HVDC LEARN

Modules for Maturing HVDC Electric Transmission Knowledge

Funded by the US Department of Energy (DOE)
within the Office of Energy Efficiency & Renewable Energy (EERE)

HVDC-Learn Short Course

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Module 1c: Power Electronics 101

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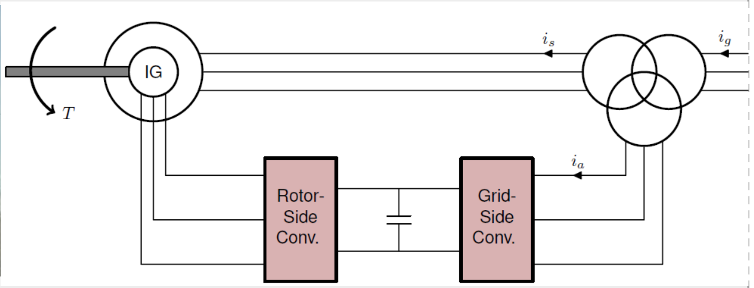
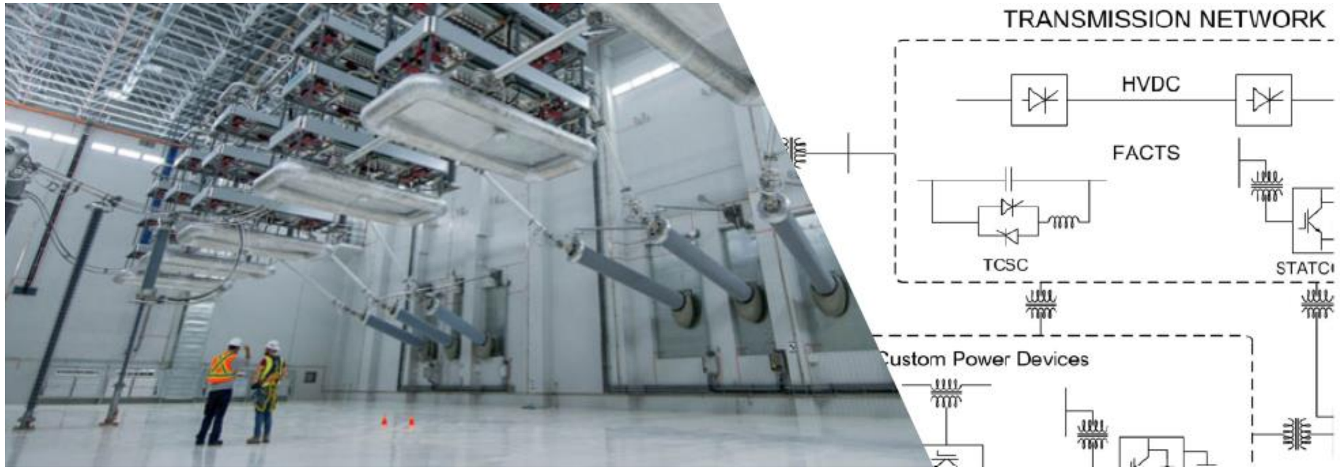
Power Electronics Interfaces

- Most generation forms involve some sort of electronic circuits.
  - Synchronous generators:
    Alternator-supplied rectifier excitation systems, e.g., AC1C
    Static excitation systems, e.g., ST1C. *
  - Inverter-based resources (IBRs): solar and most wind, EV chargers, BESS.

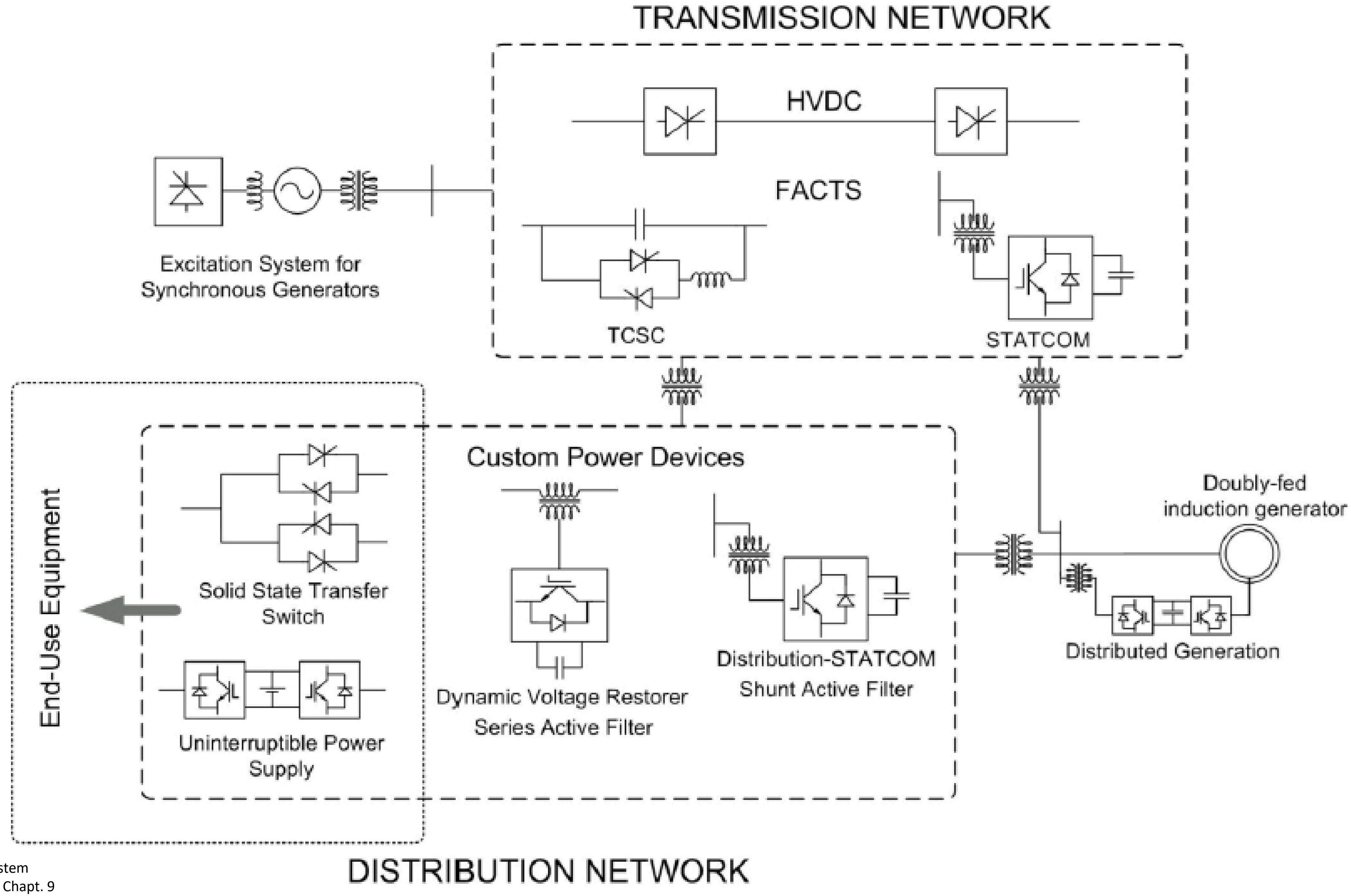


- Power electronics is the heart of many grid-enhancing technologies (GET)
  - FACTS (flexible AC transmission systems)
  - HVDC



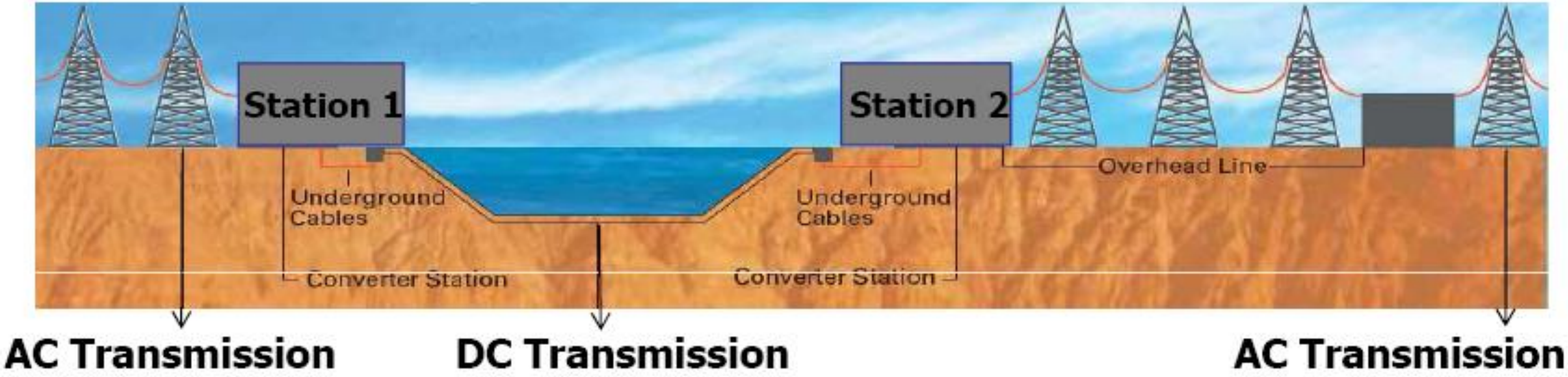
- Power electronics discusses how one form of power is converted to another; this could involve changes in voltage magnitude, shape, frequency, and power quality.

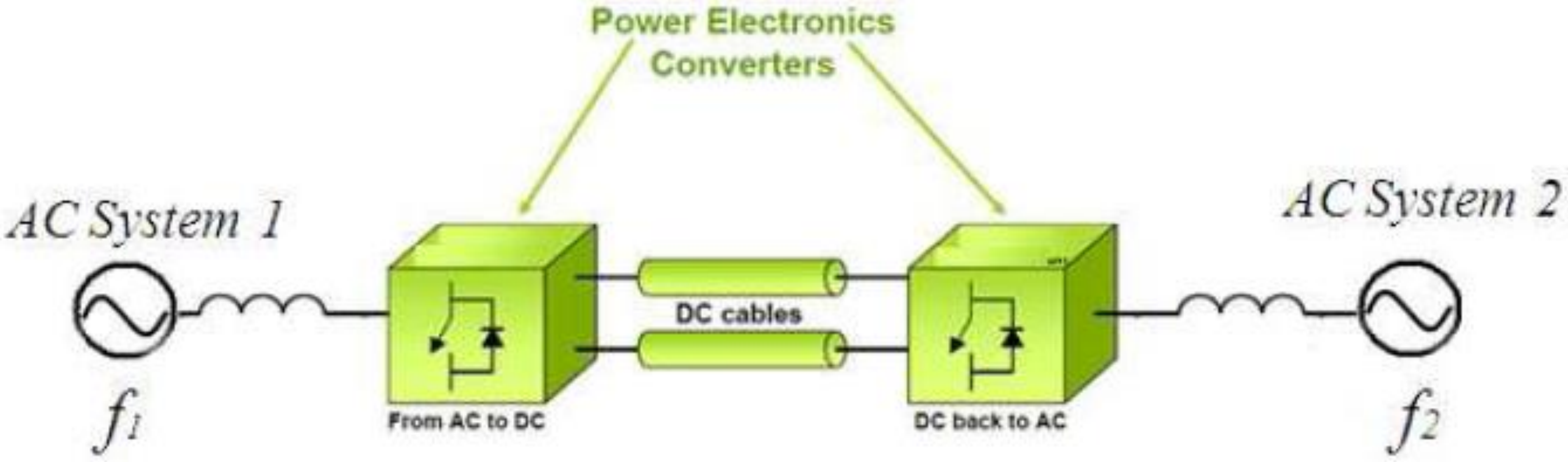
See IEEE Std 421.5-2016 Recommended Practice for Excitation System Models for Power System Stability Studies



S. Filizadeh, et al, Power system transients, J. Martinez, ed., Chapt. 9

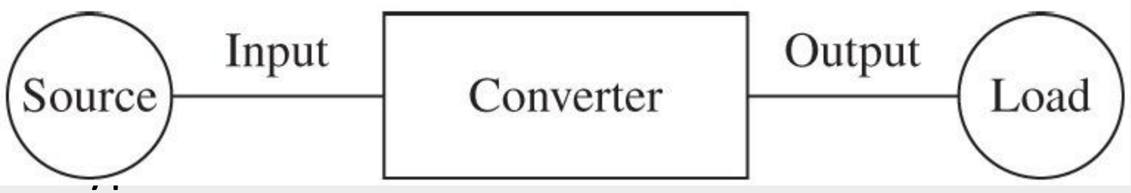
Power Electronics for HVDC Systems





EPRI HVDC-FACTS Conference in 2013

Converters

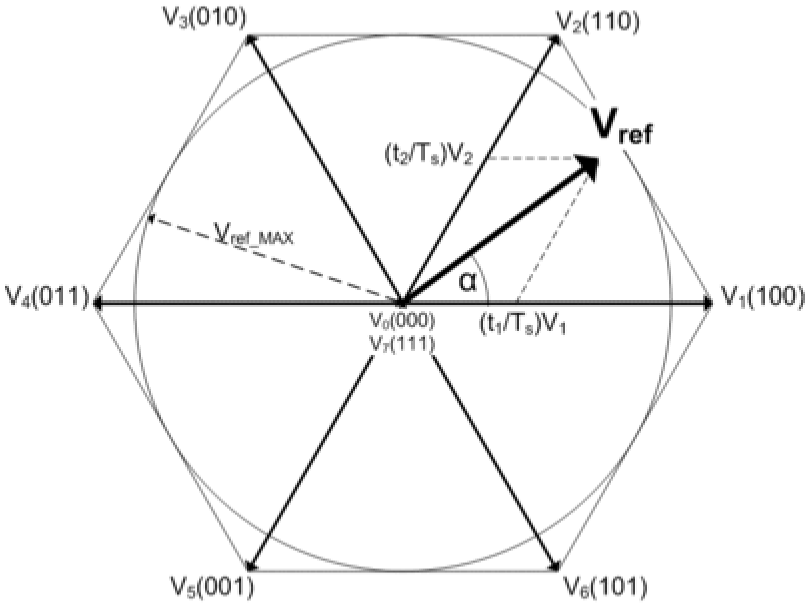


- DC-DC Conversion
  - Change voltage magnitude
- AC-DC Rectification
  - Produce dc voltage from an ac source
- DC-AC Inversion
  - Produce a sinusoidal voltage with controllable magnitude and frequency
- AC-AC Conversion
  - Change voltage magnitude and frequency

- There are also two main technologies:
  - LCC
  - VSC
  - This module talks about switching power converters, VSC,

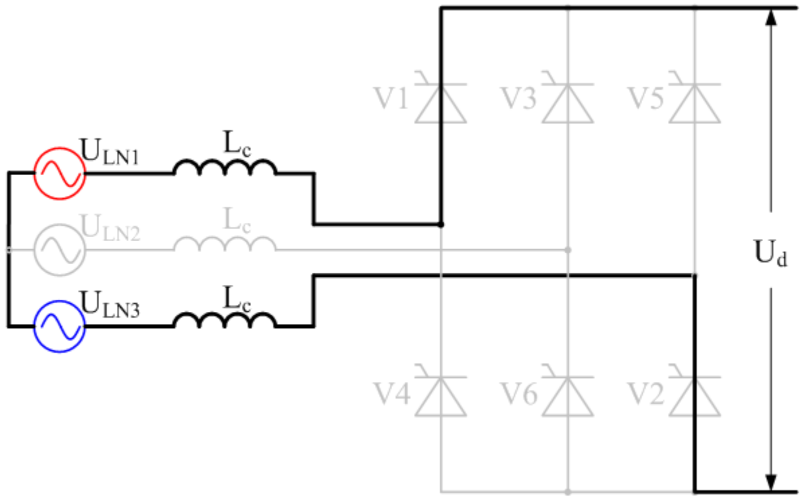
Topics

- DC-DC converter basics
- Transitioning to a DC-AC converter
- Modulation methods

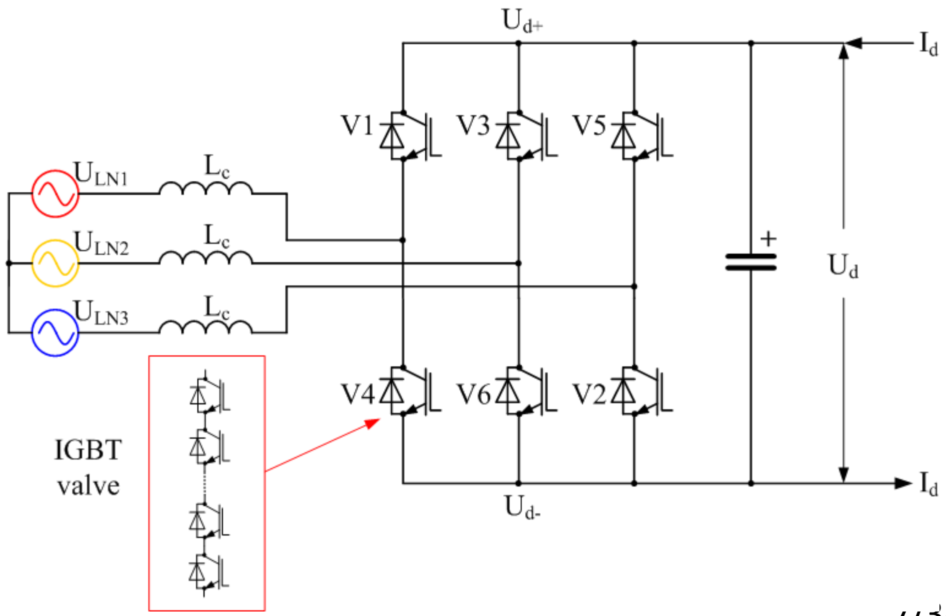


Commutation in Converters

- **Line commutated (naturally commutated)**
  - The AC system dictates the process, e.g., thyristors turn off when the current drops to zero and turn on when forward biased and gate signal is present.
  - Always consumes reactive power Q.
  - Earlier generation of HVDC and FACT, but still an active technology due to higher ratings.



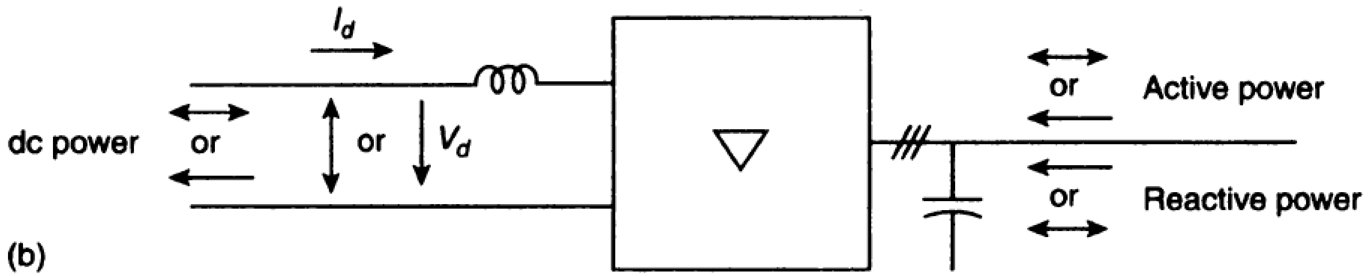
- **Forced commutated (self commutated)**
  - The gate circuitry determines both turn on and off processes---fully controllable switches
  - Can consume or generate reactive power Q.
  - This course focuses on forced commutated converters.



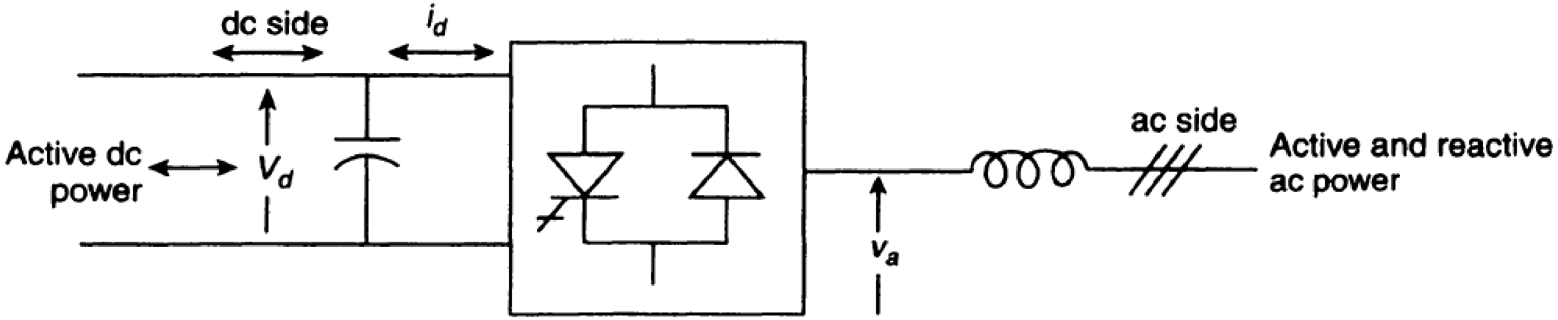
https://en.wikipedia.org/wiki/File:6_pulse_bridge_with_inductance.png

Classification of Converters

- **Based on DC-Side Source Type**
  - Current-Sourced Converter (CSC)

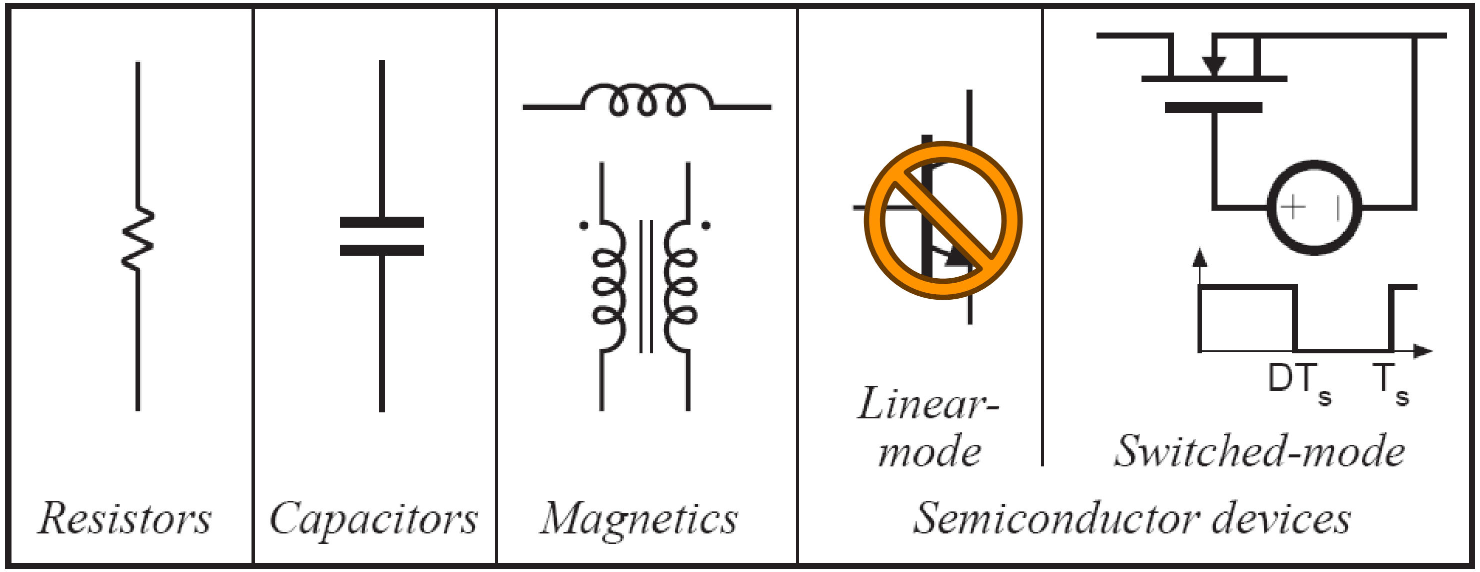


  - Voltage-Sourced Converter (VSC)
    - (spelling per Dennis Woodford of Electranix)

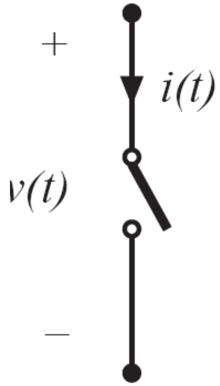


Figures are from Hingorani.

How to Build a Converter? Available Elements

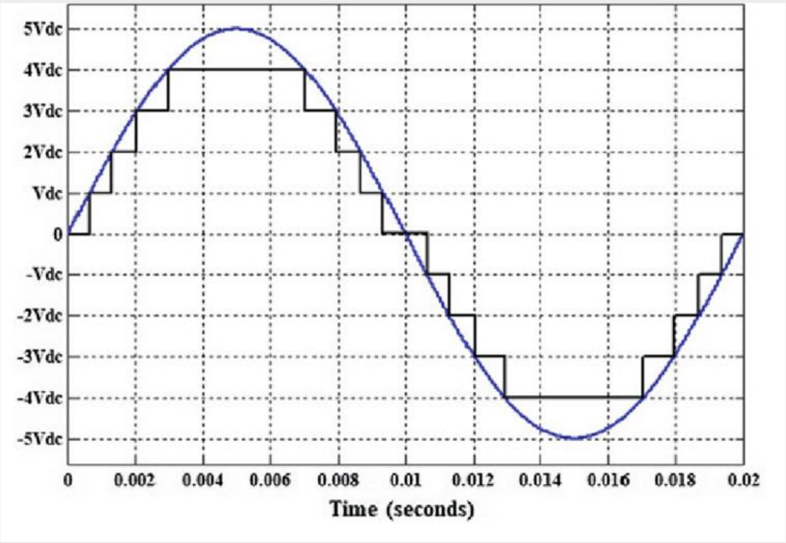


- **Considerations:**
  - Efficiency
    - Ideal switch: $p(t) = 0$
  - Size and weight
  - Power quality

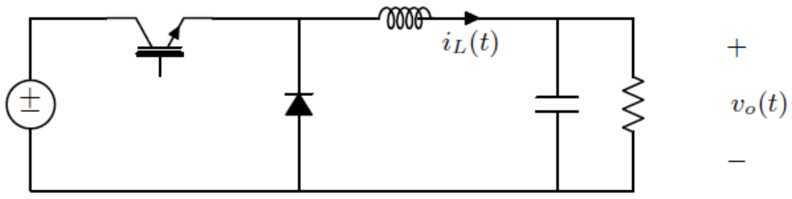


Our Approach to Studying DC-AC Conversion

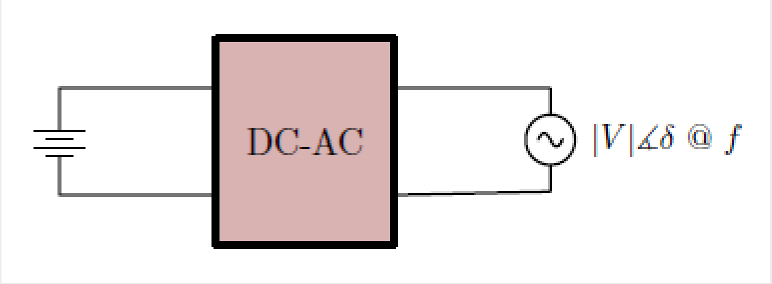
- See a sine wave as a sequence of DC values



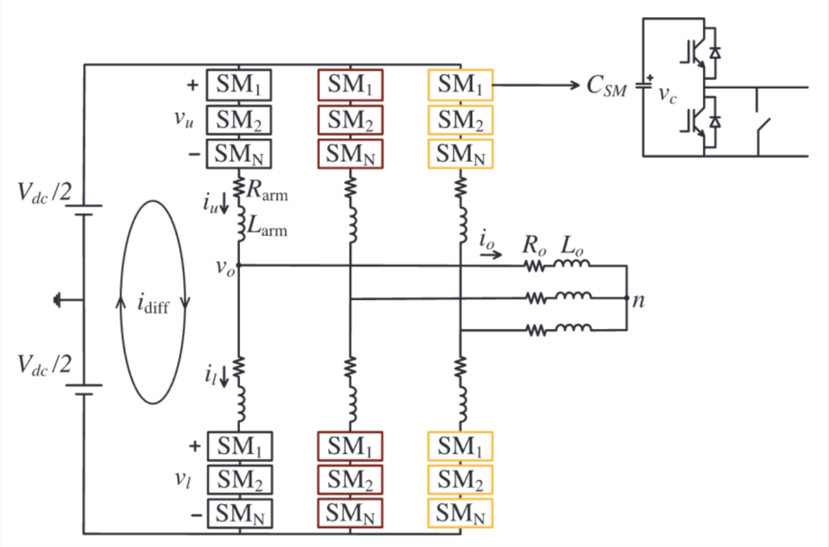
- Learn DC-DC Conversion



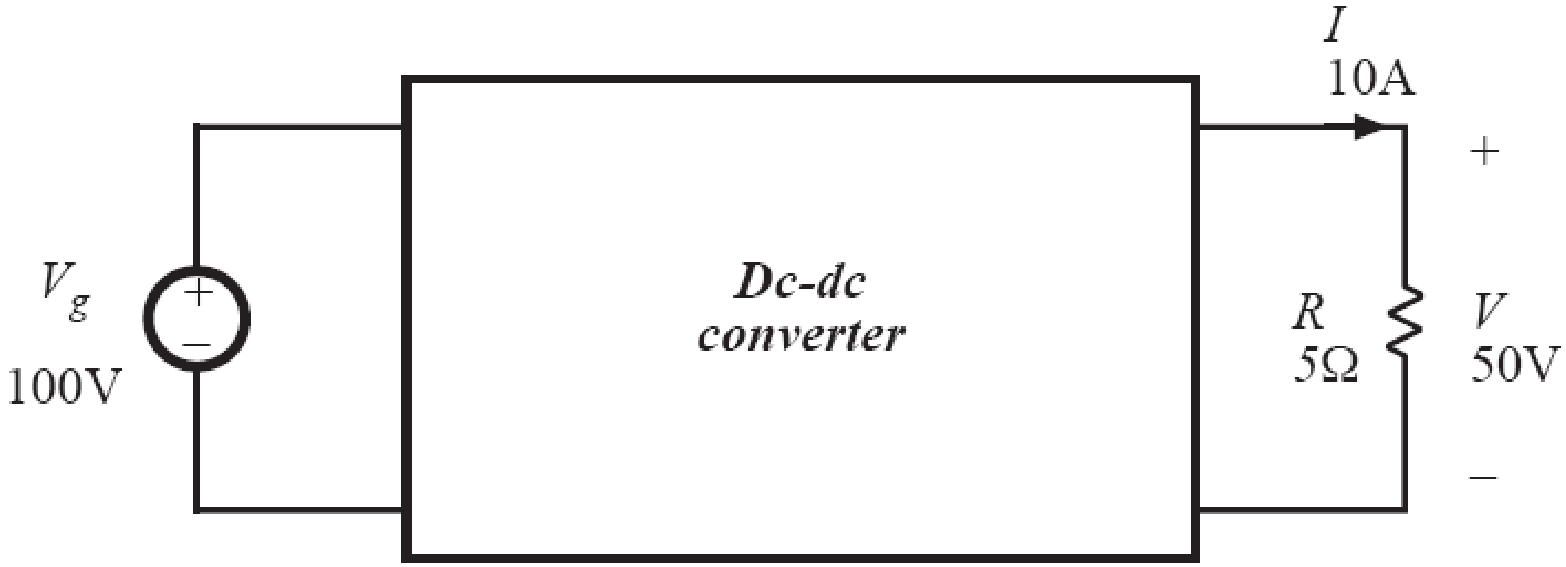
- Extend to DC-(variable dc(t)), i.e., DC-AC conversion.



- The next two modules will discuss VSCs and MMCs in details.



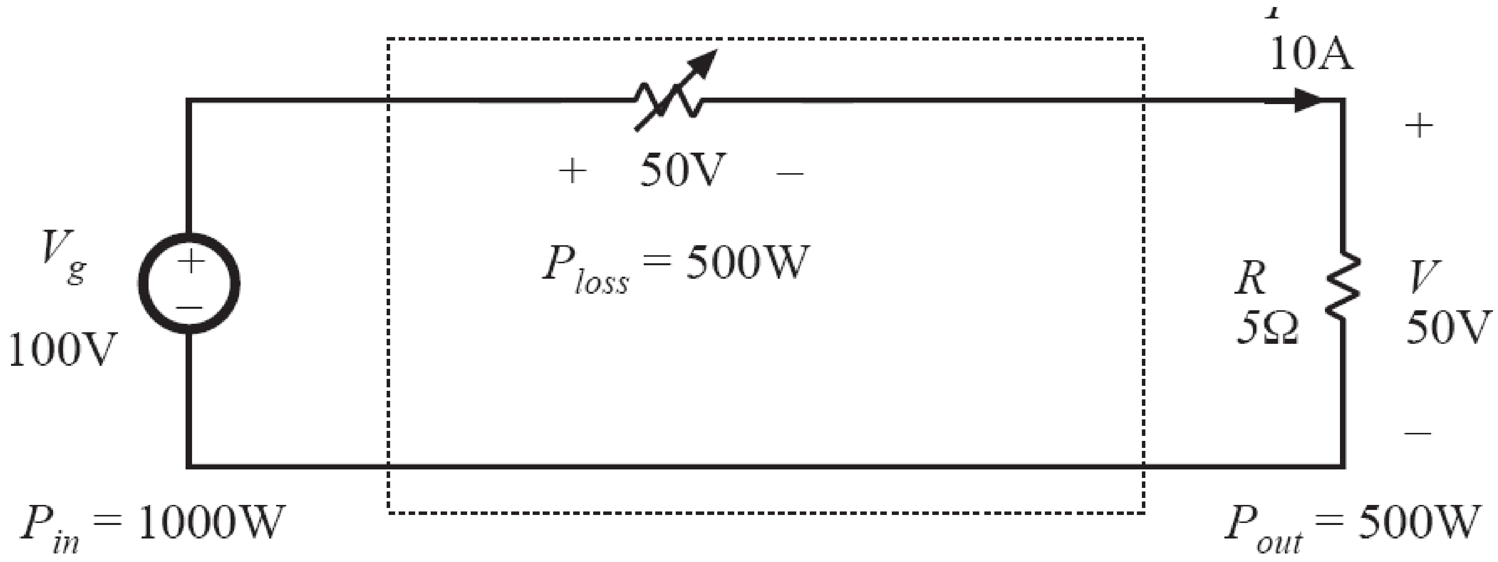
DC-DC Converter Example



- **Input: 100 V**
- **Output: 50 V, 10 A, 500 W**

Realization of a DC-DC Converter: Solution #1

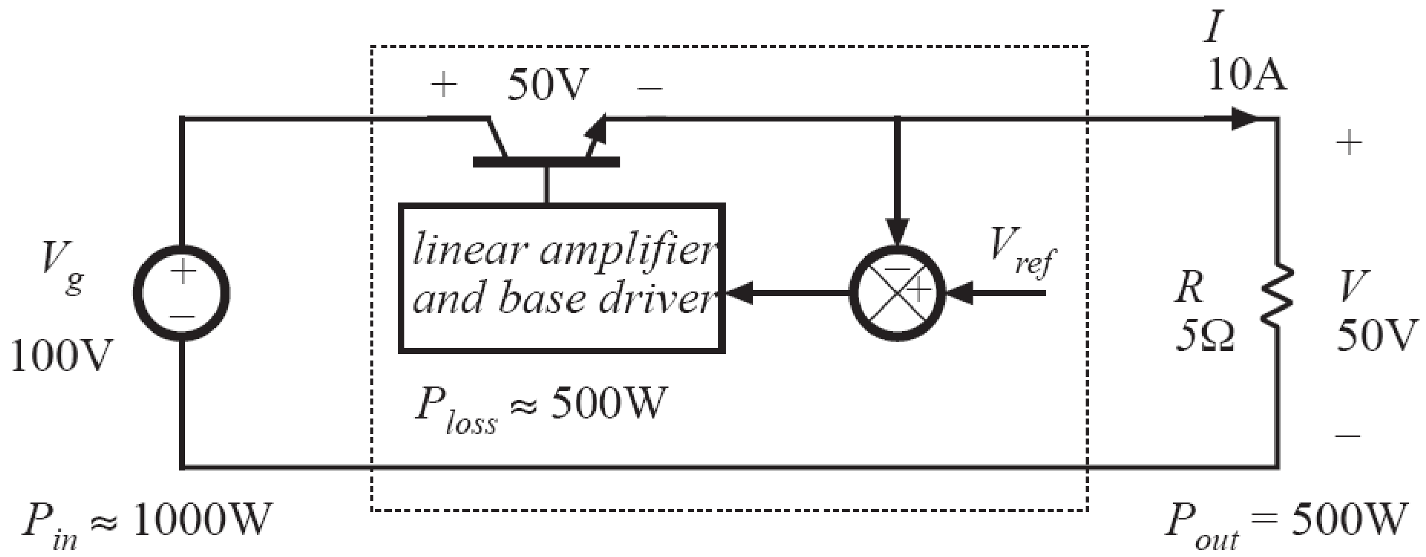
- **Resistive Voltage Divider**



- **Shortcomings:**
  - Efficiency is awfully low (and variable)
  - Output voltage can not be easily changed or regulated.
  - Loading of output will change the voltage.

Realization of a DC-DC Converter: Solution #2

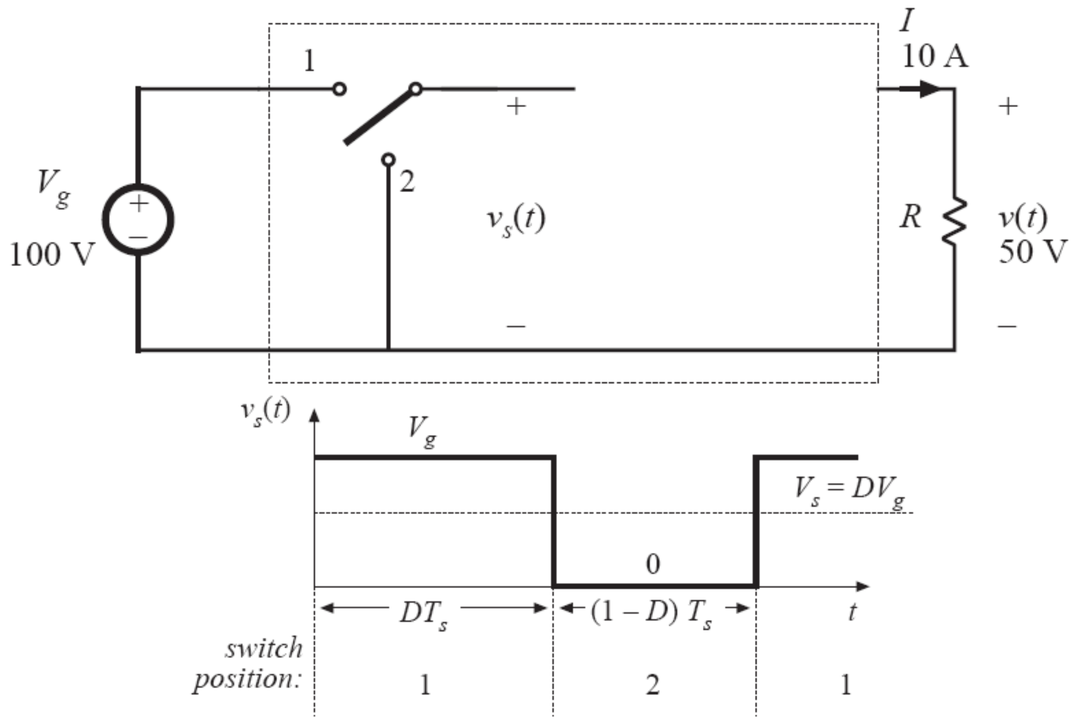
- **Transistor in Active Region**



- **Shortcomings:**
  - Efficiency is awfully low (and variable)
- **Solved these shortcomings of the previous solution:**
  - Output voltage can not be easily changed or regulated - SOLVED
  - Loading of output will change the voltage - SOLVED

Realization of a DC-DC Converter: Solution #3

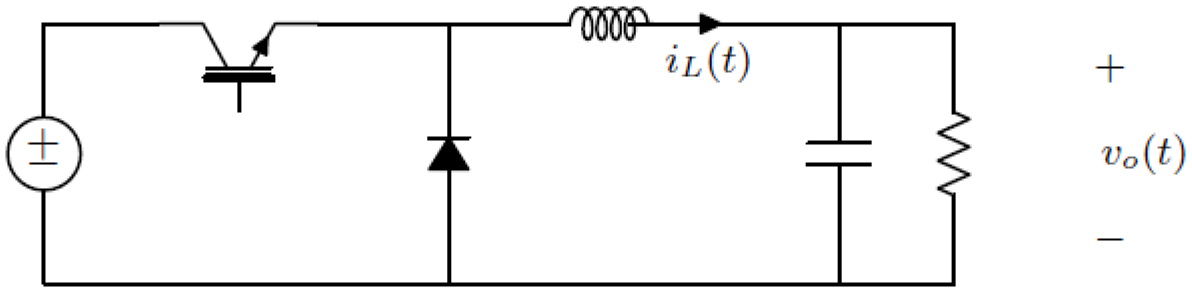
- **Power Electronics**



- **Shortcoming**
  - Chopped voltage, need to look at the average value
- **Solved these shortcomings of the previous solution:**
  - Efficiency is awfully low (and variable) - SOLVED
  - Output voltage can not be easily changed or regulated - SOLVED
  - Loading of output will change the voltage - SOLVED

Recap

- **Let's add a low-pass filter: an LC branch.**
- **While at it, let's also add a diode. Why?**

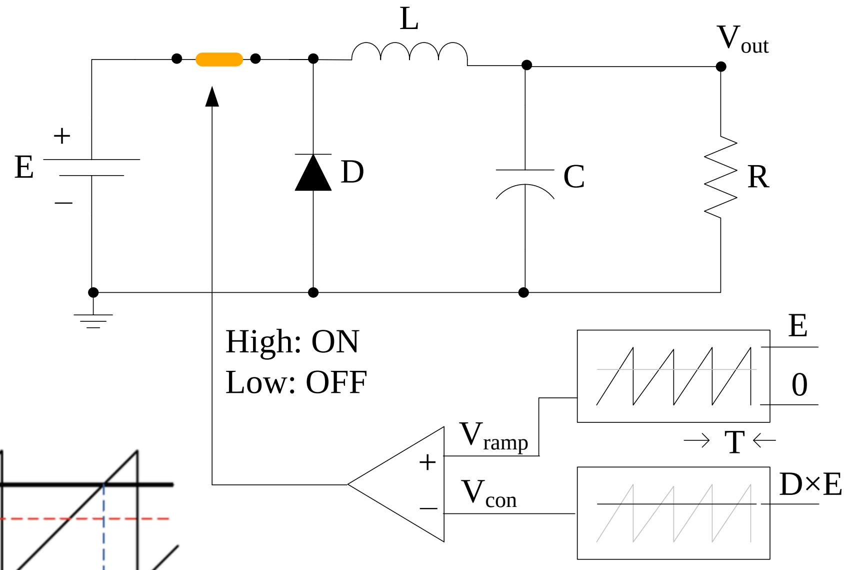


- **This is now called a buck converter.**

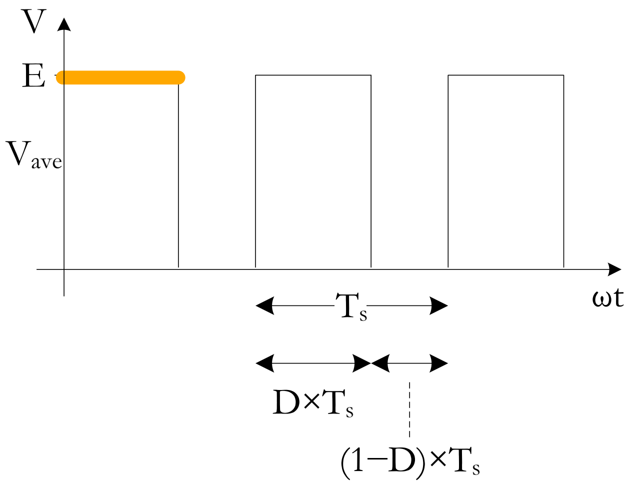
$$v_o(t) = d(t)V_s$$

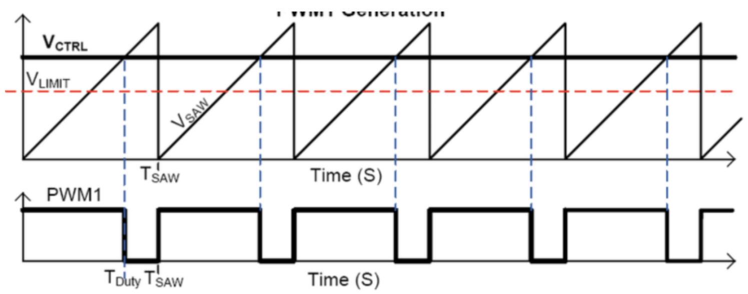
Time-Averaging of Voltages

- **The underlying idea is simple.**
- **Back to the simple idea of dc converters (Buck converter):**
  - There are only two voltage levels: 0 and *E*.
  - So we control the converter by applying the voltage *E* only for part of the sampling time.
    - On state



PWM



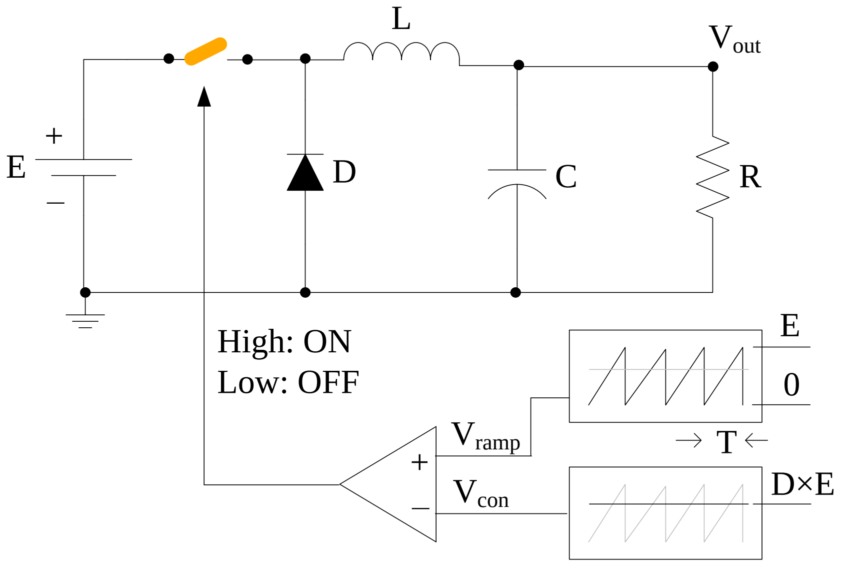


$$V_{\text{ave}} = \frac{T_{\text{ON}} \times E +}{T_s} = .$$

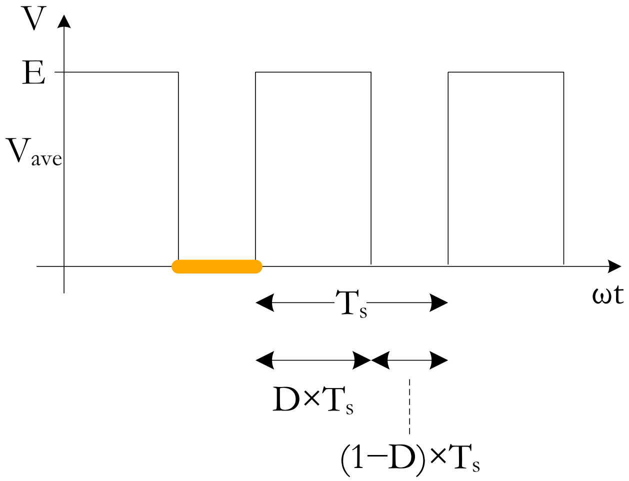
Master's Thesis Presentation, University of Manitoba, Winnipeg, MB; August 7, 2007.

Time-Averaging of Voltages

- **The underlying idea is simple.**
- **Back to the simple idea of dc converters (Buck converter):**
  - There are only two voltage levels: 0 and *E*.
  - So we control the converter by applying the voltage *E* only for part of the sampling time.
    - On state
    - Off state



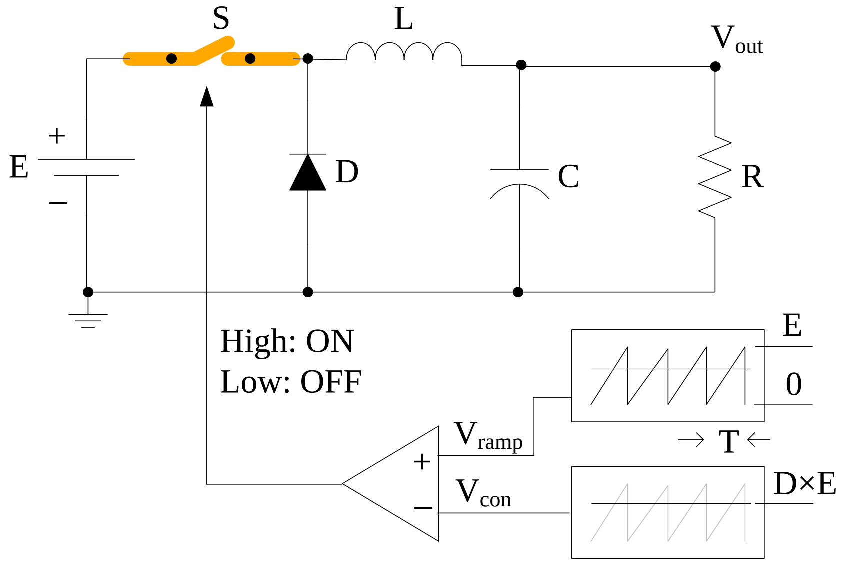
PWM



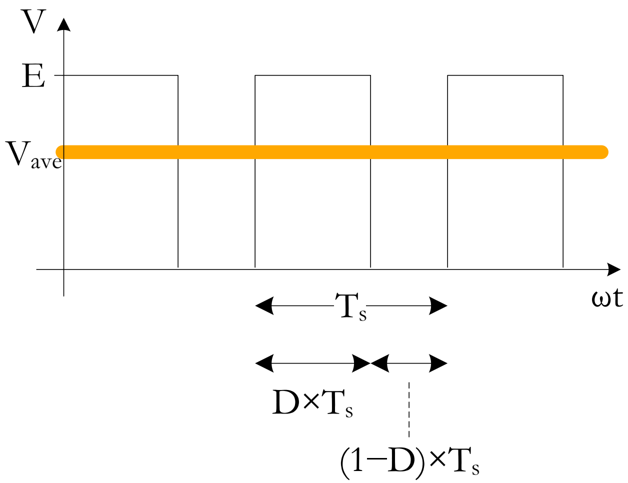
$$V_{\text{ave}} = \frac{+ T_{\text{OFF}} \times 0}{T_s} =$$

Time-Averaging of Voltages

- **The underlying idea is simple (whew!)**
- **Back to the simple idea of dc converters (Buck converter):**
  - There are only two voltage levels: 0 and *E*.
  - So we control the converter by applying the voltage *E* only for part of the sampling time.
    - On state
    - Off state
    - Average



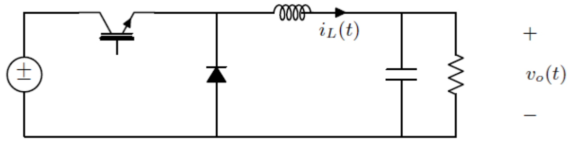
PWM

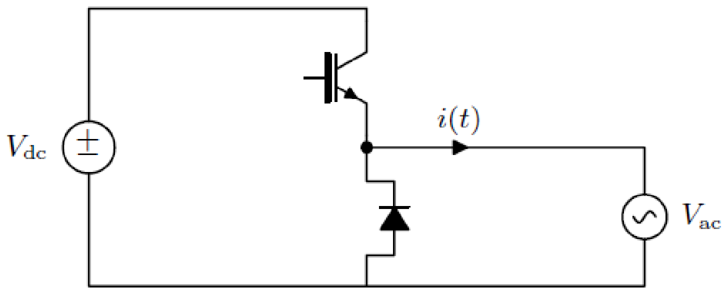


$$V_{\text{ave}} = \frac{T_{\text{ON}} \times E + T_{\text{OFF}} \times 0}{T_s} = D \times E$$

But this is (variable) DC. How do we get AC?

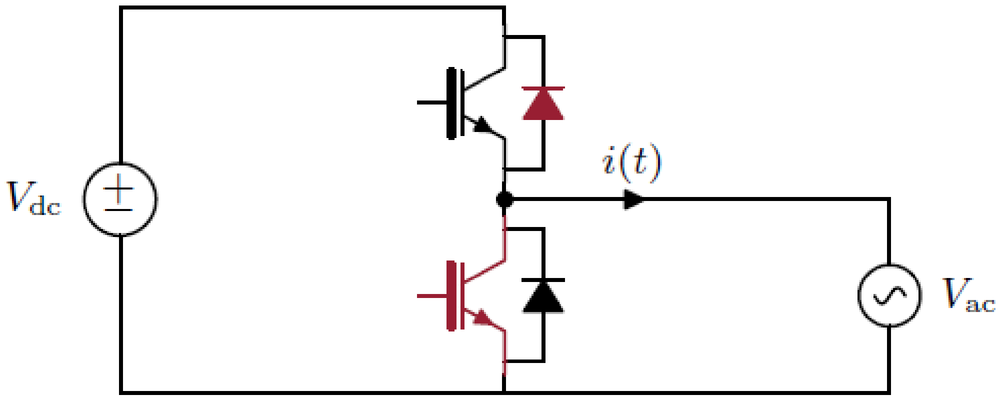
- **Redraw**



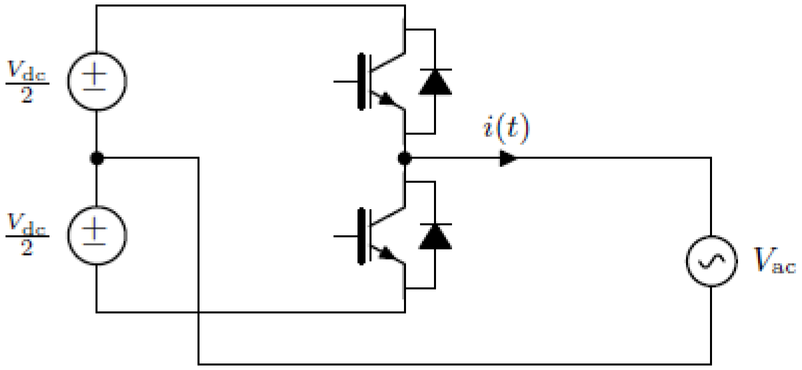


  - We need +/- current and voltage

- **Bidirectional Current**

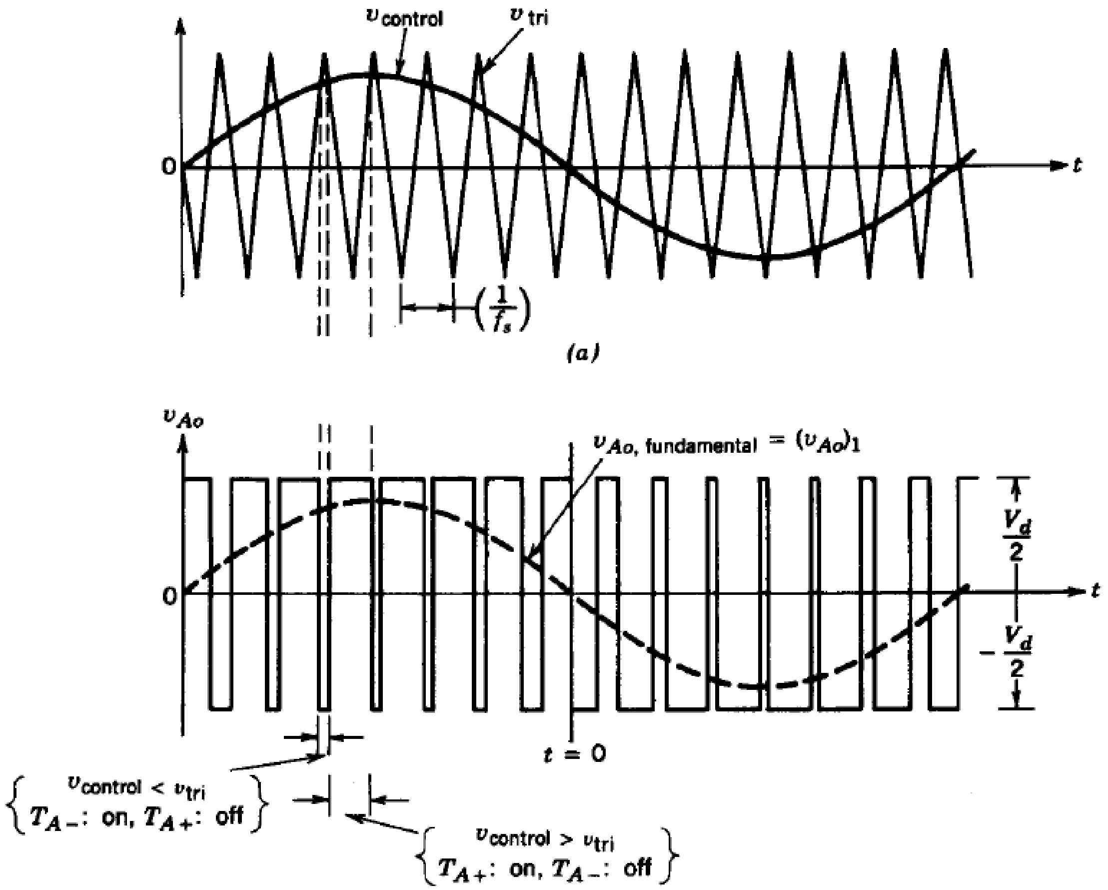


- **Bidirectional Voltage**

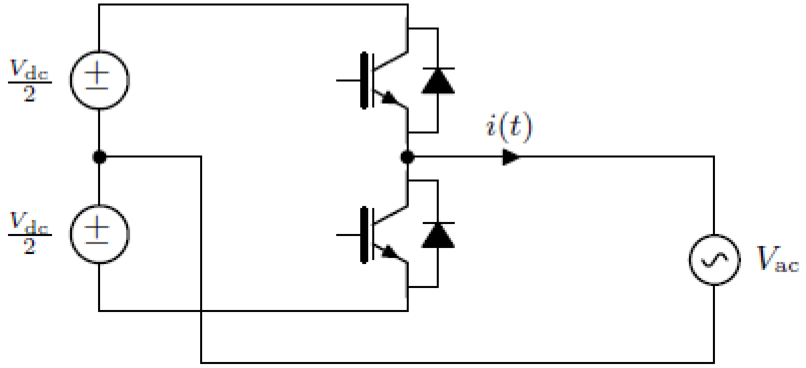


AC Waveform Generation

- **To generate a sinusoidal output, change duty cycle: D→d(t)**
- **Pulse-width modulation (PWM)**
  - Compare the reference (control) waveform (sinusoidal) to a triangular waveform of much higher frequency.
  - Not the only method: Space vector modulation (SVM), Selective harmonic elimination (SHE), Square wave, PWM+3rd



Half-Bridge Converter



- **AC voltage, on average, is**

$$V_o = d(t)\frac{V_{dc}}{2} + d'(t)\frac{(-V_{dc})}{2}$$

$$V_o = \frac{V_{dc}}{2}\left(d(t) - d'(t)\right) = (2d-1)\frac{V_{dc}}{2}$$

- **The relationship of the output voltage to the modulating signal m(t).**

$$m(t) = 2d(t) - 1$$

$$m(t) = m\cos(\omega_0 t + \theta_0)$$

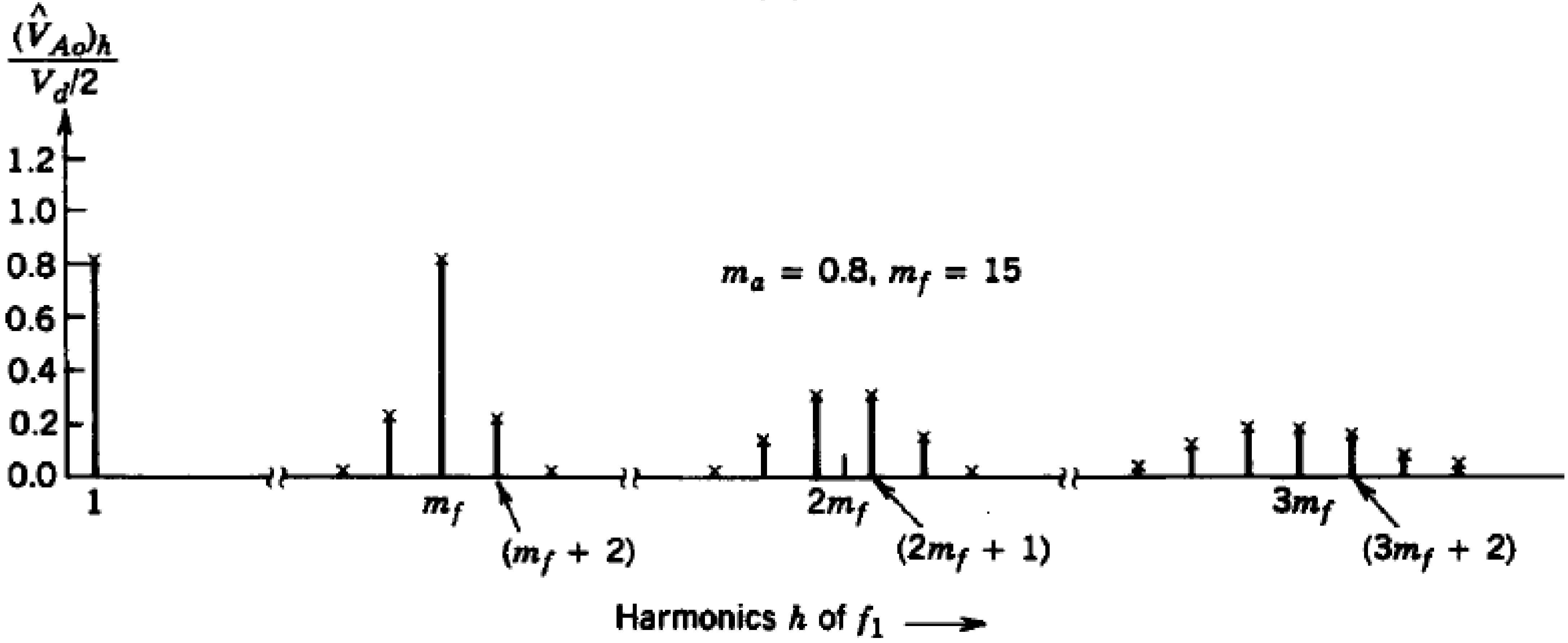
$$d(t) = \frac{1}{2} + \frac{1}{2}m\cos(\omega_0 t + \theta_0)$$

$$V_{o(t)} = \frac{m(t)V_s}{2} + harmonics$$

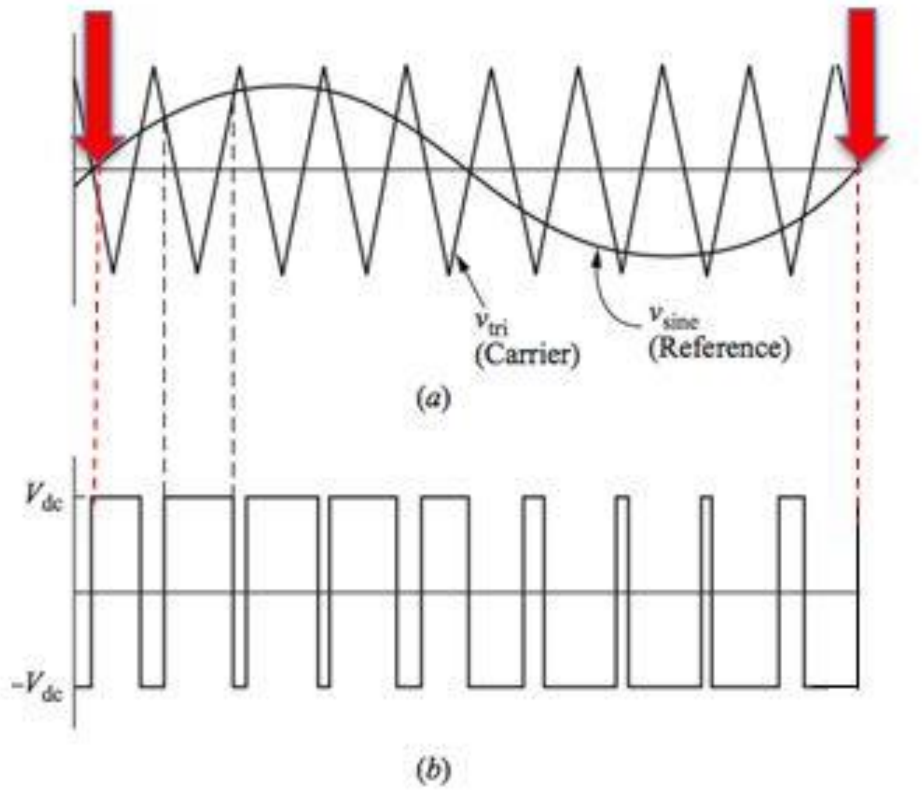
- **It is up to the switching strategy (e.g., PWM) to eliminate harmonics. Harmonics can also be reduced using external circuits (e.g., filters or transformer connection).**

VSC with PWM

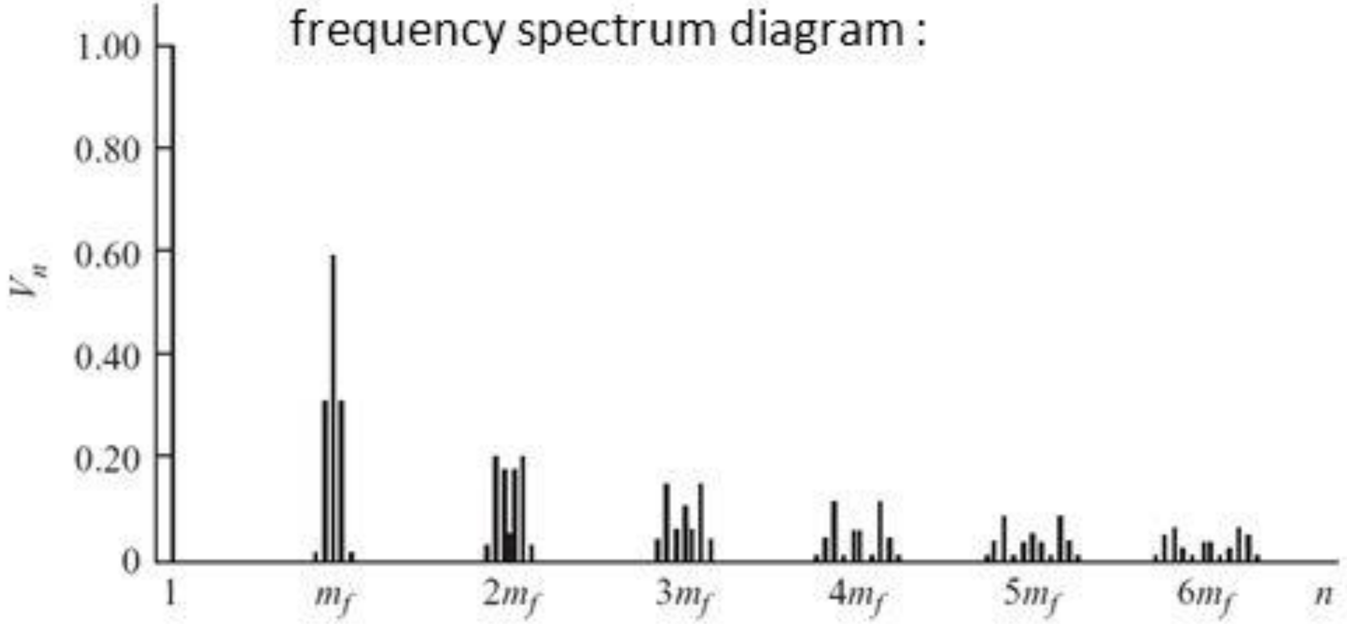
- **Harmonics**



Harmonics



Graphically, this can be represented using frequency spectrum diagram :



OR using a normalized Fourier coefficients table:

	m_a=1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$n=1$	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$n=m_f$	0.60	0.71	0.82	0.92	1.01	1.08	1.15	1.20	1.24	1.27
$n=mf\pm2$	0.32	0.27	0.22	0.17	0.13	0.09	0.06	0.03	0.02	0.00

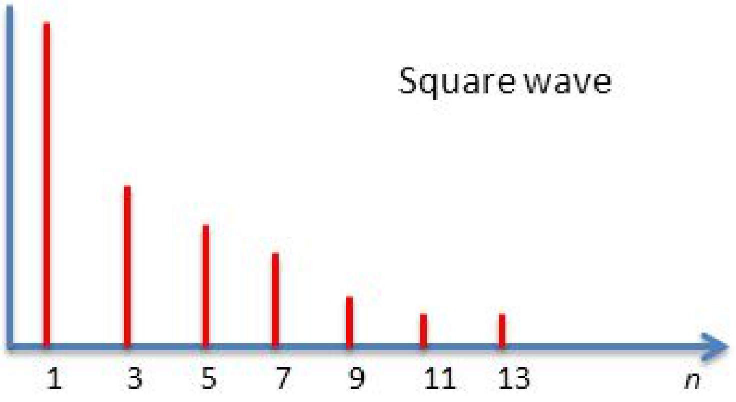
Looking Back: Why PWM?

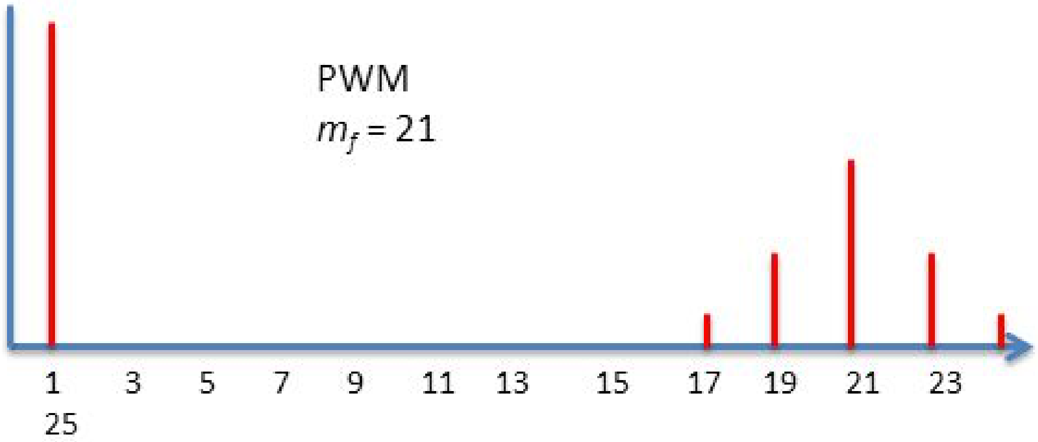
SQUARE-WAVE

- Contains harmonics at relatively low frequency: 3rd, 5th, 7th, 9th, etc.
- In order to improve the THD_V , a low pass filter can be employed → filter will be bulky since cutoff frequency is low → difficult to remove harmonics since at the same time must ensure fundamental component is not attenuated.

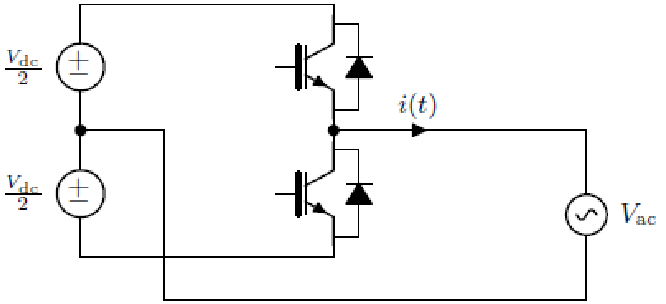
PWM

- Harmonics appear around m_f which is further away from fundamental.
- To improve THDV, filter with higher cutoff can be used → smaller in size → easier to filter out harmonics.

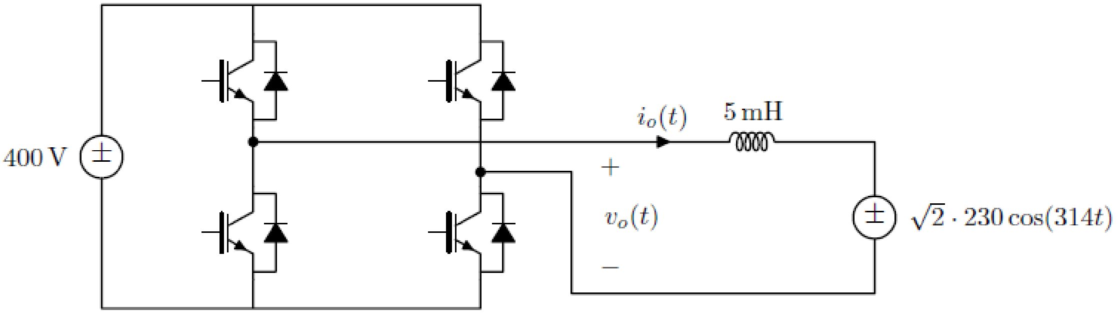


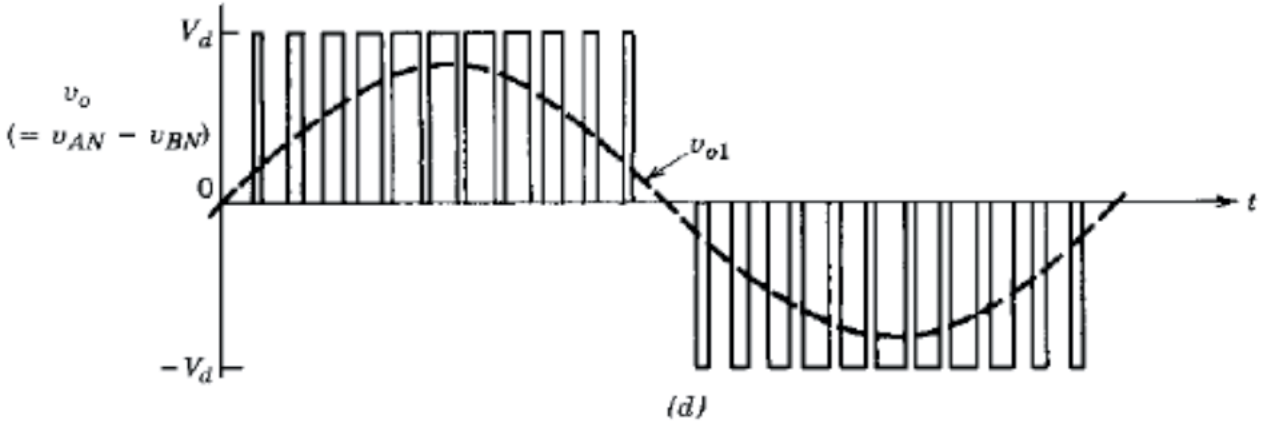


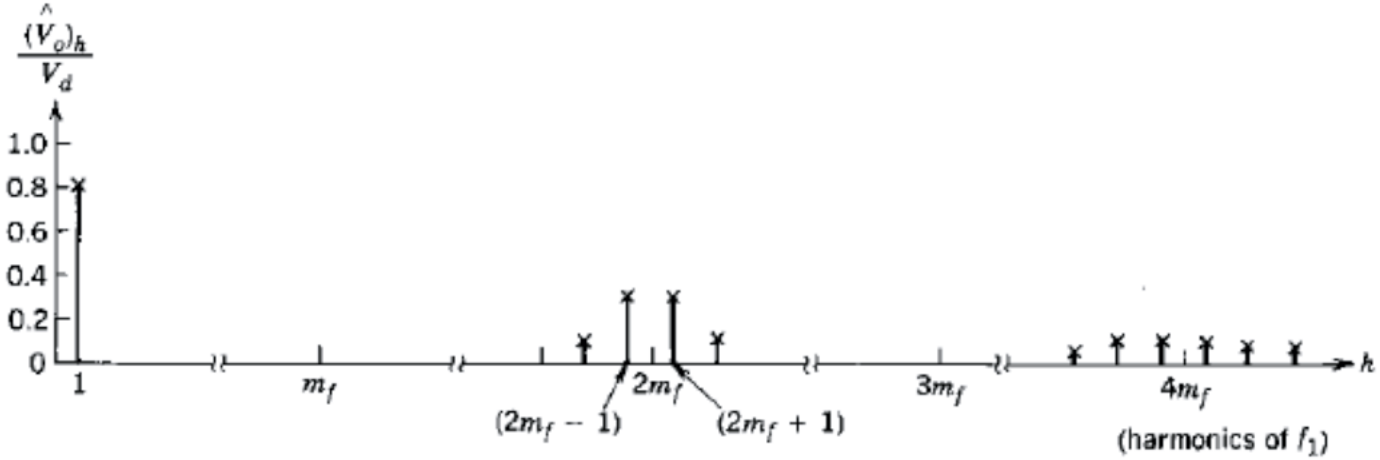
Adding More Legs (That is, Phases)



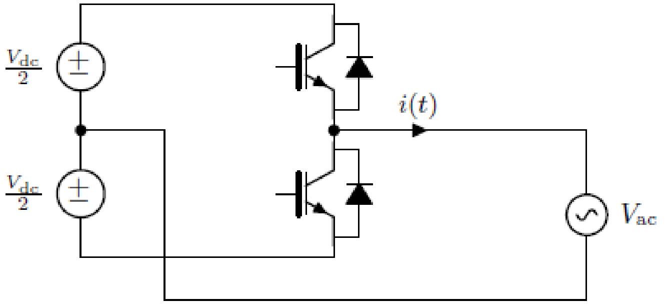
- **Two Legs: Full Bridge**
  - No need to the DC midpoint
  - Twice the voltage
  - Less harmonics (at $2m_f$, $4m_f$, etc)



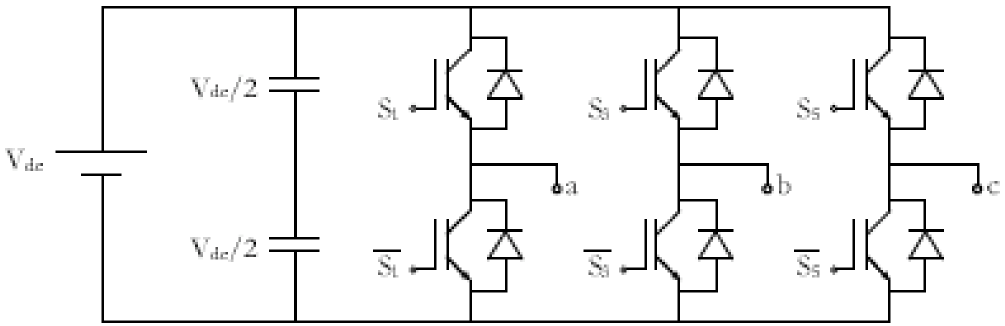


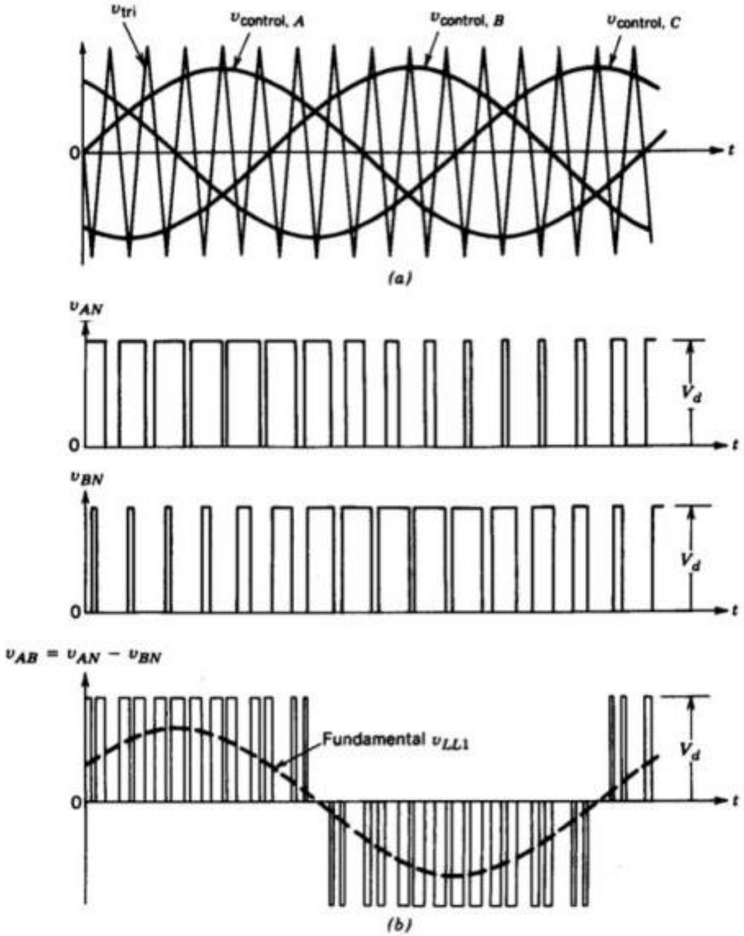


Adding More Legs (That is, Phases)



- **Three Legs: Voltage-Sourced Converter**





m_f is chosen to be multiple of 3 so that the harmonic at multiple of 3, including m_f (and its multiple) are suppressed (or canceled out) in the line-line voltage

h \ m_a	0.2	0.4	0.6	0.8	1.0
1	0.122	0.245	0.367	0.490	0.612
$m_f \pm 2$	0.010	0.037	0.080	0.135	0.195
$m_f \pm 4$				0.005	0.011
$2m_f \pm 1$	0.116	0.200	0.227	0.192	0.111
$2m_f \pm 5$				0.008	0.020
$3m_f \pm 2$	0.027	0.085	0.124	0.108	0.038
$3m_f \pm 4$		0.007	0.029	0.064	0.096
$4m_f \pm 1$	0.100	0.096	0.005	0.064	0.042
$4m_f \pm 5$			0.021	0.051	0.073
$4m_f \pm 7$				0.010	0.030

Note: $(V_{LL})_h/V_d$ are tabulated as a function of m_a where $(V_{LL})_h$ are the rms values of the harmonic voltages.

Modulation Methods: Carrier-Based PWM

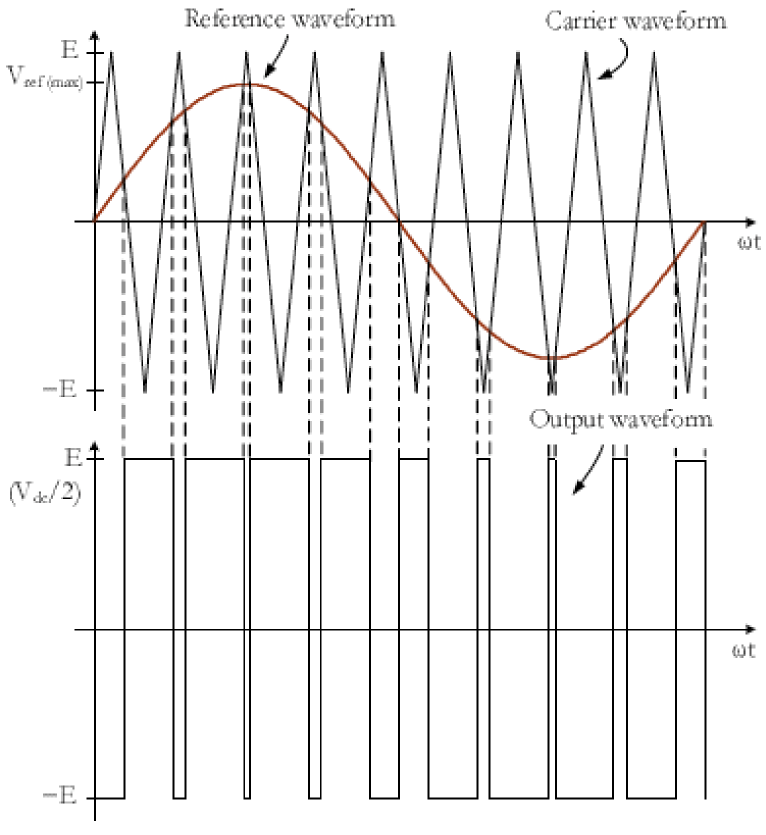


Fig. 3.1. Output voltage generated by two-level SPWM.

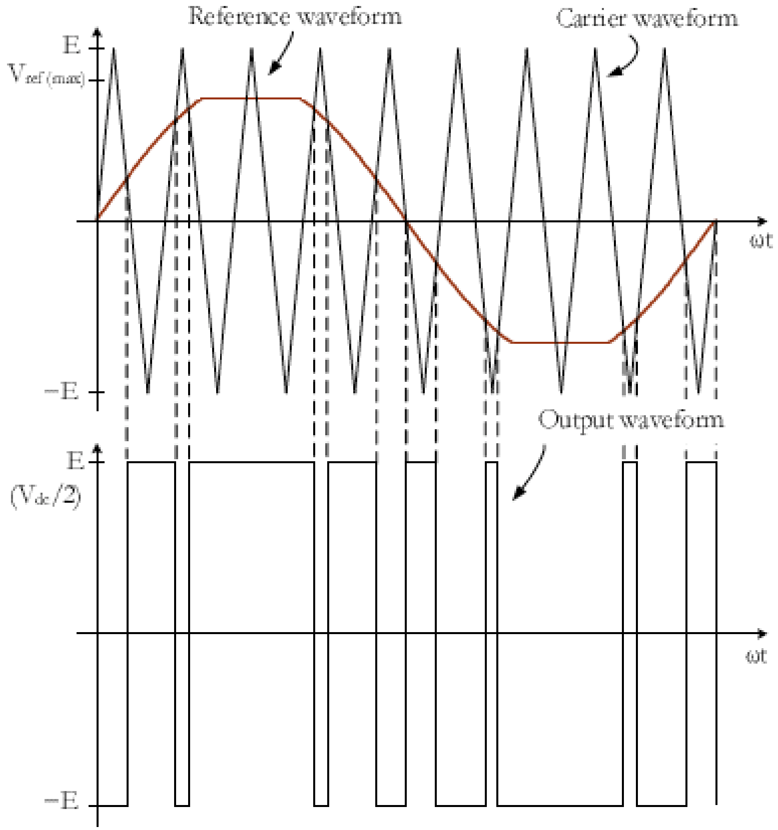


Fig. 3.3. 60-degree modulation.

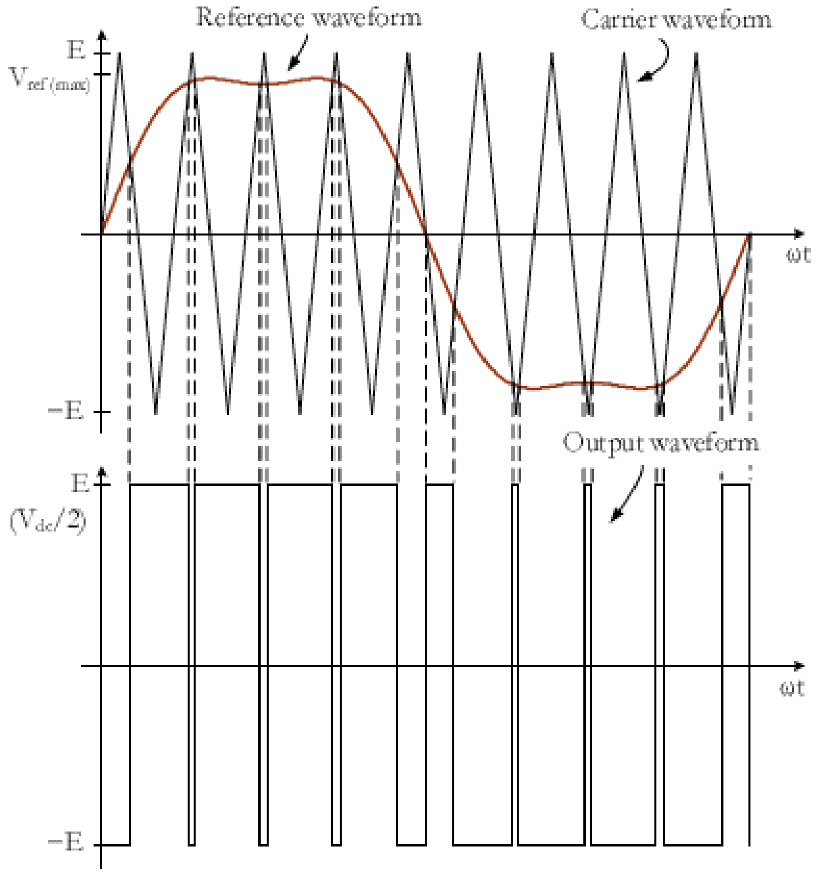


Fig. 3.5. Third-harmonic injection PWM.

Modulation: Selective Harmonic Elimination

- **Very high-power applications**

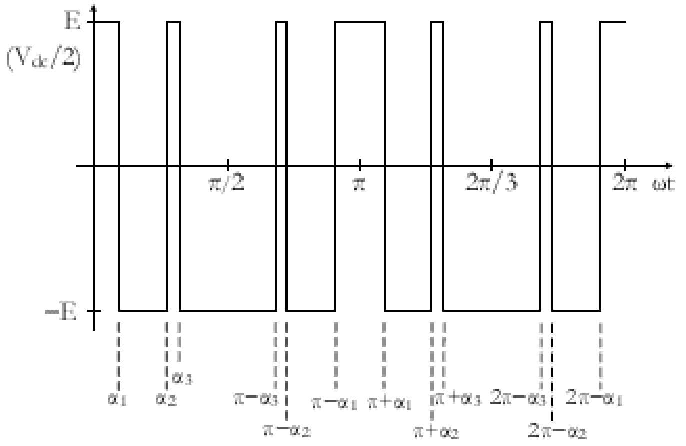
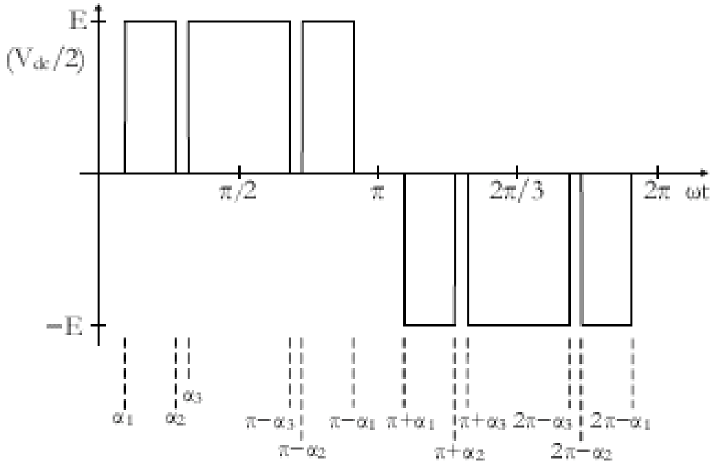


Fig. 3.7. Output voltage of a two-level, three–switching-angle SHE.

$$v(\quad v(t) = \frac{2V_{dc}}{\pi}\Big\{(1 - 2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3 + \cdots)\sin\omega t$$
$$+ (1 - 2\cos 3\alpha_1 + 2\cos 3\alpha_2 - 2\cos 3\alpha_3 + \cdots)\frac{\sin 3\omega t}{3}$$
$$+ (1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 - 2\cos 5\alpha_3 + \cdots)\frac{\sin 5\omega t}{5} + \cdots\Big\}$$
$$= \frac{2V_{dc}}{\pi}\left\{\sum_{n=1,3,\ldots}^{\infty}\frac{\sin n\omega t}{n}\left[1 + 2\sum_{k=1}^{m}(-1)^k\cos n\alpha_k\right]\right\}$$

$$v_{1,\mathrm{rms}} = \frac{\sqrt{2}V_{dc}}{\pi}(1 - 2\cos\alpha_1 + 2\cos\alpha_2 - 2\cos\alpha_3)$$
$$0 = (1 - 2\cos 5\alpha_1 + 2\cos 5\alpha_2 - 2\cos 5\alpha_3)$$
$$0 = (1 - 2\cos 7\alpha_1 + 2\cos 7\alpha_2 - 2\cos 7\alpha_3)$$

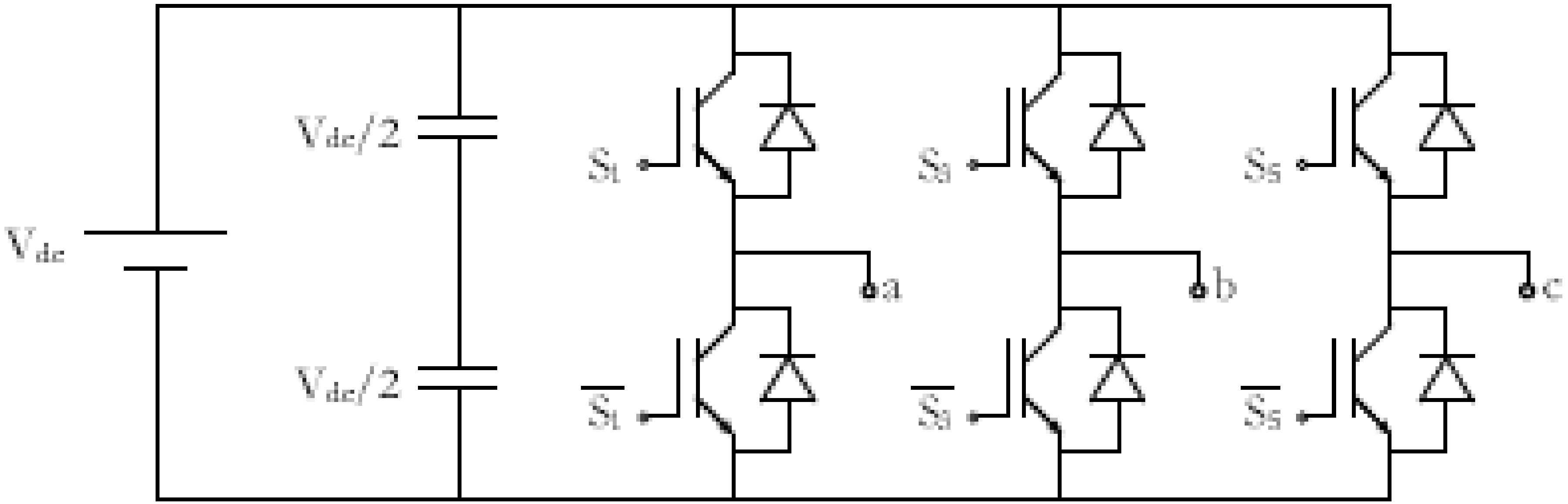


$$v(t) = \frac{2V_{dc}}{\pi}\Big\{(\cos\alpha_1 - \cos\alpha_2 + \cos\alpha_3 + \cdots)\sin\omega t$$
$$+ (\cos 3\alpha_1 - \cos 3\alpha_2 + \cos 3\alpha_3 + \cdots)\frac{\sin 3\omega t}{3}$$
$$+ (\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3 + \cdots)\frac{\sin 5\omega t}{5} + \cdots\Big\}$$
$$= \frac{2V_{dc}}{\pi}\left\{\sum_{n=1,3,\ldots}^{\infty}\frac{\sin n\omega t}{n}\left[\sum_{k=1}^{m}(-1)^{k+1}\cos n\alpha_k\right]\right\}$$

$$v_{1,\mathrm{rms}} = \frac{\sqrt{2}V_{dc}}{\pi}(\cos\alpha_1 - \cos\alpha_2 + \cos\alpha_3)$$
$$0 = (\cos 5\alpha_1 - \cos 5\alpha_2 + \cos 5\alpha_3)$$
$$0 = (\cos 7\alpha_1 - \cos 7\alpha_2 + \cos 7\alpha_3)$$

Modulation Methods: Space Vector Modulation

- **There are 8 converter states depending on the status of switches. No two switches can be on or off at the same time in each leg.**

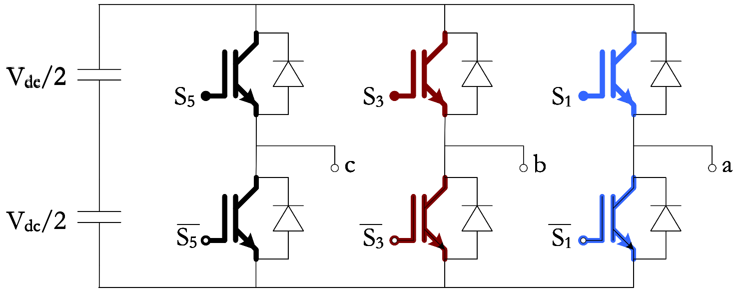


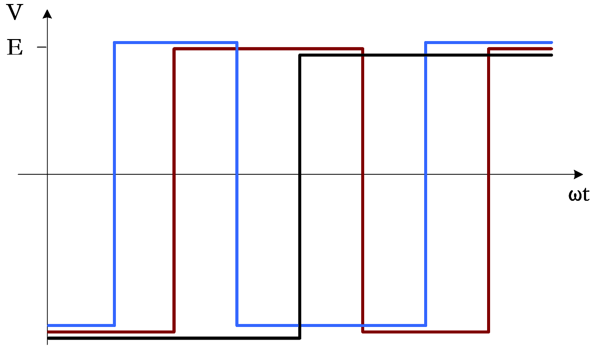
$$\mathbf{V} = \frac{2}{3}\left(v_a(t) + v_b(t)e^{j2\pi/3} + v_c(t)e^{-j2\pi/3}\right)$$

Time-Averaging of Voltages (4/6)

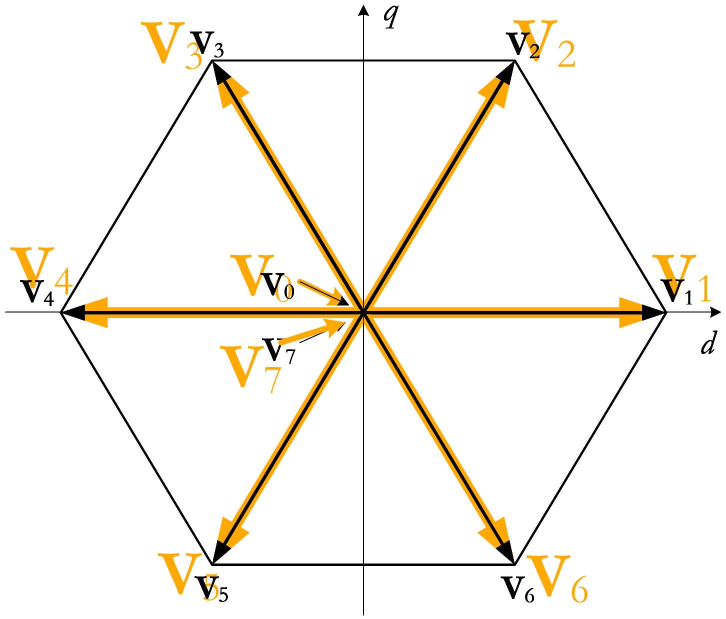
- **The same states, now also on the hexagon:**
  - zero

, one, two, three, four, five, six, seven





cba							
000	001	011	010	110	100	101	111
0	1	2	3	4	5	6	7



hide SVs

Construction of Output Voltages

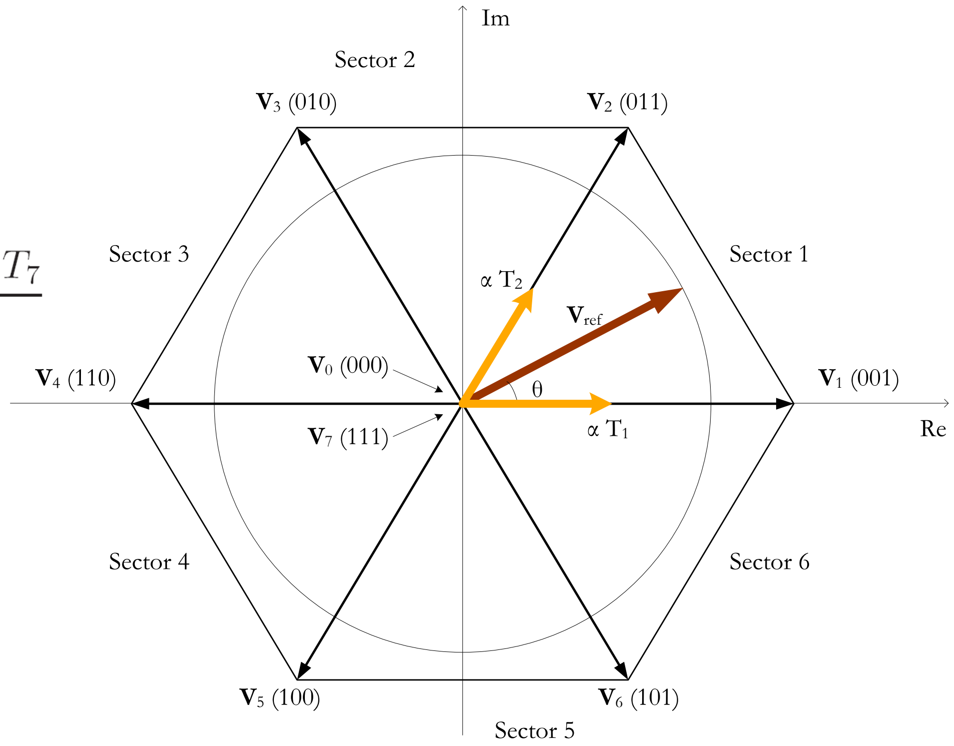
- **The reference vector, in sector 1, can be projected onto $\mathbf{V}_1$ and $\mathbf{V}_2$.**
  – If the converter is placed in $\mathbf{V}_1$ state for T_1 and in $\mathbf{V}_2$ for T_2, the average voltage over the sampling period T_s can be found from

$$\mathbf{V} = \frac{2}{3}\left(v_a(t) + v_b(t)e^{j2\pi/3} + v_c(t)e^{-j2\pi/3}\right) \qquad V_{ref} = \frac{\mathbf{Z}_0 \times T_0 + \mathbf{A}_1 \times T_1 + \mathbf{A}_2 \times T_2 + \mathbf{Z}_7 \times T_7}{T_s}$$

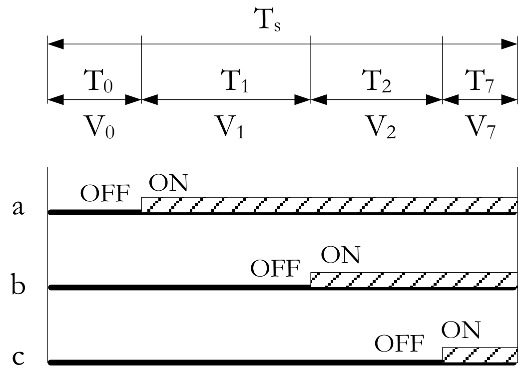
$$T_1 = \frac{\sqrt{3}}{2} \times T_s m \sin(\pi/3 - \theta)$$

$$T_2 = \frac{\sqrt{3}}{2} \times T_s m \sin(\theta)$$

$$T_0 + T_7 = T_z = T_s - T_1 - T_2$$



  – Zero vectors do not change the average value,
    - The rest of T_s is filled with them.
  – Both $\mathbf{V}_0$ and $\mathbf{V}_7$ generate zero line voltages,
    - So both can be used interchangeably and in any order.



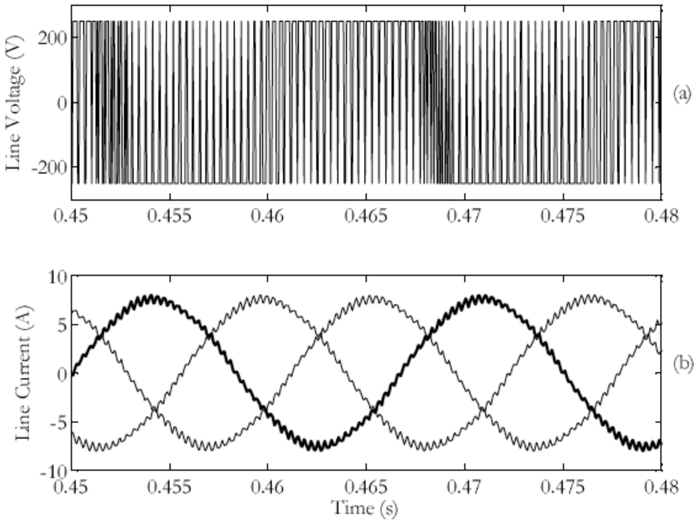


Fig. 5.3. Line voltage and currents of the two-level space-vector modulator.

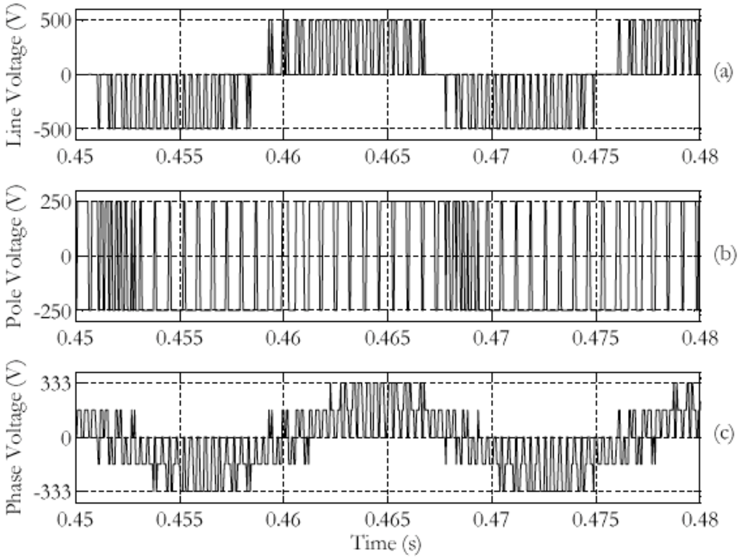
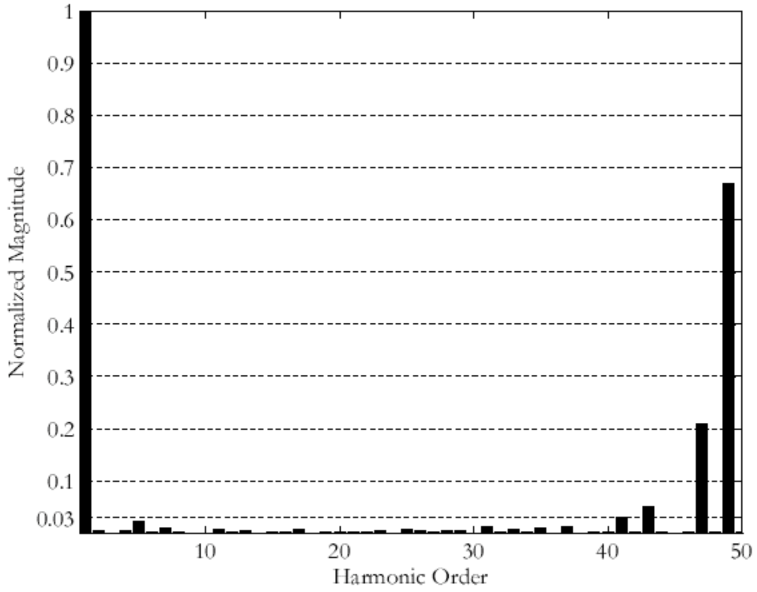


Fig. 5.4. Generated (a) line voltage, (b) pole voltage, and (c) phase voltage, $F_{sn} = 48$ and $m = 0.8$.



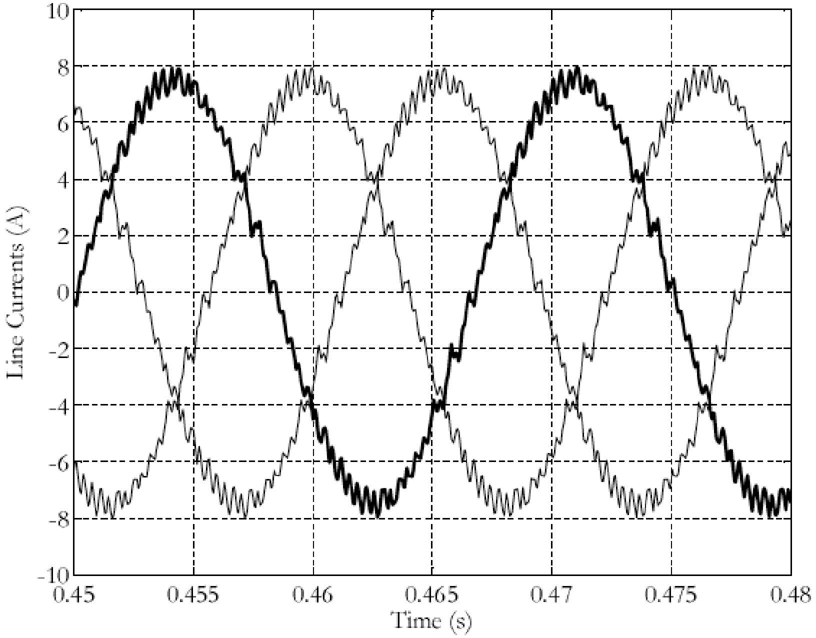


Fig. 5.6. Line currents, $F_{sn} = 48$ and $m = 0.8$.

Bigger Picture

- **Buck-boost--ness is in the eye of the beholder.**
  - Converters may be built to be "reversible." (bidirectional flow of power)
- **An inverter (DC-AC) is essentially a DC-DC converter with a slowly varying duty cycle.**
  - Variations of $d(t)$ should be much slower than the switching frequency.
- **An AC voltage controller (AC-AC) is essentially a rectifier (AC-DC) with slowly varying delay angle α.**
  - Variations of $\alpha(t)$ should be much slower than input frequency.
- **Ultimately, DC-DC and AC-AC can be thought of as controllable transformers.**
  - Can change effective resistance (impedance) seen by the source (utility).

Module 1c: Power Electronics 101

Fundamentals of Switching Power Converters

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HVDC Learn

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Bradley Department of ECE

POWER AND ENERGY CENTER

Resilient Renewable Energy Grid Adaptation Laboratory (REGAL)

This work is supported by the U.S. Department of Energy.

Waveform Generation

- **SVM can put the inverter in one of the 8 states, each giving a certain voltage space vector.**
- **The reference voltage (va, vb, vc) can also be expressed as an SV based on**

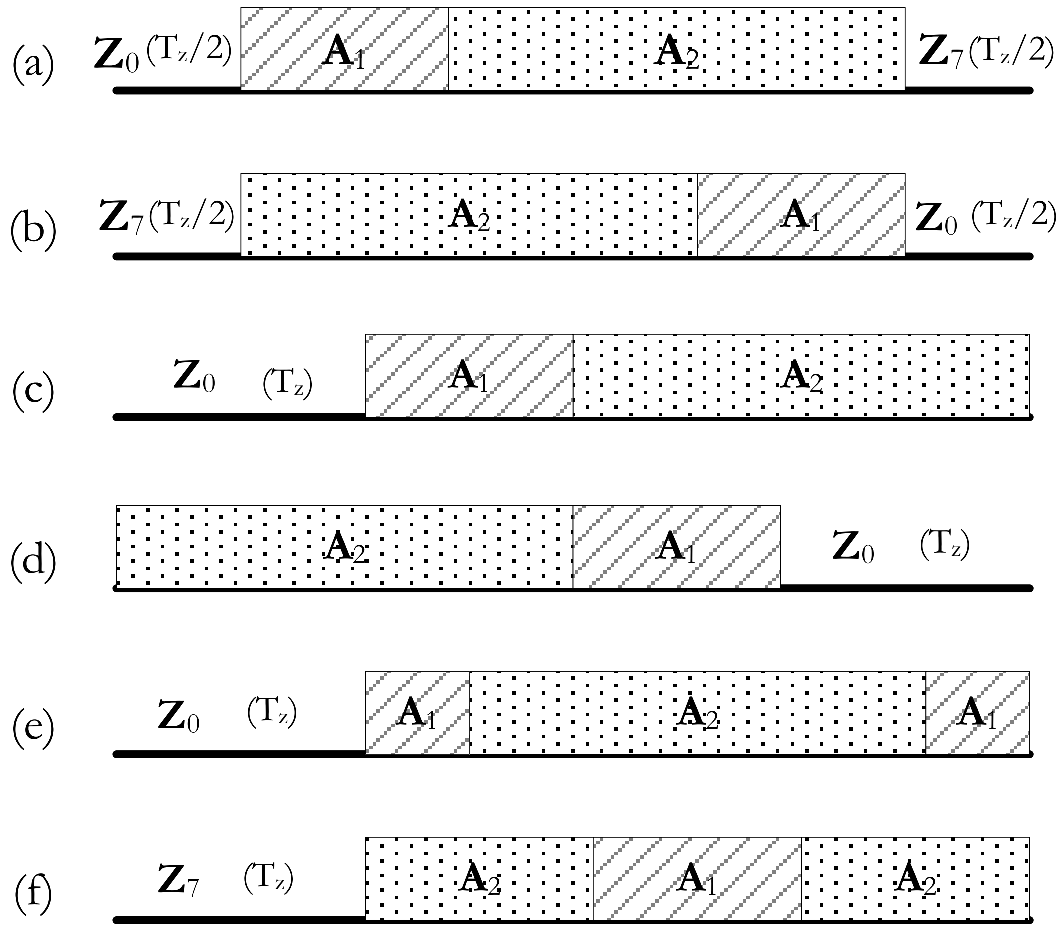
$$\mathbf{V} = \frac{2}{3}\left(v_a(t) + v_b(t)e^{j2\pi/3} + v_c(t)e^{-j2\pi/3}\right)$$

- **The idea is now simple. For each sampled value of the reference voltage, we try to find out how we can reconstruct it based on the 8 (7) SVs. This is a simple decomposition problem.**

Degrees of Freedom

- **Individual shares of Z0, Z7**
- **Placement of SVs within the sampling period**

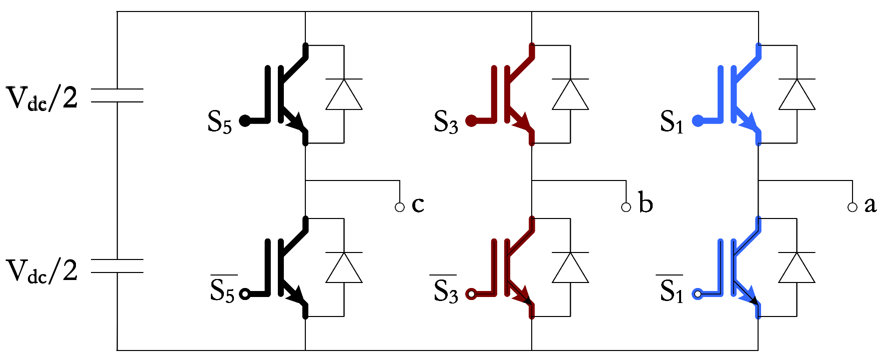
- **These can be decided based on the different harmonic performance requirements, number of switchings allowed, and losses.**

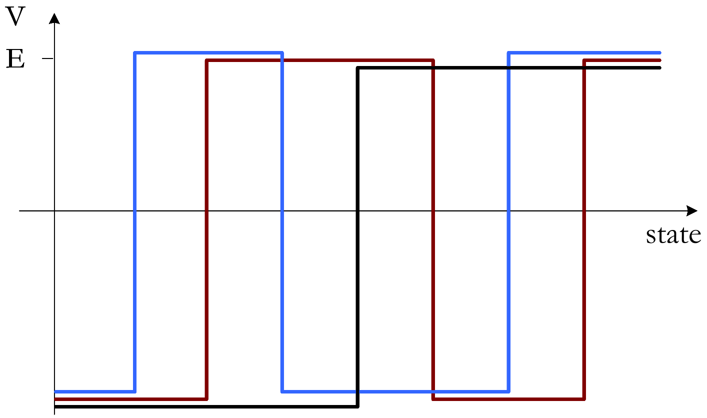


Time-Averaging of Voltages (2/6)

- **Applying the same idea for ac converters (three-phase):**
  - Likewise, the number of combinations of switching states of the VSC is limited.
  - In a two-level VSC, there are three “independent” switches, hence $2^3 = 8$ states.
    - zero

, one, two, three, four, five, six, seven





cba							
000	001	011	010	110	100	101	111
0	1	2	3	4	5	6	7

Time-Averaging of Voltages (3/6)

- **Now, we define a reference vector, through one-to-one transformation of the individual phase voltages**

$$\mathbf{V} = \frac{2}{3}\left(v_{\mathrm{a}}(t) + v_{\mathrm{b}}(t)e^{j2\pi/3} + v_{\mathrm{c}}(t)e^{-j2\pi/3}\right)$$

  - This reference vector uniquely determines the desired three-phase waveforms.
  - Therefore, we don’t need the three-phase reference waveforms we started with.

- **This leaves us with a graphical representation of the system**
  - A hexagon, with the 6 states of the system (active) as its vertices, and two zero states in the origin.

EIGHT SPACE-VECTORS AND THEIR CORRESPONDING PHASE- AND LINE-VOLTAGES

State	c, b, a	**V**	Pole Voltage, multiply by $V_{dc}/2$			Phase Voltage		
			V_{az}	V_{bz}	V_{cz}	V_{an}	V_{bn}	V_{cn}
0	**000**	0	−1	−1	−1	0	0	0
1	**001**	$2V_{dc}/3e^{j0}$	1	−1	−1	$2V_{dc}/3$	$-V_{dc}/3$	$-V_{dc}/3$
2	**011**	$2V_{dc}/3e^{j\pi/3}$	1	1	−1	$V_{dc}/3$	$V_{dc}/3$	$-2V_{dc}/3$
3	**010**	$2V_{dc}/3e^{j2\pi/3}$	−1	1	−1	$-V_{dc}/3$	$2V_{dc}/3$	$-V_{dc}/3$
4	**110**	$2V_{dc}/3e^{j\pi}$	−1	1	1	$-2V_{dc}/3$	$V_{dc}/3$	$V_{dc}/3$
5	**100**	$2V_{dc}/3e^{j4\pi/3}$	−1	−1	1	$-V_{dc}/3$	$-V_{dc}/3$	$2V_{dc}/3$
6	**101**	$2V_{dc}/3e^{j5\pi/3}$	1	−1	1	$V_{dc}/3$	$-2V_{dc}/3$	$V_{dc}/3$
7	**111**	0	1	1	1	0	0	0

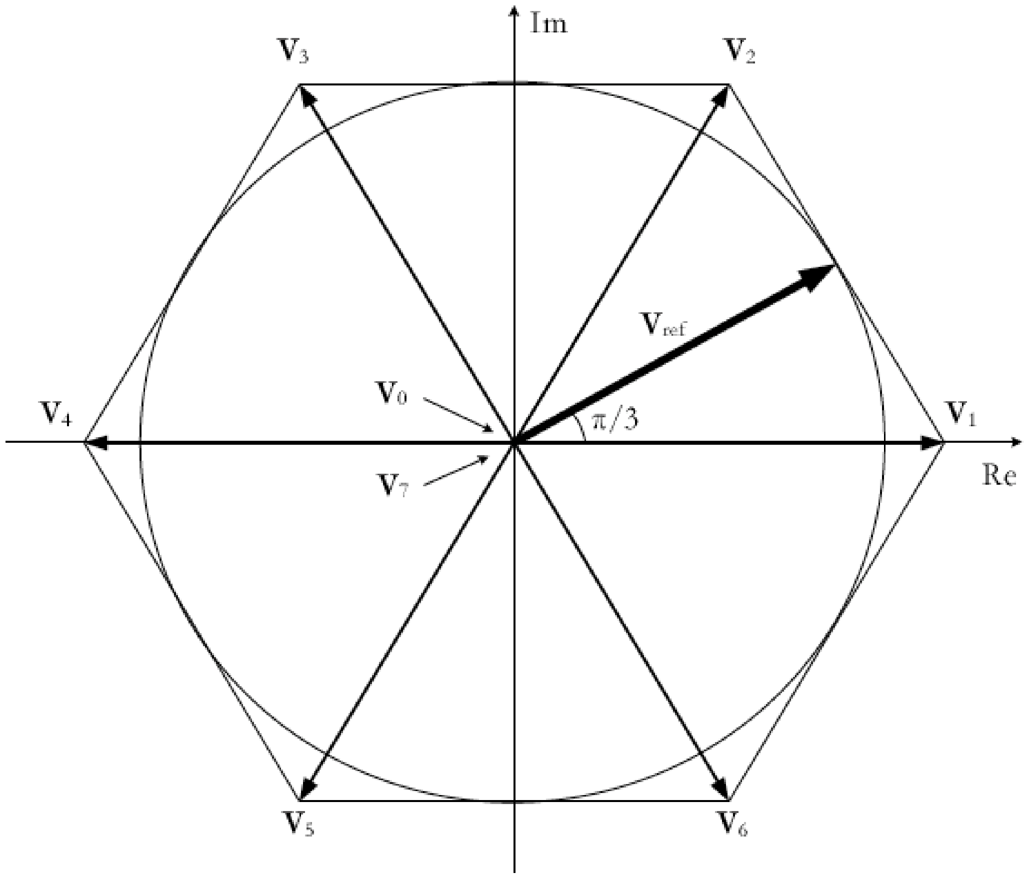
Algorithm for SVM

Algorithm 4.1 Summary of implementation of two-level SVM

1. Decide on the SVM **sequence** to be used.
2. Repeat, for each **sample** of the reference vector $\mathbf{V}_{\mathrm{ref}}$
   2.1. Find the **sector** in which $\mathbf{V}_{\mathrm{ref}}$ lies, and $\mathbf{A}_1$ and $\mathbf{A}_2$, based on θ.
   2.2. **Project** the reference vector to the first sector.
   2.3. Find **time shares** T_1, T_2, and T_z, based on m and θ.
   2.4. If $T_z > 0$, **share** T_z between T_0 for $\mathbf{Z}_0$ and T_7 of $\mathbf{Z}_7$, based on the sequence;
   otherwise, set $T_z = 0$ and adjust T_1 and T_2 based on the **overmodulation** strategy.
   2.5. **Place** the VSC in states $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{Z}_0$, and $\mathbf{Z}_7$ in the order determined by the chosen sequence.

Overmodulation

- **Maximum m is 1.15. Already 15% more than PWM.**



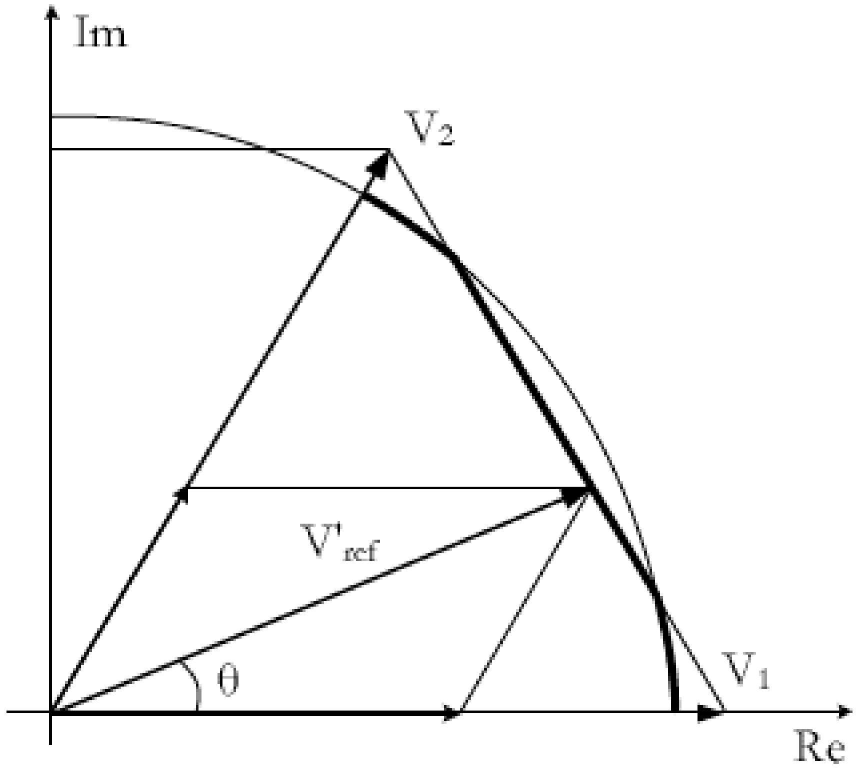


Fig. 4.5. Maximum length of the reference vector for $m = 1.15$.

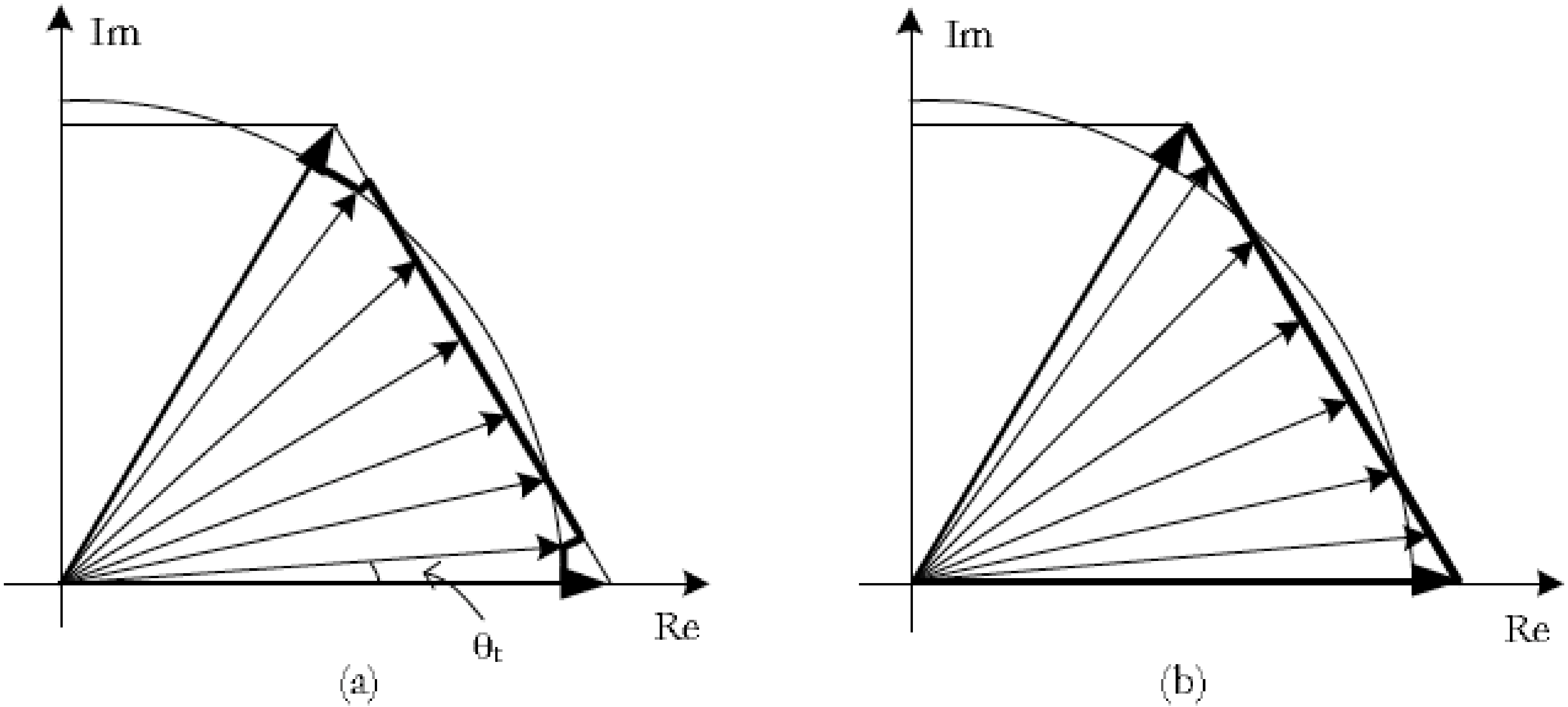


Fig. 4.8. Two-zone overmodulation approach, (a) zone I, and (b) zone II.

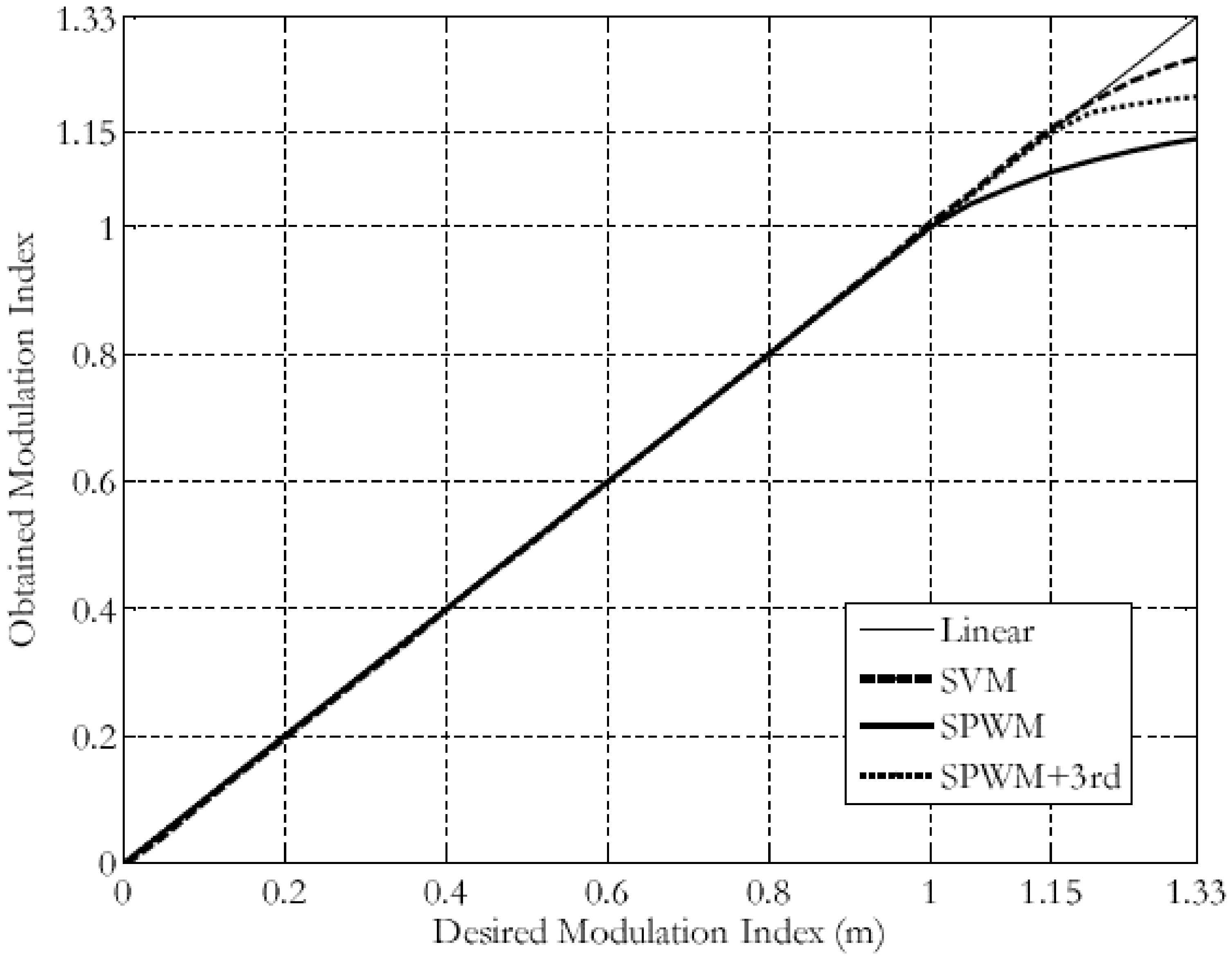
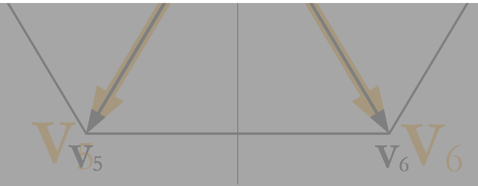


Fig. 4.7. Comparison of the degree of overmodulation in PWM methods.

Time-Averaging of Voltages (4/6)

- **The same states, now also on the hexagon:**
  - zero

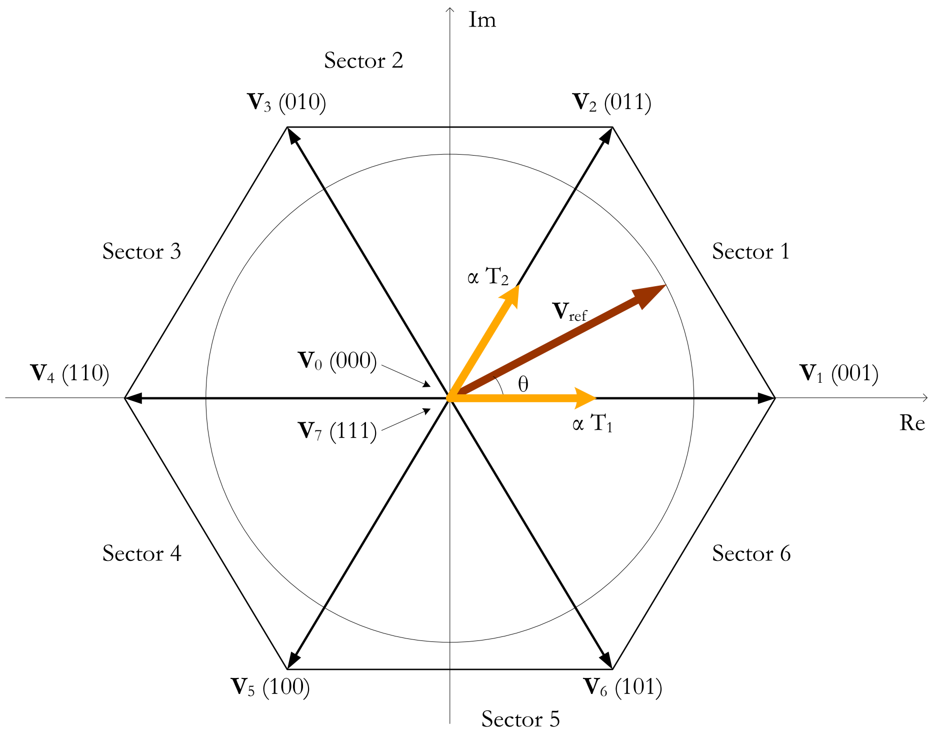
State	c, b, a	**V**	Pole Voltage, multiply by $V_{dc}/2$			Phase Voltage		
			V_{az}	V_{bz}	V_{cz}	V_{an}	V_{bn}	V_{cn}
0	**000**	0	-1	-1	-1	0	0	0
1	**001**	$2V_{dc}/3e^{j0}$	1	-1	-1	$2V_{dc}/3$	$-V_{dc}/3$	$-V_{dc}/3$
2	**011**	$2V_{dc}/3e^{j\pi/3}$	1	1	-1	$V_{dc}/3$	$V_{dc}/3$	$-2V_{dc}/3$
3	**010**	$2V_{dc}/3e^{j2\pi/3}$	-1	1	-1	$-V_{dc}/3$	$2V_{dc}/3$	$-V_{dc}/3$
4	**110**	$2V_{dc}/3e^{j\pi}$	-1	1	1	$-2V_{dc}/3$	$V_{dc}/3$	$V_{dc}/3$
5	**100**	$2V_{dc}/3e^{j4\pi/3}$	-1	-1	1	$-V_{dc}/3$	$-V_{dc}/3$	$2V_{dc}/3$
6	**101**	$2V_{dc}/3e^{j5\pi/3}$	1	-1	1	$V_{dc}/3$	$-2V_{dc}/3$	$V_{dc}/3$
7	**111**	0	1	1	1	0	0	0



hide SVs

Time-Averaging of Voltages (5/6)

- **Using the aforementioned transformation,**
  - The reference vector can be shown in the *dq*-plane; and
  - The reference vector **is rotated** in the plane with the desired output frequency;
    - (and virtually **sampled** with the sampling frequency).
  - Now, we can go back to the idea of time-averaging.



Time-Averaging of Voltages (6/6)

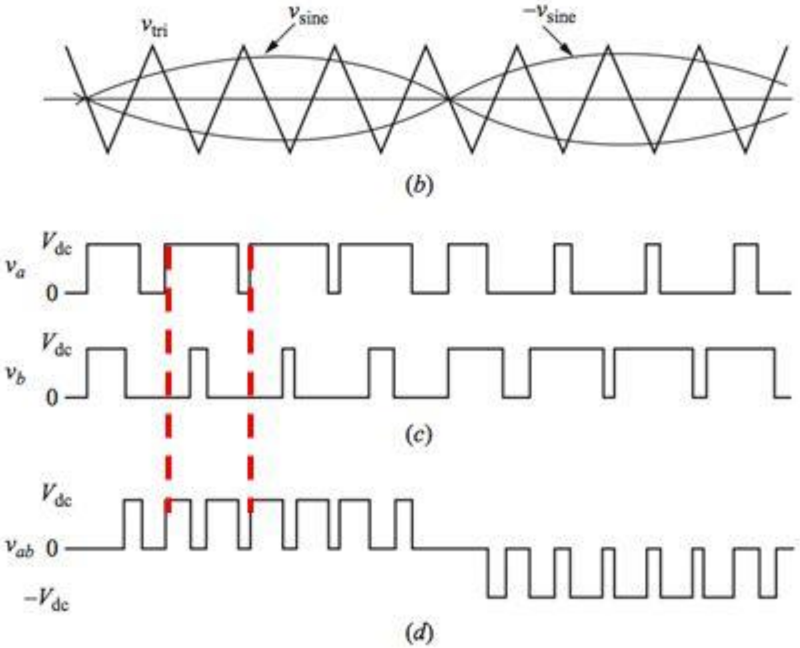
- **As the legs cannot be continuously switched on and off,**
  - This reference vector is sampled;
  - The “sampled” reference vector is only changed at each T_s (a new “virtual” sample is taken);
  - This is an approximation to the actual vector
    - and gets better as the sampling interval T_s gets smaller, at the expense of losses.

- **So in each T_s, we need to construct the reference vector,**
  - There are only 6 active states for the converter (just like voltages 0 and E in the Buck converter example);
  - The reference vector is synthesized by the adjacent space vectors.

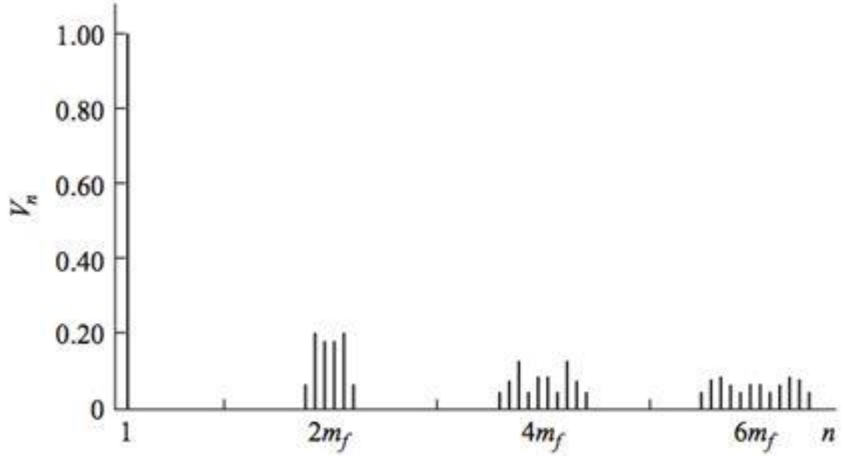
INVERTERS

Harmonics in PWM single-phase inverter : Unipolar switching scheme

- The frequency of the output voltage is doubled.
- If m_f is chosen as even integer then the first cluster of harmonics appear around $2m_f$ (the harmonic at $2m_f$ itself is zero)



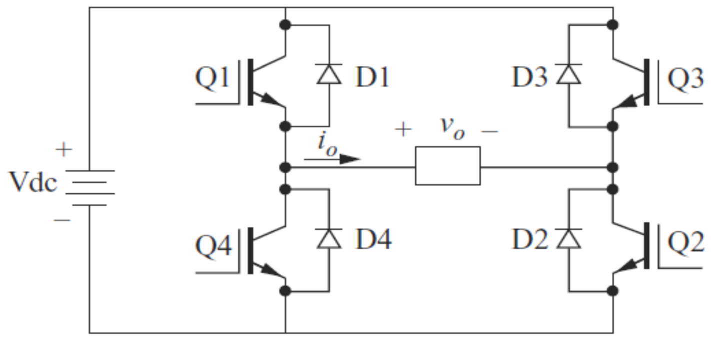
Graphically, this can be represented using frequency spectrum diagram :



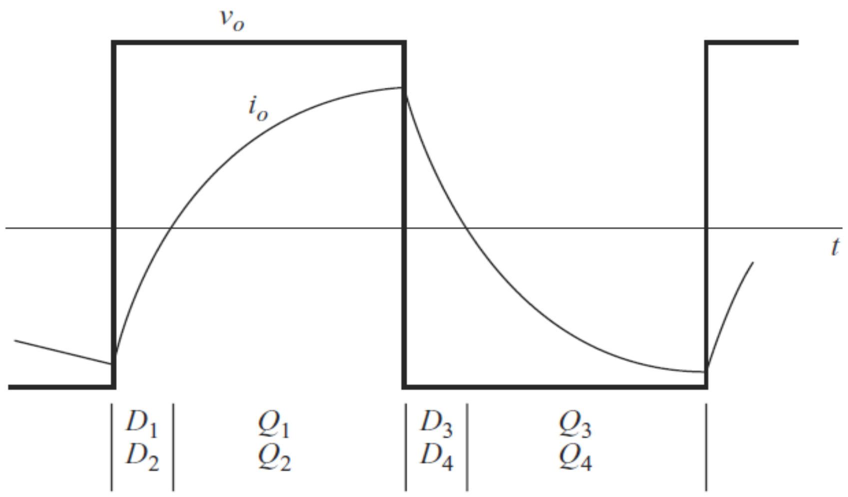
Or using a normalized Fourier coefficients table:

	m_a=1	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$n=1$	1.00	0.90	0.80	0.70	0.60	0.50	0.40	0.30	0.20	0.10
$n=2m_f\pm1$	0.18	0.25	0.31	0.35	0.37	0.36	0.33	0.27	0.19	0.10
$n=2m_f\pm3$	0.21	0.18	0.14	0.10	0.07	0.04	0.02	0.01	0.00	0.00

VSC – Need for Diodes



(a)



(b)