

# Reliability Meets Big Data: Opportunities and Challenges

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## Abstract

Reliability field data such as that obtained from warranty claims and maintenance records have been used traditionally for such purposes as generating predictions for warranty costs and optimizing the cost of system operation and maintenance. In the current (and future) generation of many products, the nature of field reliability data is changing dramatically. In particular, products can be outfitted with sensors that can be used to capture information about how and when and under what environmental and operating conditions products are being used. Today some of that information is being used to monitor system health and interest is building to develop prognostic information systems. There are, however, many other potential applications for using such data. In this paper we review some applications where field reliability data are used and explore some of the opportunities to use modern reliability data to provide stronger statistical methods to operate and predict the performance of systems in the field. We also provide some examples of recent technical developments designed to be used in such applications and outline remaining challenges.

**Key Words:** Condition-based maintenance, Dynamic covariates, Early warning, Materials state awareness, Prognostics, System health management, Warranty prediction

# 1 The Next Generation of Reliability Data

## 1.1 Background

Due to changes in technology, the next generation of reliability field data will be much richer in information. Today it is possible to install sensors and smart chips in a product to measure variables such as use rate, system load, and various environmental variables. In addition to the time series use rate/environmental data, we also can expect to see further developments in sensors that will provide information about component or system physical, chemical, or performance degradation or other indicators of imminent failure. We will refer to such data as System Operating/Environmental (or SOE) data. Additionally, many of these products/systems contain communications capabilities (e.g., an IP address and direct Internet or wireless connections to the Internet) that allow data to be uploaded automatically or on demand. For many products all SOE information is kept in a data center. If there is no communications capability within a product, some SOE information is stored on board and can be retrieved during maintenance operations.

## 1.2 Examples of systems providing big data

Some examples of particular systems and products providing SOE data include the following.

### Locomotive engines

Modern locomotive engines contain sensors that measure operational variables such as oil pressure, oil temperature and engine coolant temperature over time. This information is used to control an engine during normal operation. These time series data can also be used to warn of operating conditions that could cause serious damage to the system (e.g., loss of oil pressure). Section 9.9 of Hahn and Doganaksoy (2008) describe an application in which an algorithm was developed to a particular engine subsystem that had failed, based on signatures in the multivariate time series data. In addition to the engine-status variables, other operational information such as ambient temperature, air pressure, and GPS location can also be captured and downloaded.

### Aircraft engines

Aircraft engines, similar to locomotive engines, have sensors to obtain information (such as temperature in different parts of the engine) that is used in the normal operation of the system. Other sensors provide information about such variables as oil temperature, debris in oil, and vibration that could be used to indicate unsafe engineering operating conditions. For a detailed description of aircraft engine health monitoring systems, see Volponi and Wood (2011).

## **Automobiles**

Today's automobiles can be outfitted with sophisticated data acquisition and communications capabilities. For example GM's OnStar system is marketed for its safety and operational features, but the system is also capable of collecting and uploading important operational and environmental variables similar to those described above for locomotives.

## **Power distribution transformers**

A catastrophic failure of a power distribution transformer can cause serious damage and lengthy power outages for a large number of electricity consumers. Traditionally such transformers would be inspected periodically (e.g., once every six months) to provide some degree of assurance and protection. One important diagnostic check is an assessment of the chemical compounds in the transformer's cooling oil. These tests are known as dissolved gas analysis or DGA tests. Different amounts of combinations of gases in the oil provide a signature of possible transformer faults (e.g., arcing that could lead to catastrophic failure). Spurgeon et al. (2005) provide information on such analyses. Now, however, it is possible to have transformers that are not only outfitted with sensors to provide environmental and operation information in real time, but also with a device that will do periodic DGA (e.g., once each day or even every hour).

## **CT scanners and other large medical systems**

In addition to various operational parameters that indicate the current state of the system and suitability for continued use, large medical systems also record variables like the number of uses and the amount of power at each use. This covariate information could be utilized in conjunction with a lifetime model to predict the remaining useful life of critical components in the system. Such information can be used to detect system problems, optimize maintenance schedules, and minimize unplanned maintenance actions. The information could also be used to accurately predict longer-term life characteristics of life-limiting components.

## **Wind turbines**

The amount of electricity generated by wind has grown at approximately 30% per year in the past ten years. Giant wind farms have grown up across the US and in many other countries around the world. Texas, California, and Iowa lead the US in wind energy production. High reliability of the wind-turbine system components is extremely important because of the high cost of making repairs. Wind turbine systems generally contain numerous sensors (or arrays of sensors) in both the supporting structure (especially at the top), on the blades, and inside the turbine nacelle itself, to provide

information on such physical characteristics as stresses affecting critical components including relative movement in the system over time, the sway of the structure (due to force of winds), and vibration in the turbine's moving parts. As with other systems outfitted with such sensors, data can be sent to a centralized location or locations (e.g., the owner of the systems and the manufacturer/maintainer of the systems). Ciang, Lee, and Bang (2008), Antoniadou et al. (2012), and Faulkner, Cutter, and Owens (2012) describe the use of such sensors for system health monitoring.

### **Solar energy power inverters**

Power inverters are electronic devices that are used, for example, to convert direct current voltage from solar cell arrays to alternating current that can be attached directly to electrical distribution systems connected to the power grid. In times of high power usage, less power has to be purchased from the grid supply. In times of low power usage (e.g., weekends), power can be sold back to the grid. Advanced power inverter models provide various data streams that can be used to monitor power output over time as well as other operational and environmental variables.

### **Other systems and products providing SOE data**

Other systems collecting similar operational and environmental information include gas turbines, farm implements and large construction equipment, high-end printers/copiers, high-end computers, high-end batteries (e.g., those used in some uninterruptible power supplies), some home entertainment systems, and even smart phones.

## **1.3 Distinguishing feature of modern field reliability (SOE) data**

The common feature among these systems is that they can deliver periodically a potentially large vector of dynamic covariate values (i.e., a vector time series). As we describe and illustrate in the rest of this paper and some related technical papers, there is great potential for using this additional information to achieve higher reliability and availability of systems at lower cost.

## **1.4 Overview**

The rest of this paper is organized as follows. Section 2 describes and contrasts traditional field reliability data with the modern field reliability provided by many of today's systems and products. Section 3 briefly describes current uses of modern reliability (or SOE) data and the many opportunities for further cost-effective uses of such data. Section 4 reviews different kinds of response variables that determine or define system health. Section 5 outlines

the general goals and approaches that we suggest for taking advantage of SOE data. In Sections 6, 7, and 8 we provide a summary of the particular models and statistical methods that we have implemented in three different case studies involving the use of SOE data to describe failure processes and to predict future failures across an entire fleet of systems and for individual systems within a larger fleet. Section 9 outlines some practical concerns and challenges and Section 10 makes some concluding remarks and outlines some areas for further research.

## 2 Warranty, Maintenance, and Field-tracking Data

### 2.1 Traditional field data and its limitations

Although laboratory reliability testing is often used to make product design decisions, the “real” reliability data come from the field, often in the form of warranty returns or, specially-designed field-tracking studies.

Warranty data are the most common type of field data. Warranty databases were initially created for financial-reporting purposes, but more and more companies are finding that their warranty database is a rich source of reliability information. Perhaps six to eight months after a product has been introduced into the market (sooner if warranty costs have already been higher than expected), managers begin to ask about warranty costs over the life-cycle of the product. Two common problems with warranty data are that good failure mode information is often not available (there is usually some kind of code in the database, but it is usually of limited use to determine the actual cause of failure) and warranty data are generally heavily censored, because information obtained after a product is out of warranty is limited. Thus, even though companies should be concerned about reliability of their products far beyond the end of a warranty period, operationally, little data is available.

Maintenance data provide the second major source of field reliability data. Companies and organizations keep detailed records of costs of maintenance for their fleets of assets (e.g., information about the reliability for a fleet of automobiles or locomotives or transformers). Like warranty data, maintenance data may lack important engineering information because the reporting rules and databases were designed for financial reporting rather than for answering engineering questions. For example, if a valve seat is replaced in a locomotive that has 16 valve seats, the data may not record which valve seat was replaced. This makes it difficult to infer whether valve seats in the same position are being replaced repeatedly or not.

For some products, careful field tracking provides good reliability data. For example, manufacturers are required to keep accurate records about installation and causes of failures

of medical devices such as pacemakers and defibrillators.

## **2.2 Modern field data and its complexity**

In contrast to traditional field data where typically the only information on a unit would be failure times (for units that had failures) and perhaps some information about date of manufacturing and cost of failure, modern systems will often be providing huge amounts of data on current system operating parameters, environmental parameters, and indicators of system health. Generally these data come in the form of a vector of data, sampled periodically (e.g., every ten minutes) or on demand. In some cases there is a default vector of information, but the system can be instructed (perhaps less frequently or when troubleshooting) to provide information on additional variables that might be of interest.

To solve a problem using data that are available, it is often necessary to integrate data from “data islands” from different sources both within and external to a company or organization. For example to predict warranty costs one would need information from the warranty-returns database, the production database, engineering design information (e.g. about changes in part numbers over time) and perhaps customer data on product utilization.

## **3 Applications and Potential Applications of Big Reliability Data to System Reliability Prediction and Maintenance**

The most important applications for owners and operators of systems and fleets of systems would be to prevent in-service failures, unplanned maintenance, and especially system failures that could cause serious loss of property or lives. There are also needs to plan for system retirement and predictions of future financial needs and obligations (e.g., warranty costs). This section describes some of the particular applications of field data and explains how and why modern SOE data will be able to do a much better job.

### **3.1 Systems health management, condition-based maintenance, prognostics and short-term prediction of system failure**

Today, the most common use of SOE data is in system-health monitoring (SHM) (also known as systems health management). There is an enormous amount of literature in this particular area including several journals and annual conferences devoted to the subject, focusing on

the use of sensor technology and strategies for using the sensor data to detect unusual and undesirable system states. For example, process monitoring and signal-detection algorithms can detect unsafe operating conditions or precursors to system failure that can be used to protect a system by shutting it down or by reducing load to safe levels. In some applications (e.g., aircraft engines, wind turbines, and power distribution transformers), system health/use rate/environmental data from a fleet of systems in the field can be returned in real time to a central location for real-time process monitoring and especially for prognostic purposes. An appropriate signal in these data might provoke rapid action to avoid a serious system failure (e.g., by reducing the load on an unhealthy transformer or running a locomotive at a lower load until a repair can be made). Also, should some issues relating to system health arise at a later date, it would be possible to sort through historical data that have been collected to see if there might have been a detectable signal that could be used in the future to provide an early warning of the problem.

With additional modeling capabilities, SOE data can also be used for prognostic purposes to provide short-term predictions about the remaining life of a system. For example vibration sensors can indicate the beginning of abnormal wear and thus a change in degradation rate. A prediction of remaining life of the wearing component would be needed to schedule timely maintenance and protect the overall system from a costly in-service failure.

Relatedly, system owners/maintainers can use SOE data to plan maintenance actions based on need instead of less efficient time-based schedules. Such maintenance programs are known as condition-based maintenance (CBM). When properly implemented, CBM can lead to both higher reliability and lower costs. In some applications, however, relying on CBM will require that certain redundancies or fault-tolerances be build into a system.

Carden and Fanning (2004), and Jarrell, Sisk, and Bond (2003) describe SHM methods and prognostics for mechanical systems. Pecht (2008) gives a detailed overview of such methods for electronic systems.

### 3.2 Early warning of emerging reliability issues

There are sometimes large gaps between predictions made from product-design reliability prediction models (supplemented by limited reliability testing) and reality. These differences are often caused by unanticipated failure modes. Algorithms for early detection of emerging reliability issues (e.g., Wu and Meeker 2002) are being implemented in software and have the potential to save companies large amounts of money. More recent work in this area includes Yashchin (2010) and Lawless, Crowder, and Lee (2012). The availability of SOE data on individual units in the field has the potential to tremendously strengthen the ability to discover and even diagnose the cause(s). For example, knowing that several early failures

were due to overuse of products in a harsh environment could prevent what might otherwise have been a false alarm that there was a serious emerging reliability problem. Conversely, if failures were known to have come from systems operating in an ordinary environment, such early failures could provide a strong warning of an emerging issue.

### 3.3 Prediction of remaining life of individual systems

Even when a system is in a state of normal operation, there is often a need or desire to predict the remaining longer-term life of the system (or the remaining life of its most important life-limiting components). Technically, the distribution of remaining life of a system is defined as the conditional probability of failing at a future time, given survival until the present time  $t_c$ . If  $F(t)$  is the cumulative distribution function (cdf) of the failure time for a new unit, the cdf of *remaining* life for a unit that has survived until time  $t_c$  is,

$$G(t) = \frac{F(t_c + t) - F(t_c)}{1 - F(t_c)}, \quad t > 0.$$

Estimates of the  $\alpha/2$  and  $1-\alpha/2$  quantiles of  $G(t)$  would provide an approximate prediction interval for remaining life. Better approximate prediction intervals can be obtained by using Monte Carlo simulation to calibrate (i.e., to adjust the approximate prediction procedure so that it has a coverage probability that is closer to the desired nominal value). In a study of distribution transformer life (Hong, Meeker, and McCalley 2009), the only differentiation is with respect to system age, and weak covariates (i.e., covariates that do not explain a large proportion of the lifetime variability) like manufacturer and date of manufacturing. As shown in Hong, Meeker, and McCalley (2009), when there is not strong covariate information to differentiate among different systems in the fleet that are at risk to fail, then generally prediction intervals for individual units will be extremely wide, but still potentially useful for some purposes. If SOE data providing information on such variables as system load, temperature, and shock histories had been available (as it will in the future), then more precise prediction intervals would have been possible.

### 3.4 Prediction of retirements/replacements in a fleet of systems

Building on earlier work in Chapter 12 of Meeker and Escobar (1998) and Escobar and Meeker (1999), Hong, Meeker, and McCalley (2009) develop more general methods for using traditional field reliability data to generate predictions on the retirements or replacements of the members of a fleet of systems. They illustrate the methods on a fleet of distribution power transformers, predicting the number failing in each year out to a horizon of twenty years. They also show how to obtain statistically correct prediction intervals even when dealing with



messy field data [i.e., data with a combination of censoring and truncation (see page 41 of Meeker and Escobar 1998) ]. With the prediction of the remaining life of individual systems, covariate information in SOE data would provide both improved precision and accuracy for population predictions, although the improvements will be less dramatic than predictions for individual systems.

### 3.5 Prediction of warranty returns

Companies are required by law to maintain cash reserves to pay expected warranty costs for their products. Doing so requires the use of warranty cost prediction methods. There are financial implications for inaccurate predictions. Many companies find it satisfactory to use either basic time series prediction methods or to simply assume that warranty costs will be a certain percentage of sales, based on previous experience. Especially, however, when a new emerging reliability issue has been identified, statistical methods based on field-failure data (e.g., Escobar and Meeker 1999, Lawless and Fredette 2005, Hong and Meeker 2010) can be used to produce predictions for the additional warranty costs. Field data also provide important feedback that can be used to assess the adequacy of reliability prediction methods and to give engineers information on how to design future products. Traditionally, most warranty predictions have been based on modeling failures relative to time in service. Hong and Meeker (2010), however, had access, for a large fraction of the product population, to amount-of-use data for individual units that had failed under warranty and those that had not. They demonstrated the important advantages of basing warranty predictions on amount of use in applications (like theirs) where the failure mechanisms are driven by the amount of use.

Use rates and environmental conditions are important sources of variability affecting product lifetimes. The most important differences between carefully controlled laboratory accelerated test experiments and field reliability results are due to uncontrolled field variation (unit-to-unit and temporal) in variables like use rate, load, vibration, temperature, humidity, UV intensity, and UV spectrum. Historically, use rate/environmental data have, in most applications, not been available to reliability analysts. Incorporating use rate/environmental data into statistical analyses will provide more accurate predictions.

### 3.6 Prediction of maintenance costs

Manufacturers in some industries offer to sell to their customers maintenance agreements to provide maintenance that is necessary to keep the products operating. Indeed, in some markets (e.g., aircraft engines) manufacturers make more profit on their maintenance agree-

ments than on the product itself. The ability to accurately predict maintenance costs is essential to assure profitability of such agreements. Companies that have a combination of good product reliability and that can accurately predict maintenance costs will have an important competitive advantage.

## 4 Different Responses in Reliability Field Data

### 4.1 Failure time field reliability data

The vast majority of field reliability datasets have the response as the failure time of units that failed and the running times of units that have not failed. Ideally, time would be defined using a scale appropriate to the failure mechanism(s), but operationally, in most applications, all that is available is time in service or time since manufacture with some kind of adjustment for the gap between manufacturing and installation. In a rough sense, the amount of information in the data is proportional to the number of failures.

### 4.2 Degradation field reliability data

In modern high-reliability applications, often we see few or no hard failures (where a product suddenly stops operating) in reliability testing. Suppose that 100 units had been put on test and run for 2000 hours of operation for a product that is expected to last through 20,000 hours. If there are no failures at the end of the test, there would be little or no information for quantifying reliability (depending on assumptions that one might be willing to make). If, however, we could monitor, over time, a degradation (or a performance) variable that is closely related to failure (e.g., length of a fatigue crack or light output from a laser) on all of the units, there would be a large amount of reliability information, especially if physics-of-failure knowledge provides additional information about the degradation mechanism (e.g., to allow a degradation path to be extrapolated in time). Also, in some reliability applications hard failures are rare. In such applications, the product's performance degrades over time and degradation is the natural response. Examples include the gloss and color of automobile coatings and the light output of solid-state lighting (i.e., LEDs).

Over the past 30 years, the use of degradation data in reliability testing has received much attention. It is also possible, in some applications, to use degradation as the response for units in the field. There are a number of other advantages for using such *repeated-measures degradation* data for reliability assessment. In addition to providing additional information about reliability and the opportunity to make reliability inferences with few or even no failures, degradation data provide information that is much richer for building and assessing

the adequacy of physical/chemical models used for test acceleration.

Some engineers (e.g., Murray 1993) had been using informal “simple” methods of analysis that fit models to the sample path for individual units and extrapolated these until some failure level, providing “pseudo failure” data that could be analyzed by common life data analysis methods. A more appropriate method, used for example in Lu and Meeker (1993) and Meeker, Escobar, and Lu (1998), uses a random effects model to describe unit-to-unit variability. Such a model along with a soft-failure definition, will induce a failure time distribution that can be estimated directly from the degradation data. In some areas of application (and more commonly in field data), it is necessary to model the stochastic behavior in the sample paths over time. Lawless and Crowder (2004), for example, use such a model.

If an appropriate degradation variable can be measured, degradation data, when properly analyzed, can provide much more information because there are quantitative measurements on all units (not just those that failed). Indeed, it is possible to make powerful reliability inferences from degradation data even when there are no failures. It is, of course, not always possible to find a degradation variable that corresponds to a failure mode of concern.

### 4.3 Recurrence field reliability data

In some reliability applications involving repairable systems, the available maintenance data are recurrence data, providing information on a sequence of events on individual systems from a fleet of systems. Nelson (2003) describes many examples and provides statistical methods for such data when covariate information is not available. The presentation in Cook and Lawless (2007) is at a higher technical level and does include models for covariates and time-varying (i.e., dynamic) covariates, along with many examples from biological and medical statistics.

## 5 Extending Existing Models for Reliability to Take Advantage of System Operating/Environmental

Traditional reliability field datasets are almost always of manageable size. Even if the number of product units is large, the number of failures is typically relatively small, and both the failure times and the censoring time, when there are no covariates, can be binned into intervals as long as the interval lengths are small with respect to the overall spread in the data.

The biggest challenge that we face in using SOE data is to develop appropriate models

that will effectively use the information in SOE data for the various applications described in Section 3. Generally this will involve two modeling efforts. First, there is need to build a regression like model that will relate the response to dynamic covariates. Second, when predictions or other inferences about the future are desired, it will be necessary to develop a model for the dynamic covariates themselves. In the following three sections we briefly outline such efforts, motivated by particular examples for lifetime data, degradation data, and recurrence data, respectively.

## 6 Example 1: Modeling and Field Failure Prediction Using Failure Time Data with Dynamic Covariate Information

The work in Hong and Meeker (2013) was motivated by an application involved with a product called Product D2, a product with electronic and mechanical components that is used in homes and offices. For this product, dynamic information is available on the product use rate for those units that are connected to a network. The dataset contains  $n = 1,800$  units with 69 failures and the maximum length of the observation period is 70 weeks. Figure 1 shows plots of the failure-time data and the corresponding use-rate trajectories (i.e., number of uses per week as a function of system age). These plots are similar to those in Figure 1 of Hong and Meeker (2013) but for a different subset of the data. The major goal of Hong and Meeker (2013) is to provide a general method that utilizes the failure-time data and dynamic covariate information to generate better field-failure predictions. This goal was realized, as described in Hong and Meeker (2013).

To briefly describe the models and methods, some notation is needed. Let  $T$  be the time to failure random variable and let  $\mathbf{X}(t) = \{X(s) : 0 \leq s \leq t\}$  be the covariate process history. The corresponding data are denoted by  $\{t_i, \delta_i, \mathbf{x}_i(t_i)\}$  where  $t_i$  is the failure time and the event indicator  $\delta_i = 1$  if the unit failed and 0 otherwise. The observed individual covariate history is  $\mathbf{x}_i(t_i) = \{x_i(s) : 0 \leq s \leq t_i\}$ .

Models for time-to-event data with dynamic covariate information and for covariate process are needed. The cumulative exposure (CE) model was used to describe the effect of a dynamic covariate on the failure-time distribution. For the CE model, each unit accumulates an unobservable amount of CE  $u(t) = \int_0^t \exp[\beta x(s)] ds$ , which depends on the random covariate path  $\mathbf{X}(\infty) = \mathbf{x}(\infty)$ . Here,  $\beta$  is an acceleration coefficient. When the amount of CE reaches a random threshold  $U$  at time  $T$ , the unit fails and  $T$  is the failure time for the unit. The CE threshold  $U$  has the baseline cdf  $F_0(u)$ . The Weibull distribution was used

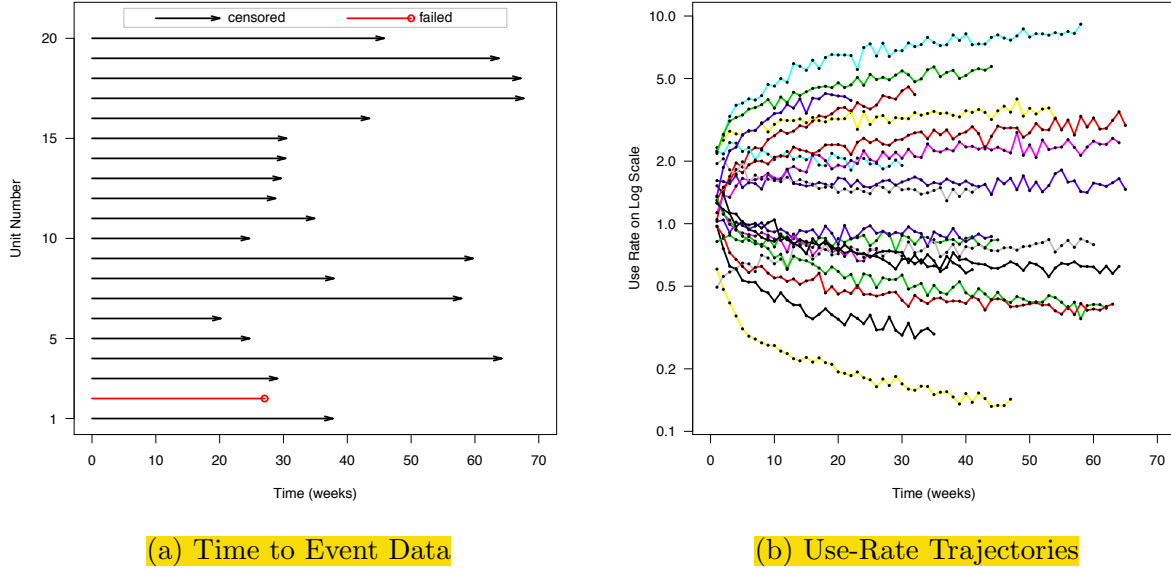


Figure 1: Plots of the failure-time data and the corresponding use-rate trajectories for a subset of the data.

for baseline cdf in the Product D2 application. The maximum likelihood (ML) approach is used to estimate the unknown model parameters. The ML estimates imply that the effective amount of extrapolation for predicting future failures is much less with the dynamic use-rate model. A linear mixed effects model is used to describe the covariate process  $X(t)$ , which is needed for the purpose of prediction. The random effect in the mixed model can account for unit to unit and temporal variability in units' covariate processes. The parameters in the mixed model are also estimated by ML approach.

The parametric cumulative exposure model used in Hong and Meeker (2013) is the same as the model that is commonly used in step-stress accelerated testing, described in Nelson (1990, Chapter 10). The basic idea behind this model had been expressed also in, for example, in Sedyakin (1966) and Cox and Oakes (1984). Robins and Tsiatis (1992) uses the same cumulative exposure model with a semiparametric estimation method that does not require specification of the underlying distribution for  $U$ .

Both individual and population predictions are considered in Hong and Meeker (2013). The distributions of remaining life (DRL) for the surviving units are the key elements for such predictions. In particular, the DRL for unit  $i$  is the distribution of  $T_i$ , given the current time in service  $t_i$  and  $\mathbf{X}_i(t_i) = \mathbf{x}_i(t_i)$  and it is calculated by

$$\rho_i(s; \boldsymbol{\theta}) = \mathbf{E}_{\mathbf{X}_i(t_i, t_i+s) | \mathbf{X}_i(t_i) = \mathbf{x}_i(t_i)} \{ \Pr[t_i < T_i \leq t_i + s | T_i > t_i, \mathbf{X}_i(t_i), \mathbf{X}_i(t_i, t_i + s)] \}.$$

Here  $\boldsymbol{\theta}$  is the collection of parameters,  $s$  is the number of time units after the data freeze date

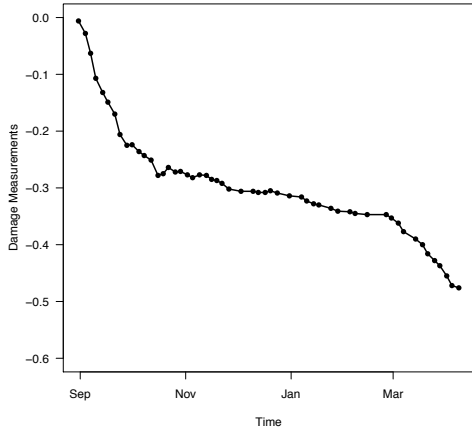
(DFD), and  $\mathbf{X}_i(t_1, t_2) = \{X_i(s) : t_1 < s \leq t_2\}$  is the covariate history for unit  $i$  from time  $t_1$  to time  $t_2$ . Simulation-based approaches were developed to evaluate  $\rho_i(s; \hat{\boldsymbol{\theta}})$  and compute pointwise confidence intervals (CIs) for  $\rho_i(s; \hat{\boldsymbol{\theta}})$ . The numerical results in Hong and Meeker (2013) show that heavily used units tend to have much higher risk for failures, based on the estimated DRL. The population prediction is based on  $N(s) = \sum_{i \in RS} I_i(s)$ , which gives the number of field failures at  $s$  time units after the DFD. Here  $RS$  is the risk set and  $I_i(s) \sim \text{Bernoulli}[\rho_i(s; \boldsymbol{\theta})]$ . The point prediction for  $N(s)$  can be obtained by  $\rho(s; \hat{\boldsymbol{\theta}}) = \sum_{i \in RS} \rho_i(s; \hat{\boldsymbol{\theta}})$ , which is also based on the estimated DRL. To quantify multiple sources of uncertainties, such as distributional uncertainty and statistical uncertainty, prediction procedures were developed for both individual and population predictions in Hong and Meeker (2013).

Extensive simulations were done in Hong and Meeker (2013) to demonstrate the advantages of incorporating dynamic data into statistical modeling and predictions. In general, when there are temporal trends in dynamic covariates (i.e., monotone and/or seasonal trends), the dynamic use-rate model often has advantage in field prediction, for both individual and population predictions. The advantage is especially pronounced for individual unit predictions.

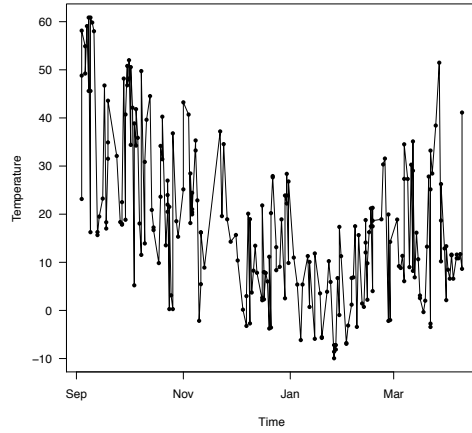
## 7 Example 2: Modeling and Field Failure Prediction Using Degradation Data with Dynamic Covariate Information

The major goal of Hong et al. (2012) is to develop general models for analyzing degradation data with dynamic covariate information. Their modeling of degradation data allows for unit-to-unit and temporal variability in covariates. The motivating application is from the National Institute of Standards and Technology (NIST) outdoor weathering data. In a study of the service life of organic coatings under outdoor environments, 36 specimens were placed in outdoor environmental chambers, each starting at a different time of the year. Due to different starting times, the dynamic covariate profiles vary from unit to unit and thus different units have different rates of degradation. Figure 2 shows the plots of the observed degradation path for a sample exposed outdoors, the time series of temperature and relative humidity (RH), and the Ultraviolet (UV) exposure as a function of time and wavelength. The outdoor temperature, RH, and UV exposure were recorded automatically by sensors. For modeling convenience, the UV exposure was summarized by the scientifically-motivated UV dosage, which is proportional to the number of photons absorbed into the coating.

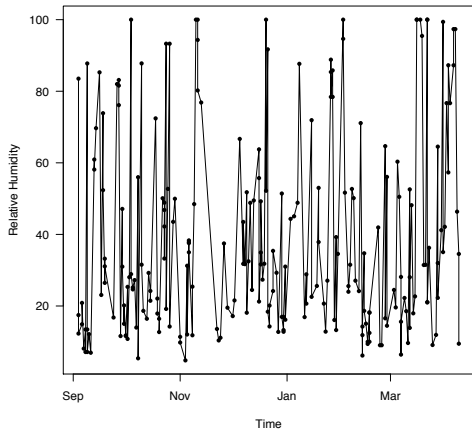
Let  $y_i(t_{ij})$  be the degradation measurements at time  $t_{ij}$  for unit  $i$ . The covariate informa-



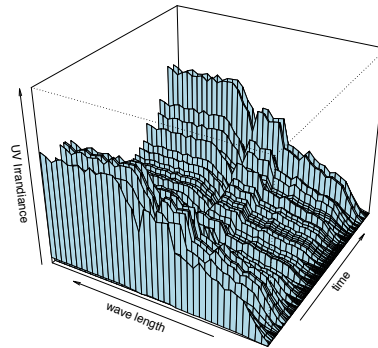
(a) Degradation Measurements



(b) Temperature



(c) RH



(d) UV

Figure 2: Plots of the observed degradation path for a sample exposed outdoors, the time series of temperature and RH, and the UV exposure as a function of time and wavelength.

tion at time  $t$  is denoted by  $X(t) = [X_1(t), \dots, X_p(t)]'$  where  $p$  is the number of covariates. We denote the history by  $\mathbf{X}(t) = \{X(s) : 0 \leq s \leq t\}$ . The observed history for unit  $i$  is  $\mathbf{x}_i(t_{in_i}) = \{x_i(s) : 0 \leq s \leq t_{in_i}\}$ . A general additive model for the observed degradation path is proposed in Hong et al. (2012). In particular,

$$y_i(t_{ij}) = D[t_{ij}; \mathbf{x}_i(t_{ij})] + R(t_{ij}; w_i) + \varepsilon_i(t_{ij}). \quad (1)$$

The first term in model (1)  $D[t_{ij}; \mathbf{x}_i(t_{ij})] = \beta_0 + \sum_{l=1}^p \int_0^{t_{ij}} f_l[x_{il}(\tau); \beta_l] d\tau$  incorporates the dynamic covariates into the degradation path through a covariate effect transformation function  $f(\cdot)$ . For example,  $f_l[x_{il}(\tau); \beta_l]$  represents the effect of covariate  $x_{il}(\tau)$  at time  $\tau$  on the degradation path. Thus, the cumulative effect of  $x_l$  is given by  $\int_0^t f_l[x_l(\tau); \beta_l] d\tau$ . Shape-restricted splines are proposed to construct the functional form of  $f(\cdot)$ . The second term  $R(t; w_i)$  is the random component for unit-to-unit variability and the third term is the error term. The estimation of parameters in (1) is not trivial because both shape-restricted splines and random effects are involved. An iterative algorithm was proposed by using the mixed primal-dual bases algorithm to solve the generalized least squares problem under constraints. The numerical results show that the UV dosage causes relatively large amounts of damage relative to temperature and RH. A model for the covariate process is needed for reliability prediction. The covariate process can be modeled by  $X(t) = m(t; \boldsymbol{\eta}) + a(t)$  where  $m(t; \boldsymbol{\eta})$  is the mean structure with parameter  $\boldsymbol{\eta}$ . The covariate process in the NIST data was modeled by sine waves for the mean structure and by a second order vector autoregressive time series model for the error term.

The cdf of the degradation-induced time to failure is important for reliability inference. Based on the model, the actual path is  $D[t; \mathbf{X}(\infty)] + R(t; w)$  and the first crossing time is  $t_{\mathcal{D}} = \min\{t : D[t; \mathbf{x}(\infty)] + R(t; w) = \mathcal{D}_f\}$ . The cdf of  $T = T[\mathcal{D}_f, \mathbf{X}(\infty), w]$  is

$$F(t; \boldsymbol{\theta}) = \mathbf{E}_{\mathbf{X}(\infty)} \mathbf{E}_w \Pr \{T[\mathcal{D}_f, \mathbf{X}(\infty), w] \leq t\}, \quad t > 0, \quad (2)$$

where  $\boldsymbol{\theta}$  denotes all unknown parameters. A simulation based procedure was developed to evaluate  $F(t; \boldsymbol{\theta})$ . The conditional cdf is also useful in applications, especially for CBM and SHM. The conditional distribution for individual with covariate history  $\mathbf{X}_i(t_{in_i}) = \mathbf{x}_i(t_{in_i})$  is

$$\rho_i(s; \boldsymbol{\theta}) = \mathbf{E}_{\mathbf{X}_i(\infty) | \mathbf{X}_i(t_{in_i}) = \mathbf{x}_i(t_{in_i})} \mathbf{E}_w \Pr \{T[\mathcal{D}_f, \mathbf{X}(\infty), w] \leq t_{in_i} + s | T > t_{in_i}\}, \quad s > 0,$$

which gives the failure probability at a future time, conditional on  $\mathbf{X}_i(t_{in_i}) = \mathbf{x}_i(t_{in_i})$ . A simulation based procedure was developed to evaluate  $\rho_i(s; \boldsymbol{\theta})$ . To illustrate the use of these methods, Hong et al. (2012) applied these models and procedures to the NIST outdoor weathering data.



## 8 Example 3: Field Failure Prediction Based on Multi-Level Repair and System Usage Information

In this section, we introduce a model that can be used for field failure prediction based on multi-level repair and system usage information. Repairable systems in the field often receive repair actions at different levels. For example, a truck may have an engine replaced or have a component of the engine replaced. Thus we may consider three levels: system (truck), sub-system (engine), and component level. At the system level, the system usage and environmental information can be available. At sub-system and component levels, the repair information can also be available through maintenance records.

We start with the introduction of a repairable system model for one level. A useful reference for this topic is Cook and Lawless (2007). Let  $0 < T_1 < \dots < T_i < \dots$  be the event times from a repairable system. Let  $N(t)$  be the counting process and  $\mathcal{F}_t$  be the event history up to time  $t$ . The event intensity for the counting process is defined as

$$\lambda(t|\mathcal{F}_{t-}) = \lim_{\Delta t \rightarrow 0} \frac{\Pr\{N(t + \Delta t) - N(t) = 1 | \mathcal{F}_{t-}\}}{\Delta t}.$$

The cumulative event intensity function is defined as  $\Lambda(t) = \int_0^t \lambda(u|\mathcal{F}_{u-})du$ . The nonhomogeneous Poisson process (NHPP) and the renewal process (RP) are the two most commonly used models for repairable systems. The NHPP corresponds to a minimal repair, in which the repair is as bad as old. The RP corresponds to a perfect repair, in which the unit is as good as new after the repair. The trend renewal process (TRP) model provides an alternative that not only includes both NHPP and RP as special cases but also captures situations that are in-between NHPP and RP (Lindqvist, Elvebakk, and Heggland 2003). The TRP is characterized by a trend function  $\Lambda(t) = \int_0^t \lambda(u)du$  and a renewal distribution  $F$ . It has the property that  $\Lambda(T_{i+1}) - \Lambda(T_i) \stackrel{iid}{\sim} F$ ,  $i = 1, 2, \dots$ . The event intensity function of a TRP is  $\lambda(t|\mathcal{F}_{t-}) = h\{\Lambda(t) - \Lambda[T_{N(t-)}]\}\lambda(t)$ . The event intensity function of the TRP process has two factors. The factor  $\lambda(t)$  reflects the overall system deterioration. The factor  $h\{\Lambda(t) - \Lambda[T_{N(t-)}]\}$  reflects the effect of each repair. After each repair, there is a change in the intensity function. The behavior of the change is determined by the hazard function of the renewal distribution  $F$ .

To model the event intensity for the component replacement that uses information from different levels, a multi-level TRP model is proposed. We consider a fleet of  $n$  systems. Let  $N_{i1}(t)$  be the number of component replacements, up to time  $t$  for unit  $i$  and excluding sub-system replacements. Let  $N_{i2}(t)$  be the number of sub-system replacements, up to time  $t$  for unit  $i$ . Let  $N_i(t) = \sum_{j=1}^2 N_{ij}(t)$  be the total number of replacements up to time  $t$  for unit  $i$ . The last follow up time for each unit is denoted by  $\tau_i$ . The event times

of event type  $j$  for unit  $i$  are denoted by  $0 < t_{ij1} < \dots < t_{ijk} < \dots < t_{ij, N_{ij}(\tau_i)} < \tau_i$ . For notational convenience, the event times regardless of the types for unit  $i$  are denoted by  $0 < t_{i1} < \dots < t_{ik} < \dots < t_{i, N_i(\tau_i)} < \tau_i$ . Let  $X_i(t) = [X_{i1}(t), \dots, X_{ip}(t)]'$  be the covariate process. The event history up to time  $t$  for the dataset is  $\mathcal{F}_t = \{N_{ij}(s), X_i(s) : i = 1, \dots, n, j = 1, 2, \text{ and } 0 \leq s \leq t\}$ . The event history for sub-system replacements up to time  $t$  is  $\mathcal{F}_t^s = \{N_{i2}(s), X_i(s) : i = 1, \dots, n, \text{ and } 0 \leq s \leq t\}$ . The event intensity for component replacements is modeled as

$$\begin{aligned}\lambda_i(t|\mathcal{F}_{t-}) &= h \{ \Lambda_i(t|\mathcal{F}_{t-}^s) - \Lambda_i [T_{N_i(t-)} | \mathcal{F}_{t-}^s] \} \lambda_i(t|\mathcal{F}_{t-}^s), \\ \lambda_i(t|\mathcal{F}_{t-}^s) &= h^s \{ \Lambda_i(t) - \Lambda_i [T_{N_i(t-)}] \} \lambda_i(t), \\ \lambda_i(t) &= \lambda_b(t) \exp[\beta' X_i(t)],\end{aligned}$$

where  $\Lambda_i(t|\mathcal{F}_{t-}^s) = \int_0^t \lambda_i(u|\mathcal{F}_{u-}^s) du$  and  $\Lambda_i(t) = \int_0^t \lambda_i(u) du$ . Here  $\lambda_i(t)$  describe the general system trend that consists of a baseline trend described by  $\lambda_b(t)$ , and the trend can be described by covariate process  $X_i(t)$ . The event intensity for component replacement will be adjusted by the factor  $h^s \{ \Lambda_i(t) - \Lambda_i [T_{N_i(t-)}] \}$  if there is sub-system replacement. That is a replacement in subsystem will reduce the risk of component replacement. The event intensity will be further adjusted by  $h \{ \Lambda_i(t|\mathcal{F}_{t-}^s) - \Lambda_i [T_{N_i(t-)} | \mathcal{F}_{t-}^s] \}$  if there is a component replacement. Figure 3 illustrates the intensity function of the multi-level TRP model for a simulated event history [the vertical solid (dot-dashed) lines show the sub-system (component) repair times]. **The ML approach can be used to do the parameter estimation and statistical predication procedure can also be developed. The estimated intensity function can be used as the basis for the field failure prediction and other decision making.**

## 9 Practical Concerns and Challenges

Periodically downloading and saving SOE data for units in a product population will produce **massively large datasets**. For example, suppose there are 500 thousand units providing 100 single precision (4-byte) numerical values every half hour **for a period of five years**. The total size of the dataset would be on the **order of 2 terabytes!** Of course such large datasets would overwhelm modern desktop computers and software. With traditional field reliability data lacking covariate information, extremely large populations can be represented in small data files by proper binning of observations. For example, the bearing cage dataset, representing 1,703 aircraft engines given in Table C.5 of Meeker and Escobar (1998) has only 25 records and contains fewer than 500 bytes. With dynamic covariate information, such binning will be less effective in reducing the size of a dataset. Strategies will need to be developed to make analyses manageable (both in terms of storage and computational time). Examples of such strategies include the following:

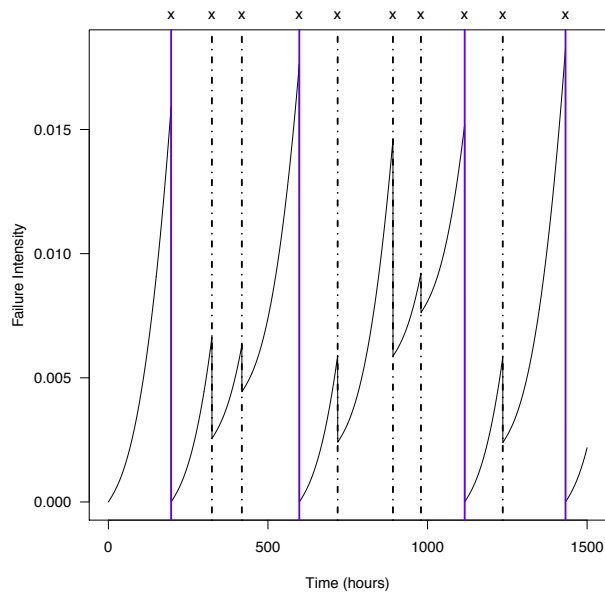


Figure 3: Illustration of the intensity function of the multi-level TRP model for a simulated event history [the vertical solid (dot-dashed) lines show the sub-system (component) repair times].

- The original environmental data (weather and solar) used in Hong et al. (2012) had been collected at 12 minute intervals. Compressing the data down to daily averages was more than adequate for their modeling purposes.
- In some cases, especially when there is a huge number of observations, it might be possible to use a subset of the entire dataset. Predictions made based on the subset could then be checked on the remainder of the data, in the spirit of cross validation. One problem with this approach is that there is often a huge number of units in the field that are at risk to fail, but only a limited number of failures. A general alternative approach is described in Guha et al. (2012) where a dataset is divided into manageable parts, analyzed separately, and then the results are combined to produce a composite analysis.
- Large product populations can be stratified in logical subgroups that are not only more manageable, but more homogeneous, allowing for better modeling. For example in a warranty prediction problem, a population with many hundreds of thousands of fielded units was stratified into what were called “part-number genealogy groups,” according to part number changes in critical components that had been made over the manufacturing time of the product. The more homogeneous datasets allowed for the

generation of more accurate overall predictions.

- Hong and Meeker (2013) show that predictions can be seriously biased when covariate for individual units change over time and just a simple average of the covariate time series is used in a simpler model. Handling individual covariate histories for all units in the field, for a large product population, may be intractable. One alternative we are pursuing is to use cluster analysis to divide the product population into groups and to use a simple static description of the covariate behavior for each group. If the purpose of the analysis is prediction for each individual, one could augment this procedure by having a scalar or low-dimensional per-unit adjustment to the covariate description.

## 10 Concluding Remarks and Areas for Further Research

The future possibilities for using use rate/environmental data in reliability applications are unbounded. Lifetime models that use rate/environmental data have the potential to explain much more variability in field data than has been possible before. The information can also be used to predict the future environment lifetimes of individual units. This knowledge can, in turn provide more precise estimates of the life time of individual products. As the cost of technology drops, cost–benefit ratios will decrease, and the number of potential applications will grow. We can expect that the potential will be realized after an appropriate set of general statistical methods have been developed and implemented in easy-to-use software.

There is a large number of open areas for further research in this area. These include the following.

- Our discussion and thinking has focused on hardware failures. Software will continue to be an increasingly important role in modern systems. Correspondingly, system reliability and availability will be increasingly dependent on the reliability of software. SOE data certainly will also provide benefits to better modeling, diagnosis, and improvements of software reliability and thus overall system reliability.
- Our examples have used a single reliability characteristic (failure time or degradation) or failure criteria. Often systems will have more than one failure mode, and it is often useful, if not necessary, to consider these modes separately. For example, Hong and Meeker (2010) showed how to make warranty predictions with multiple failure modes. Corresponding methods could be developed for products that have multiple degradation performance characteristics (e.g., color and gloss degradation of its coating

as well as smoothness-of-ride degradation resulting from wearing shock absorbers). See Wang and Coit (2004) for an example.

- We have described field reliability data with both time and degradation responses. In some applications the output of the degradation assessment test will be an image or even a sequence of images. For example, Li, Holland, and Meeker (2010) develop a crack-detection algorithm for sequence-of-image vibrothermography nondestructive evaluation method. The same sequence-of-image data can be used to estimate the size of a crack.
- Our discussion has focused on what might be called “data-driven” or empirical methods of analysis, detection, and prediction. For many purposes such an approach can be perfectly satisfactory. For other purposes they may be woefully inadequate. In particular predictive reliability inferences often require extreme amounts of extrapolation. For **example**,
  - **Predicting the fraction failing within a three-year warranty period based on six months of field-failure reports.**
  - Estimating the fraction failing after five years at a used temperature of 25 degrees C, based on accelerated tests at 50, 60, and 70 degrees C that last only three months.
  - Predicting the reliability of production units in the field based on laboratory tests on a few prototype units.

Similar kinds of **extrapolation** arise in the prediction of remaining useful life of individual fielded units. As mentioned in Section 3.3, **without strong covariate information**, such predictions **will not be precise** (e.g., Hong, Meeker, and McCalley 2009). Having SOE data and good models about how SOE data affect life may, however, provide much better predictions for individuals, as demonstrated in Hong and Meeker (2013).

Extrapolation will be more reliable if predictions are based on a combination of science-based models of reliability (e.g., knowledge of the physics of well-understood failure modes) and data are used to develop predictive models for a failure-time distribution. Because of its importance in aerospace applications, fatigue failure due to cyclic loading is probably the most widely studied and best understood failure mechanism. Models based on a combination of theory and empirical experimentation provide engineers with powerful tools to predict the growth of fatigue cracks. For example, Haldar and Mahadevan (2000) describe the use of such models in structural reliability applications. Yu and Harris (2001) provide a model based on first principles that predicts the

lifetime of roller bearings. Much work remains to be done to obtain a similar level of understanding of many other failure modes.

- Big data, by itself, will not solve all reliability problems. Especially in those cases listed above, where extrapolation is required, detailed knowledge of the physics of failure is exceedingly important to justify the extrapolation and to provide some degree of assurance for resulting predictions.
- An extension of using the combination of physics-based models and dynamic covariate information to predict remaining life is to add in-situ monitoring of the actual physical state of system components (e.g., the amount of wear, creep, or other deformation) to provide what some call “material state awareness” but the same ideas have been discussed using the terms “integrated structural health monitoring” and “integrated vehicle health monitoring,” depending on the organization and particular application. A summary of ideas from this area is given in National Research Council (2008), a proceeding from a workshop on this topic.
- Many of the applications described in this paper and particularly in this concluding section will require combining information from different sources (e.g., data, in exact physics-based knowledge and certain kinds of expert opinion). Additionally, statistical models being used will often contain multiple sources of variability. Bayesian statistical methods provide a natural approach for combining such information. Tremendous advances have been made over the past 20 years and there has been much written about the use of Bayesian methods in statistics in general and there is some work that has focused on reliability applications. Some particular examples include the following. Singpurwalla (2006) provides the theoretical framework for the use of Bayesian methods to quantify reliability and risk. Hamada et al. (2008) provides operational methods for applying Bayesian methods to a wide range of reliability application areas. Yan (2012) shows how to use a Bayesian approach to solve a complicated inverse problem to identify crack properties. Li and Meeker (2013) provide an introduction to the basic ideas of using Bayesian methods for reliability data analysis and illustrate the methods with four basic kinds of reliability data (single distribution analysis of field failure data, accelerated life testing, accelerated destructive degradation data analysis, and accelerated repeated measures degradation data analysis).

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## References

- Antoniadou, I., G. Manson, N. Dervilis, T. Barszcz, W. Staszewski, and K. Worden (2012). Condition monitoring of a wind turbine gearbox using the empirical mode decomposition method and outlier analysis. In *6th European Workshop on Structural Health Monitoring*. Proceedings available online at <http://www.ewshm2012.com/Proceedings.aspx>.
- Carden, E. P. and P. Fanning (2004). Vibration based condition monitoring: a review. *Journal of Structural Health Monitoring* 3, 355–377.
- Ciang, C. C., J. R. Lee, and H. J. Bang (2008). Structural health monitoring for a wind turbine system: a review of damage detection methods. *Measurement Science and Technology* 19, 1–20.
- Cook, R. J. and J. F. Lawless (2007). *The Statistical Analysis of Recurrent Events*. New York: Springer-Verlag.
- Cox, D. D. R. and D. Oakes (1984). *Analysis of survival data*, Volume 21. CRC Press.
- Escobar, L. A. and W. Q. Meeker (1999). Statistical prediction based on censored life data. *Technometrics* 41, 113–124.
- Faulkner, P., P. Cutter, and A. Owens (2012). Structural health monitoring systems in difficult environments—offshore wind turbines. In *6th European Workshop on Structural Health Monitoring*.
- Guha, S., R. Hafen, J. Rounds, J. Xia, J. Li, B. Xi, and W. S. Cleveland (2012). Large complex data: divide and recombine(D&R) with RHIPE. *Stat* 1, 53–67.
- Hahn, G. J. and N. Doganaksoy (2008). *The Role of Statistics in Business and Industry*. New Jersey, Hoboken: John Wiley & Sons, Inc.
- Haldar, A. and S. Mahadevan (2000). *Reliability Assessment Using Stochastic Finite Element Analysis*. New York: John Wiley & Sons.

- Hamada, M. S., A. Wilson, C. S. Reese, and H. Martz (2008). *Bayesian Reliability*. New York: Springer.
- Hong, Y., Y. Duan, W. Q. Meeker, D. L. Stanley, and X. Gu (2012). Statistical methods for degradation data with dynamic covariates information and an application to outdoor weathering data. Technical Report No. 12-3: Department of Statistics, Virginia Tech, Blacksburg, Virginia 24061. Available at [http://www.stat.vt.edu/tech\\_reports/index.html](http://www.stat.vt.edu/tech_reports/index.html).
- Hong, Y. and W. Q. Meeker (2010). Field-failure and warranty prediction based on auxiliary use-rate information. *Technometrics* 52, 148–159.
- Hong, Y. and W. Q. Meeker (2013). Field-failure predictions based on failure-time data with dynamic covariate information. *Technometrics* 55(2), 135–149.
- Hong, Y., W. Q. Meeker, and J. D. McCalley (2009). Prediction of remaining life of power transformers based on left truncated and right censored lifetime data. *The Annals of Applied Statistics* 3, 857–879.
- Jarrell, D. B., D. R. Sisk, and L. J. Bond (2003). Prognostics and condition-based maintenance: a new approach to precursive metrics. *Nuclear Technology* 145, 275–286.
- Lawless, J. F. and M. Crowder (2004). Covariates and random effects in a gamma process model with application to degradation and failure. *Lifetime Data Analysis* 10, 213–227.
- Lawless, J. F., M. J. Crowder, and K. A. Lee (2012). Monitoring warranty claims with cusums. *Technometrics* 54, 269–278.
- Lawless, J. F. and M. Fredette (2005). Frequentist prediction intervals and predictive distributions. *Biometrika* 92, 529–542.
- Li, M., S. D. Holland, and W. Q. Meeker (2010). Statistical methods for automatic crack detection based on vibrothermography sequence-of-images data(with discussion). *Applied Stochastic Models in Business and Industry* 26, 481–495.
- Li, M. and W. Q. Meeker (2013). Application of bayesian methods in reliability data analyses. *Quality Technology*. To appear.
- Lindqvist, B., G. Elvebakk, and K. Heggland (2003). The trend-renewal process for statistical analysis of repairable systems. *Technometrics* 45, 31–44.
- Lu, C. J. and W. Q. Meeker (1993). Using degradation measures to estimate a time-to-failure distribution. *Technometrics* 34, 161–174.
- Meeker, W. Q. and L. A. Escobar (1998). *Statistical Methods for Reliability Data*. New York: John Wiley & Sons, Inc.



- Meeker, W. Q., L. A. Escobar, and C. J. Lu (1998). Accelerated degradation tests: modeling and analysis. *Technometrics* 40, 89–99.
- Murray, W. P. (1993). Archival life expectancy of 3M magneto-optic media. *Journal of the Magnetism Society of Japan* 17, Supplement S1, 309–314.
- National Research Council (2008). *Proceedings of a Workshop on Materials State Awareness*, Washington, DC. National Research Council: The National Academies Press.
- Nelson, W. (1990). *Accelerated Testing: Statistical Models, Test Plans, and Data Analyses*, (Republished in a paperback in *Wiley Series in Probability and Statistics*, 2004). New York: John Wiley & Sons.
- Nelson, W. B. (2003). *Recurrent Events Data Analysis for Product Repairs, Disease Recurrences, and Other Applications*. Philadelphia: ASA-SIAM.
- Pecht, M. G. (2008). *Prognostics and Health Management of Electronics*. Hoboken: John Wiley & Sons.
- Robins, J. and A. A. Tsiatis (1992). Semiparametric estimation of an accelerated failure time model with time-dependent covariates. *Biometrika* 79(2), 311–319.
- Sedyakin, N. (1966). On one physical principle in reliability theory. *Engineering Cybernetics* 3, 80–87.
- Singpurwalla, N. D. (2006). *Reliability and Risk: A Bayesian Perspective*. New York: John Wiley & Sons.
- Spurgeon, K., W. H. Tang, Q. H. Wu, Z. J. Richardson, and G. Moss (2005). Dissolved gas analysis using evidential reasoning. *IEE Proceedings - Science, Measurement & Technology* 152, 110–117.
- Volponi, A. and B. Wood (2011). *System Health Management with Aerospace Applications*, Chapter 24. Hoboken: John Wiley & Sons.
- Wang, P. and D. W. Coit (2004). Reliability prediction based on degradation modeling for systems with multiple degradation measures. In *Proceedings of the 2004 Reliability and Maintainability Symposium*, pp. 302–307.
- Wu, H. and W. Q. Meeker (2002). Early detection of reliability problems using information from warranty data bases. *Technometrics* 44, 120–133.
- Yan, G. (2012). Bayesian approach for identification of structural crack using strain measurements. In *6th European Workshop on Structural Health Monitoring*. Proceedings available online at <http://www.ewshm2012.com/Proceedings.aspx>.

- Yashchin, E. (2010). Computational and monte carlo aspects of systems for monitoring reliability data. In Y. Lechevallier and G. Saporta (Eds.), *Proceedings of COMSTAT 2010*, pp. 253–262. Physica-Verlag.
- Yu, W. K. and T. A. Harris (2001). A new stress-based fatigue life model for ball bearings. *Tribology Transactions* 44, 11–18.