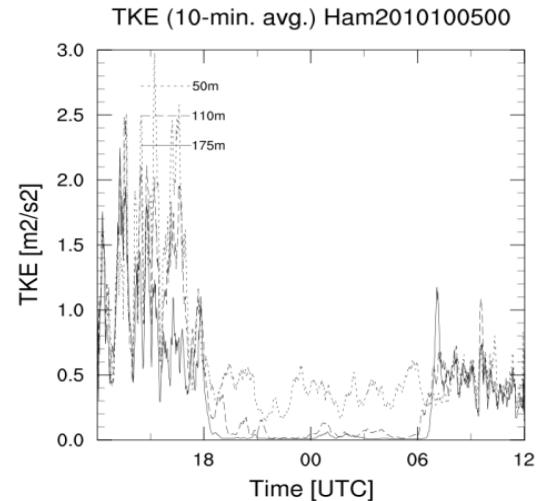
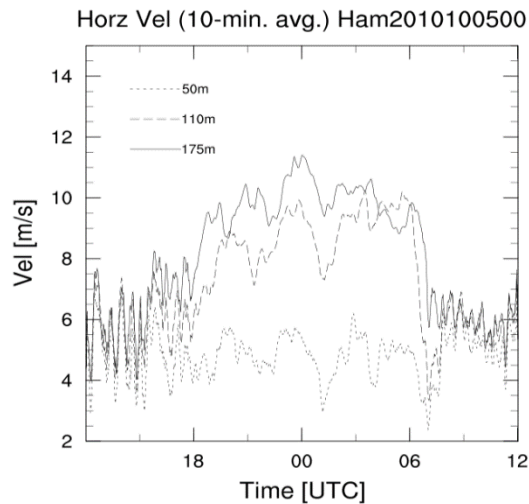
Calculate Closure Constants

$$B_1 = \frac{(TKE)^{3/2}/L}{\frac{g}{\theta_0} \overline{w\theta} + \overline{uw} \frac{\partial U}{\partial z}}$$



Example Case: 09/05/2010

Velocity
[m/s]



TKE [m2/s2]

Time-tendency

$$\frac{\partial(TKE)}{\partial t}$$

=

TKE vert. flux

$$-\frac{\partial}{\partial z} \overline{w(TKE)^{1/2}}$$

+

Shear

$$-\overline{uw} \frac{\partial U}{\partial z}$$

+

Buoyancy

$$\frac{g}{\Theta_o} \overline{w\theta}$$

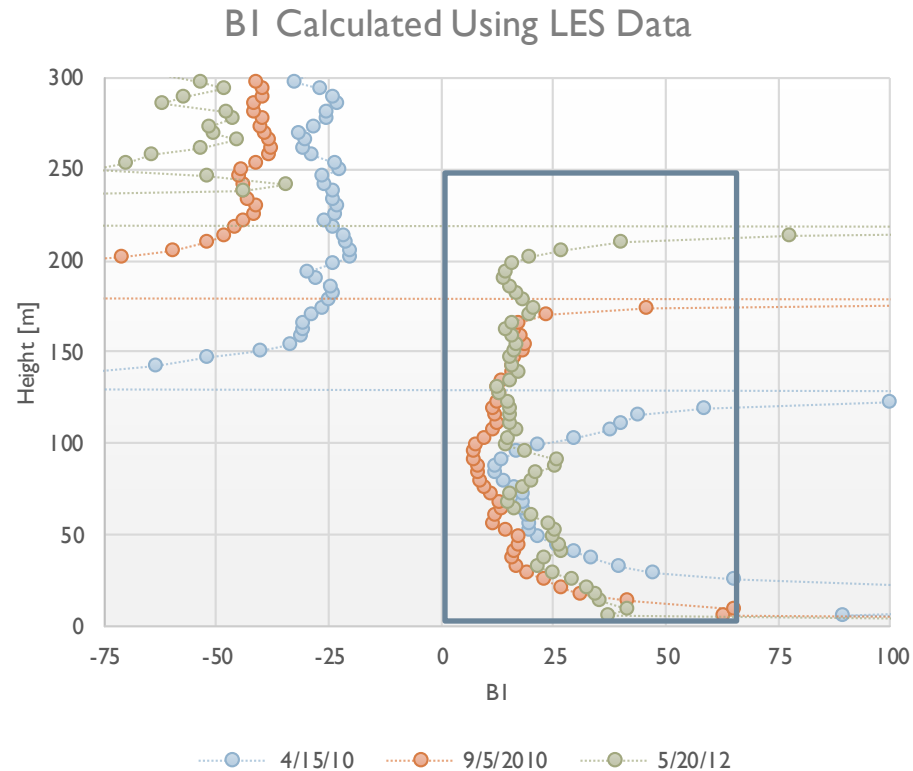
+

Dissipation

$$-\frac{1}{B_1} \frac{(TKE)^{3/2}}{L}$$

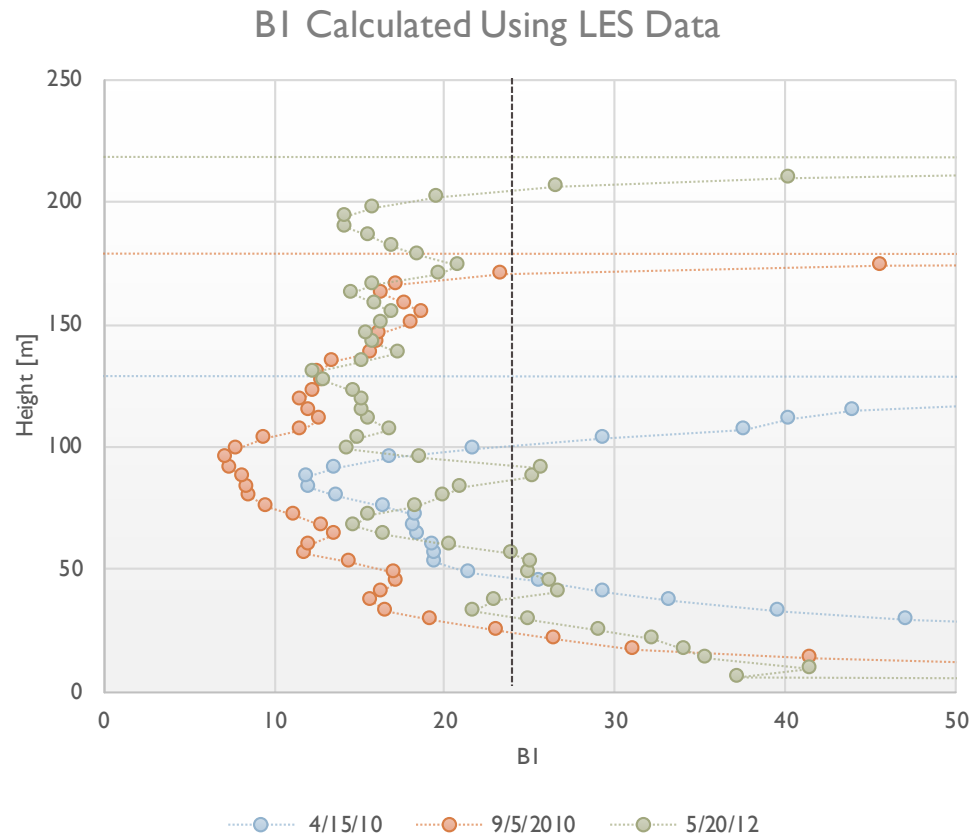
Calculate Closure Constants

$$B_1 = \frac{(TKE)^{3/2}/L}{\frac{g}{\theta_0} \overline{w\theta} + \overline{uw} \frac{\partial U}{\partial z}}$$

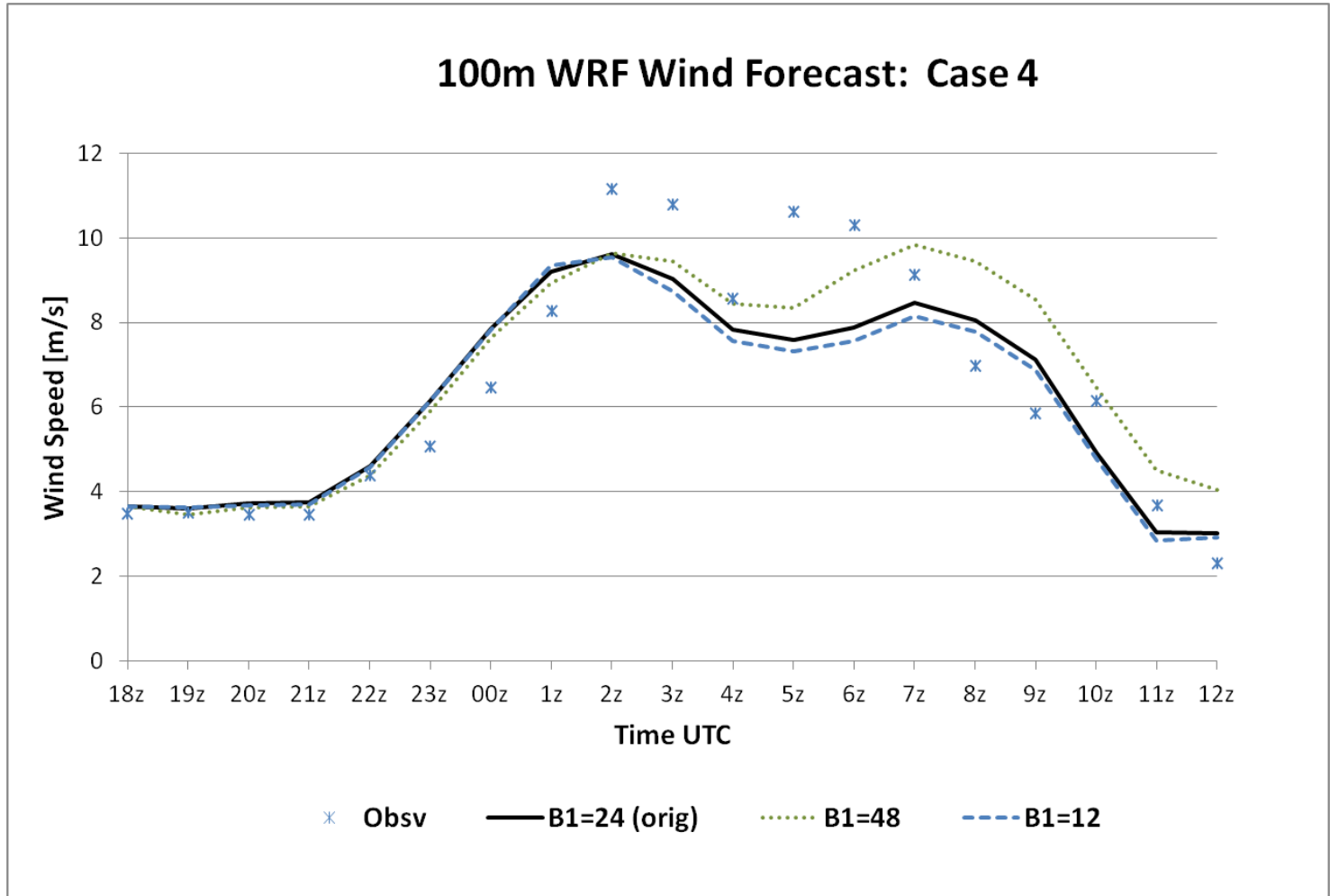


Calculate Closure Constants

BI=24
Per MYNN



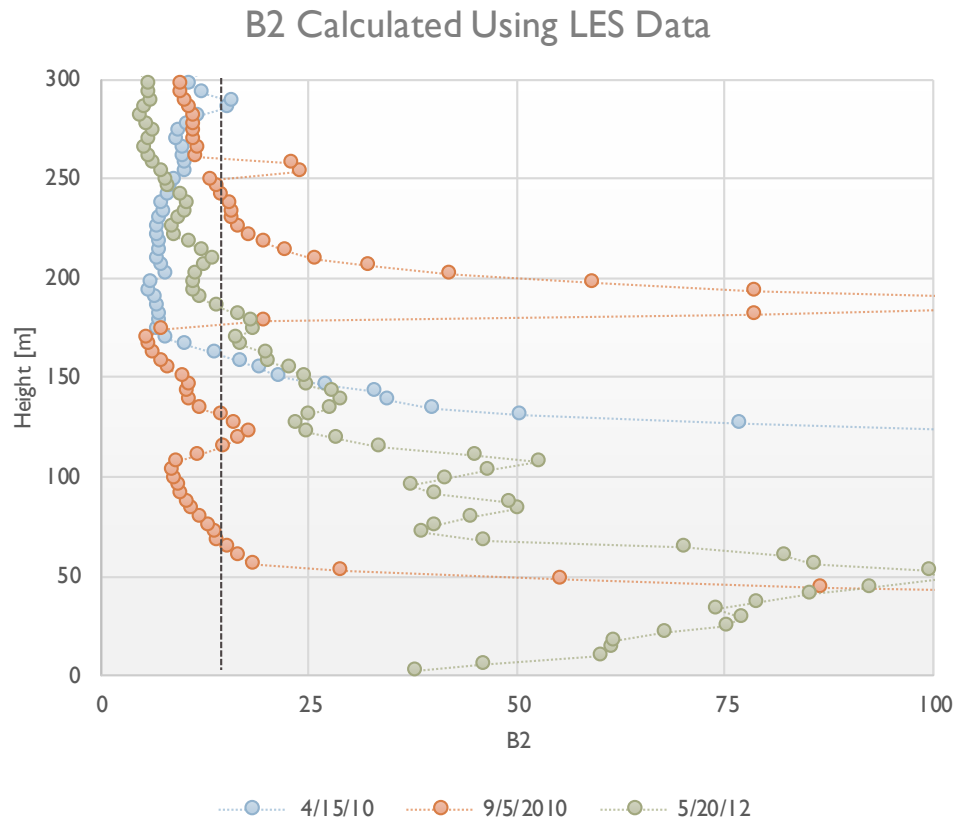
Dissipation Term Sensitivity Tests



Calculate Closure Constants

$$B_2 = \frac{(TKE)^{1/2} \overline{\theta^2} / L}{\overline{w\theta} \frac{\partial \theta}{\partial z}}$$

B2=15
Per MYNN



Sensitivity Tests

Rel. Val. B1	0.5 x B1 _{cntl}	2.0 x B1 _{cntl}
6/13/2008	0.18	9.54
10/25/2007	-2.46	12.30
9/19/2007	-0.73	0.92
Average	-1.00	7.59
<hr/>		
Rel. Val. B2	0.5 x B2 _{cntl}	2.0 x B2 _{cntl}
6/13/2008	3.46	-2.41
10/25/2007	5.69	-4.30
9/19/2007	0.21	-0.97
Average	3.12	-2.56
<hr/>		

Percentage change in wind velocity at 100m as compared to control.

MYNN Scheme: Solving for turbulent fluxes $\overline{u_i u_j}$

$$\begin{aligned} \text{Time-tendency} \quad \frac{D\overline{u_i u_j}}{Dt} &= \text{Energy redistribution} \quad \overline{p \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)} \\ &+ \text{Dissipation} \quad \nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}} \\ &+ \text{Buoyancy} \quad \overline{g u_j \theta} \\ &+ \text{Diffusion} \quad \overline{u_i u_j u_k} \\ &+ \text{Shear production} \quad \overline{u_k u_i} \frac{\partial \overline{U_j}}{\partial x_k} \end{aligned}$$

Appx.

MYNN Scheme: Solving for turbulent fluxes $\overline{u_i u_j}$

Time-tendency

$$\frac{D\overline{u_i u_j}}{Dt}$$

=

Energy redistribution

$$\overline{p\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)}$$

Appx.

+

Dissipation

$$\nu \overline{\frac{\partial u_i}{\partial x_k} \frac{\partial u_j}{\partial x_k}}$$

+

Buoyancy

$$\overline{g u_j \theta}$$

+

Diffusion

$$\overline{u_i u_j u_k}$$

+

Shear production

$$\overline{u_k u_i} \frac{\partial \overline{U_j}}{\partial x_k}$$

Calculate Closure Constants

$$\overline{u^2} = \gamma_1 q^2 + 2A_1 C_2 \frac{L}{q} \frac{g}{\Theta_o} \overline{w\theta} - 6A_1 \frac{L}{q} \overline{uw} \frac{\partial U}{\partial z} \quad (1)$$

$$\overline{v^2} = \gamma_1 q^2 + 2A_1 C_2 \frac{L}{q} \frac{g}{\Theta_o} \overline{w\theta} \quad (2)$$

$$\overline{w^2} = \gamma_1 q^2 + 2A_1 (3 - 2C_2) \frac{L}{q} \frac{g}{\Theta_o} \overline{w\theta} \quad (3)$$

$$\overline{uw} = 3A_1 \frac{L}{q} \left[-(\overline{w^2} - C_1 q^2) \frac{\partial U}{\partial z} + (1 - C_2) \frac{g}{\Theta_o} \overline{u\theta} \right] \quad (1)$$

$$\overline{u\theta} = 3A_2 \frac{L}{q} \left[-\overline{uw} \frac{\partial \Theta}{\partial z} - (1 - C_5) \overline{w\theta} \frac{\partial U}{\partial z} \right] \quad (2)$$

$$\overline{w\theta} = 3A_2 \frac{L}{q} \left[-\overline{w^2} \frac{\partial \Theta}{\partial z} + (1 - C_3) \frac{g}{\theta_o} \overline{\theta^2} \right] \quad (3)$$

Summary and Future Work

- Summary
 - For LLJ cases in a SBL, the B_1 parameter varies by vertical extent and has a value about half than currently prescribed in the MYNN scheme
 - The B_2 parameter adheres closer to what is prescribed currently in MYNN except for near-neutral lapse rates
- Future Work
 - Consider larger set of LLJ test cases
 - Derive values for remaining closure constants (C_1 - C_3 , A_1 , A_2) as appropriate for LLJ cases

Acknowledgements

- Research funding through the NSF Interdisciplinary Graduate Education and Research Training (IGERT) Program
- Partial funding by State of Iowa EPSCoR grant.
- Meteorologische Institute, Universität Hamburg for 175m tower data
- Iowa Energy Center for 200m tower data
- Thanks to Dr. Gene Takle, Dr. William Gallus, Dr. McCalley, Dr. Arritt, Dr. Sharma

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