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Observations on Low-Speed Aeroelasticity

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Abstract: A brief history of developments in the field of low-speed aeroelasticity is provided in the context of application to long-span bridge structures. The paper begins with summary of some of the significant developments in aeroelasticity for low-speed aeronautical applications in the early 20th century. The role of the pivotal Tacoma Narrows failure and subsequent investigation is introduced. The development of formal experimental and analytical tools for the prediction of long-span bridge response to wind is presented, and their roots in—but differences from—classical low-speed aeroelasticity are presented and discussed (e.g., aerodynamic admittance). The important issue of Reynolds number scaling is discussed and posed as a problem that requires resolution in future research. Details of analytical and experimental techniques are not provided herein; readers are referred to the references for developments in these areas. The intent of the paper is to emphasize the parallels between these two strongly related fields, and in so doing, highlight the role of classical aeronautical engineering in the development of state-of-the-art bridge wind engineering.

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Introduction

The vortex-excited Aeolian harp of the ancient Greeks is one of the earliest historical examples of an aeroelastic or fluid-structure interactive phenomenon. Its characteristics were closely studied in the 1870s by Strouhal (1878). Among much later examples might be cited the striking cases of wind-induced flutter of the truss wires, wings, and tails of stick-and-wire aircraft of World War I vintage. Most of the early aircraft examples of aeroelastic effects were physically observed phenomena unaccompanied by theory, but by the 1920s attempts at theoretical descriptions of aircraft-related flight-induced oscillatory phenomena had advanced considerably with the work of Frazer (Frazer and Duncan 1928), Collar, Pugsley, and Glauert (Glauert 1928) in England and Birnbaum (1924), Wagner (1925), and others in Germany. Den Hartog's (1932) transmission-line galloping analysis is a representative nonaeronautical aeroelastic example. His simple steady-flow ''galloping criterion'' associated with a negative lift gradient has served to elucidate incipient galloping conditions in numerous practical cases over some seven decades.

By the mid- and latter 1930s firm analytical descriptions of the flutter of thin airfoils in incompressible flow had become available through the work of individuals like Küssner (1929) and Schwarz in Germany and Theodorsen (Theodorsen 1935; Theodorsen and Garrick 1941), Garrick (1939), von Karman Sears (1941) in the United States. From December 1941 through 1945 the United States was at war, and engineers, including the writer, applied their skills to aircraft design, analysis, and production. Technical problems in aeroelasticity were studied intensely during this period. Together with others in different locations in the United States the writer worked on problems of aircraft structural dynamics and flutter in 1942–1946. A body of structural dynamic theory and practice was thus accumulated in those technically productive years. Particularly with the great impetus given aircraft development during World War II, studies of aeroelastic effects progressed by substantial leaps. By now, such studies have ranged throughout all flight speed regimes, from incompressible through supersonic and beyond. The present account will focus on some of the low speed effects only.

In the 1950s three American textbooks on aeroelasticity appeared: Scanlan and Rosenbaum (1951) *Aircraft Vibration and Flutter*, Fung (1955) *The Theory of Aeroelasticity*, and Bisplinghoff et al. (1955) *Aeroelasticity*. These texts set forth state-of-the-art precepts in the field of aircraft aeroelasticity that subsequently played useful roles in practical aircraft design and parallel academic background studies. The discussion of bluff-body aeroelasticity offered at a later point in this paper would be lacking in perspective without at least mention of these important summaries of pioneering studies.

With the preceding remarks as a brief introduction, the present review will proceed from these historical concerns to examine a selected few of the low-speed aeroelastic phenomena associated with bluff structures—particularly bridges—in the wind. While throughout the 20th century aircraft design advanced by concentrating on high-speed flow around streamlined objects, in more recent decades the field of civil engineering has benefited from increasing attention to the many modest yet recondite concerns of slower flows around bluff bodies.

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Few nonaeronautical aeroelastic examples have received the broad publicity given the dramatic wind-induced collapse of the Tacoma Narrows Bridge that occurred on November 7, 1940. This took place in a cross wind of some 42 mi/h under the direct observation (and filmed documentation) of a few eyewitnesses. Chief among these eyewitnesses was Professor F. B. Farquharson of the University of Washington, who was preoccupied with that event over many months, both before and after the collapse. The Tacoma Narrows Bridge disaster is widely and correctly cited as the critical triggering episode of modern bridge aeroelasticity and may now be viewed as the key motivating point of departure for studies in that field.

The initial, most important, and most accurate review of the event occurred in a detailed report (Ammanh et al. 1941) addressed to the U.S. Federal Works Agency. In that report a key section outlined the experimentally demonstrated evolution of the aerodynamic damping of a torsionally oscillating section model of the Tacoma Narrows Bridge, examined in the wind tunnel by Professor Dunn of Caltech.

In spite of the clarity of the 1941 report, it did not succeed in putting the matter to rest in the public mind. Perhaps it was read by too few. In any event, a series of Tacoma Narrows tales, begun in 1941, has continued sporadically ever since—some 60 years to date. Many of these tales veered substantially—even inventively—wide of the known facts.

Billah and Scanlan (1991) undertook, in a paper in the American Journal of Physics to redress some of the varying accounts of the destructive Tacoma Narrows event. The deck of that bridge girder consisted of a squat, H-section profile with two outstanding characteristics: a shape strongly susceptible of tripping a leading edge vortex, and a relatively weak structural stiffness in torsion that enabled twisting oscillation. These characteristics combined in strong wind to engender flutter instability. Objecting to the oft-cited but over-simplistic ‘‘resonance’’ characterization of the associated bridge motion found in numerous textbooks, we described the Tacoma phenomenon instead as an interactive wind-induced, self-excited torsional oscillation. The bridge response just prior to collapse was *not* simply a case of the well-known Karman ‘‘vortex street’’ phenomenon.

During 1944–1954, Farquharson published a set of five comprehensive reports on the original and replacement Tacoma Narrows designs (Farquharson 1949). In the period of the late 1940s and early 1950s, Prof. F. Bleich of Columbia University offered a theoretical analysis of the Tacoma phenomenon based, unfortunately, on the Theodorsen airfoil theory (clearly misapplied in that context) (Bleich 1948). During the 1950s Vincent and Farquharson, on a more practical tack, created and tested dynamic wind tunnel section models of several bridges, including the original Tacoma Narrows and Golden Gate spans. Both of the associated section models demonstrated strong torsional instability in the wind tunnel. To the present day, the Golden Gate span evidences only marginally acceptable stability under cross wind, this in spite of the addition to the deck girder of a lower-chord horizontal ‘‘wind truss’’ installed around 1955. In the early 1990s Raggett and Scanlan made wind tunnel models and performed theoretical studies that demonstrated the possibility of substantially increasing the flutter speed of the Golden Gate Bridge by adding certain deck and railing fairings.

In the late 1960s and early 1970s me and my students Sabzevari and Tomko (Scanlan and Sabzevari 1967, 1969; Scanlan and Tomko 1971) established early analytic models to parallel the action of several of the physical wind tunnel models created and studied by Vincent (1952) and Farquharson (1949) in the United States and Scruton (1952) in England. Our analytical modeling followed the natural course of first imitating the *style* of the theoretical Theodorsen airfoil model, but we soon recognized that the typical *bluff body* represented by most bridge deck sections then extant could not be expected to establish detailed flow characteristics and structural responses simulating those involving an airfoil. In fact, the net oscillatory forces engendered around the typical bluff body needed to be viewed characteristically as the net of a complex of *experimentally* determined physical forces—often associated with separated flow, rather than being closed-form smooth-flow analytical descriptions, like those portrayed in Theodorsen's theory. This situation led to the eventual experimental identification of flutter derivatives. To date, such motion-linked aerodynamic derivatives have been extensively identified, and once identified, they have been successfully applied to bridge deck model studies in many venues internationally. Their use has been efficient and valuable in numerous design studies.

Retracing the evolutionary steps involving the development of flutter derivatives, it was first observed that in spite of complex wind-structure interactions, the net motion of most bluff bridge sections, possessed of substantial inertia, displayed almost pure sinusoidal or damped sinusoidal response. This characteristic led naturally to simple means for identification of the associated oscillating wind force coefficients. Since the introduction of the free-oscillation methods of Scanlan and Tomko (1971) for bridge section models, many techniques have been exploited over the years demonstrating different alternate experimental methods for obtaining flutter derivatives (e.g., Sarkar et al. 1994).

Following the early style of airfoil aeroelastic theory, linear, first order, sectional lift, and moment effects were initially represented in the form

$$L=\frac{1}{2}\rho U^2 B(a_1\dot{h}+a_2\dot{\alpha}+a_3\alpha)$$

$$M=\frac{1}{2}\rho U^2 B^2(b_1\dot{h}+b_2\dot{\alpha}+b_3\alpha)$$

where ρ = air density; U = cross-wind velocity; B = model width, along wind; a_i, b_i = appropriate constants; and h,α = vertical and torsion coordinates, respectively.

Scanlan and Tomko (1971) reorganized these expressions into a format that incorporated the dimensionless flutter derivatives. Arrays of similar coefficients have, by the present time, appeared in several other contexts. To date at least two dozen doctoral theses from various places around the world have been written on the subject of bridge deck aeroelasticity, featuring flutter derivatives. At a somewhat more recent stage of development, in which three freedoms—h (lift), α (twist), and p (drag)—were included, a full set of 18 dimensionless flutter derivatives—H_i^*, A_i^*, P_i^* ($i=1,2,\ldots,6$)—were developed and given the following form (Singh et al. 1994):

$$L_{ae}=\frac{1}{2}\rho U^2 B\left[KH_1^*\left(\frac{\dot{h}}{U}\right)+KH_2^*B\left(\frac{\dot{\alpha}}{U}\right)+K^2H_3^*\alpha+K^2H_4^*\frac{h}{B}\right.$$
$$\left.+KH_5^*\left(\frac{\dot{p}}{U}\right)+K^2H_6^*\left(\frac{p}{B}\right)\right]$$

$$D_{ae}=\frac{1}{2}\rho U^2 B\left[KP_1^*\left(\frac{\dot{p}}{U}\right)+KP_2^*B\left(\frac{\dot{\alpha}}{U}\right)+K^2P_3^*\alpha+K^2P_4^*\frac{p}{B}\right.$$
$$\left.+KP_5^*\left(\frac{\dot{h}}{U}\right)+K^2P_6^*\left(\frac{h}{B}\right)\right]$$

$$M_{ae}=\frac{1}{2}\rho U^2B^2\left[KA_1^*\left(\frac{\dot{h}}{U}\right)+KA_2^*B\left(\frac{\dot{\alpha}}{U}\right)+K^2A_3^*\alpha+K^2A_4^*\frac{h}{B}+KA_5^*\left(\frac{\dot{p}}{U}\right)+K^2A_6^*\left(\frac{p}{B}\right)\right]$$

where H_i^*, A_i^*, P_i^* = dimensionless flutter derivatives, functions of $K=B\omega/U$, and where ω = circular flutter frequency. In this formulation, the sway or drag degree of freedom (p), not generally present for airfoils, but important for some flexible bridge spans, has been included. This effect was demonstrated to be important, for example, in the case of the current world's longest suspension span, the Akashi-Kaikyo, in Japan. In the present paper it will be sufficient to focus only on lift and moment terms involving h and α.

It has been observed that the type of experimental flutter derivatives obtained by Scanlan and Tomko (1971) have repeatedly permitted calculations that support full-bridge dynamic predictions. The paradigm for such implementations has been a close parallelism with the style and format of the Theodorsen thin-airfoil flutter theory. In this theory the concept of linear superposition of small effects, such as airfoil angle of attack, is justified both analytically and physically. In the context of bridge decks, the equivalent step of analysis must be considered an approximation only, as it is throughout the present discussion. If in this process anomalous effects appear they may be considered subjects for further experimental investigation. This paper reviews some elements of thin airfoil theory and their reinterpretation in the form of linearized bluff-body theory.

For typical full-bridge flutter analysis, acquired sectional flutter derivative forces are integrated spanwise over a large number (typically dozens) of natural structural modes. Solution of the corresponding eigenproblem then yields the wind speed of flutter. In this, the procedures, though presently substantially enhanced by computer aid, follow theoretical methods already outlined in the 1940s and 1950s or before. Details of the flutter eigenvalue problem have, as always in either bridge or aircraft cases, been computationally demanding.

The employment of aeroelastic formulations based on flutter derivatives follows the original style of aircraft flutter, in which flutter analyses written in the time domain are based on flutter derivatives expressed in the frequency domain. This "hybrid" formulation served well for the flutter case alone but did not provide for arbitrary structural motions described wholly in the time domain, such as occur under wind buffeting action.

The Wagner (1925) lift-growth or "indicial" function $\varphi(s)$ associated with a theoretical step change in airfoil angle of attack α provided a basis for time-domain lift force development under arbitrary time-dependent (small) angle of attack change α as follows:

$$L(s)=\frac{1}{2}\rho U^2BC_L'\left\{\varphi(0)\alpha(s)+\int_0^\infty\varphi'(\sigma)\alpha(s-\sigma)d\sigma\right\}$$

where

$$s=\frac{2Ut}{B}$$

$$\varphi'(s)=\frac{d\varphi}{ds}$$

$$C_L'=\frac{dC_L}{d\alpha}$$

Garrick (1939) demonstrated the following Fourier transform identity between the classic Theodorsen circulation function $C(k)$ and Wagner function $\varphi(s)$:

$$C(k)=\varphi(0)+\overline{\varphi'(s)}$$

where

$$\overline{\varphi'(s)}=\int_0^\infty\varphi'(s)e^{-iks}ds$$

This link permits writing the Theodorsen airfoil flutter theory alternately in either the time or frequency domains. A single lift-growth function $\varphi(s)$ can be shown to be sufficient to represent lift or moment expressions in the Theodorsen theory. This single function must later be generalized to several functions in bluff-body theory.

Jones (1940) offered the following excellent approximation for the Wagner lift-growth function:

$$\varphi(s)=1-ae^{-bs}-ce^{-ds}$$

with

$$\varphi'(s)=abe^{-bs}+cde^{-ds}$$

where $a=0.165$, $b=0.0455$, $c=0.335$, and $d=0.300$. A wide variety of choices for the constants a, b, c, and d is clearly available to fit other indicial functions. The Theodorsen function

$$C(k)=F(k)+iG(k)$$

can be approximated by

$$F(k)=1-a-c+\frac{ab^2}{b^2+k^2}+\frac{cd^2}{d^2+k^2}$$

$$G(k)=-k\left[\frac{ab}{b^2+k^2}+\frac{cd}{d^2+k^2}\right]$$

Flutter lift and moment components are each associated with two forms of effective wind vertical angle of attack, i.e. α and $\dot{h}/U$, so that, at flutter in complex notation, the following four force components may be written.

Lift:

$$L_h=\frac{1}{2}\rho U^2BK[H_1^*-iH_4^*]\frac{\dot{h}}{U}$$

$$L_\alpha=\frac{1}{2}\rho U^2BK^2[iH_2^*+H_3^*]\alpha$$

Moment:

$$M_h=\frac{1}{2}\rho U^2B^2K[A_1^*-iA_4^*]\frac{\dot{h}}{U}$$

$$M_\alpha=\frac{1}{2}\rho U^2B^2K^2[iA_2^*+A_3^*]\alpha$$

Each of these four functions may be associated with a separate "circulation" function $F(\,)+iG(\,)$. Assuming (via implicit linearization) that the flutter derivative and associated notation $H_i^*(k)$ and $A_i^*(k)$ hold for either airfoil or bridge deck sections, the following set of equivalences can be demonstrated:

$$K[H_1^*-iH_4^*]=C_L'[F_{Lh}+iG_{Lh}]=C_L'[\varphi_h(0)+\overline{\varphi_h'}]$$

$$K^2[iH_2^*+H_3^*]=C_L'[F_{L\alpha}+iG_{L\alpha}]=C_L'[\varphi_\alpha(0)+\overline{\varphi_\alpha'}]$$

$$K[A_1^* - iA_4^*] = C_M'[F_{Mh} + iG_{Mh}] = C_M'[\psi_h(0) + \overline{\psi_h'}]$$

$$K^2[iA_2^* + A_3^*] = C_M'[F_{M\alpha} + iG_{M\alpha}] = C_M'[\psi_\alpha(0) + \overline{\psi_\alpha'}]$$

where the following definitions and observations are in order: C_L', C_M' = slope of associated lift or moment curve; φ_h, φ_α = indicial lift functions; ψ_h, ψ_α = indicial moment functions; $F(\,), G(\,)$ = associated circulation functions.

This array of theoretical results groups together both frequency- and time-domain aspects of the linearized flutter problem, either for thin airfoil or bluff-body (bridge section).

A further observation is that *aerodynamic admittances* (χ) can be derived from these relationships. In this context four such functions will be available. For example (Scanlan and Jones 1999)

$$|\chi|^2 = K^2[H_1^{*2} + H_4^{*2}]/C_L'^2$$

which emphasizes a typical form of the link between admittances and the flutter derivatives. Such forms occur in expressions for structural buffeting by wind components u, w. For example, lift can be written

$$L = \frac{1}{2}\rho U^2 B\left[2C_L\chi_u\frac{u}{U} + C_L'\chi_w\frac{w}{U}\right]$$

for specific complex admittances χ_u, χ_w.

For quasisteady (static) lift, admittances are equal to unity. It is worth commenting that in the literature over a number of years, bridge deck bluff-body nonairfoil admittance was often incorrectly identified with the Sears airfoil admittance. The present review should correct this long-standing misinterpretation.

Summary

In the analytic theory of thin airfoil flutter, terms involving small angles of attack, such as α or $\dot{h}/U$, are linearly additive both theoretically and experimentally. This situation does not carry over in the strictest sense to bluff bodies like bridge deck sections around which flow effects are generally nonlinear. In practice, however, if the relevant structural displacements remain small, as with low-amplitude vibrations, most studies involving flutter derivatives yield reasonably accurate results for depicting overall system dynamics. Katsuchi et al. (1999) and Miyata and Yamada (1990) separately demonstrated response predictions well within 10% for a 1:100 scale model of the full Akashi-Kaikyo span.

Bridge structures with inertia large compared to the air, and with linear elastic structural characteristics, tend to respond like simple linear oscillators close to their natural frequencies, or in alternate formulation, the structural model under study may expressly be driven in prescribed sinusoidal cycles. Exploitation of like circumstances permits identification of flutter derivatives from net integrated aerodynamic force components. A particular note should be added. Some bluff bodies with box-like sections evidence flow regimes in which self-organized, periodic vortex shedding, and associated periodic body motion occur. In such situations, special care is required to interpret flutter derivatives appropriately (Scanlan 1998).

A continuing source of uncertainty in experimental bridge deck aeroelasticity is the low value of Reynolds number inherent in typical small-scale bridge models tested at low speed in atmospheric wind tunnels. Analogous circumstances also hold for building models in boundary layer wind tunnels. In such circumstances Reynolds number can be low by several orders of magnitude. This avenue of wind engineering research has—for obvious practical reasons—rarely been explored in the civil engineering context because of the sparse and inadequate means available to exploit it. The usual argument adduced with objects having ''sharp'' edges is that these permit definitive flow separations and thus ostensibly a near-Reynolds number equivalence of scaled force characteristics between small- and full-scale structural forms. This mostly unverified assumption merits thorough research or at least the establishment of appropriate means to *interpret* ostensibly analogous aerodynamic force equivalence when actual physical similarity cannot be achieved. Models of extremely small scale present a veritable kaleidoscope of flow/pressure situations (separation, reattachment, etc.) that differ from their prototype equivalents.

In low-speed atmospheric wind tunnels, proper duplication of full-scale Reynolds number effects is a practical impossibility, as is the proper realization of equivalent full-scale turbulence. Overall, these shortcomings may be even more egregious with aeroelastic phenomena. These questions—commonly dismissed or summarily treated at present—remain open for serious future resolution.

Farquharson (1949) listed a fair number of bridges that failed under wind—some as much as a century before the Tacoma Narrows episode. The Tacoma Narrows warning came at a juncture at which danger signs began to be heeded and responded to in technical depth. While numerous bridges have continued to exhibit disturbances under wind, no comparable catastrophes have since been reported. This reflects the fact that since 1940 bridge aeroelasticity, of both new and existing designs, has been seriously developed into an effective engineering art.

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