Time Constants (Section 4.14.1)

In a linear static circuit with no capacitance, i.e., an R-L circuit, the transient currents decay with time according to

\[ i(t) = i_0 e^{-t/T} \]  

(1)

where \( i_0 \) is the initial current and \( T \) is the time constant. For an R-L circuit, we may show that

\[ i(t) = \frac{1}{R} e^{-(L/R)t} \]  

(2)

where we see that \( T = L/R \). How do we think of \( T \)?

Let \( t = T \) and then we get that

\[ i(t) = i_0 e^{-T/T} = i_0 e^{-1} = 0.368i_0 \]  

(3)

Thus we see that the time constant is:

1. The time in which the current decreases to 36.8% of its initial value;
2. The time in which the current decrease equals 63.2% of its initial value;
3. The time in which the current would decrease to zero if it continued to decrease at its initial rate of decrease.

Figure 1 illustrates these three ways of thinking about \( T \).
So the time constant is a good measure of the speed of the dynamics. Low $T \rightarrow$ fast dynamics.

For a salient pole machine, we have a time constant for each rotor circuit given as the ratio of some inductance to the circuit resistance.

We can obtain the time constants under one of two conditions:
1. Stator is open-circuited.
2. Stator is short-circuited.

The procedure used in VMAF for developing these equations is as follows (see pp 133-134). \textit{I will apply it to obtain the open circuit d-axis subtransient time constant.}

1. **Flux linkage voltage equations**: Write voltage equation for the appropriate circuit using flux derivatives using (4.36) below:

   $\begin{bmatrix}
   v_d \\
   v_q \\
   -v_F \\
   v_G = 0 \\
   v_D = 0 \\
   v_Q = 0
   \end{bmatrix}
   =
   -
   \begin{bmatrix}
   r & 0 & 0 & 0 & 0 & 0 \\
   0 & r & 0 & 0 & 0 & 0 \\
   0 & 0 & r_F & 0 & 0 & 0 \\
   0 & 0 & 0 & r_G & 0 & 0 \\
   0 & 0 & 0 & 0 & r_D & 0 \\
   0 & 0 & 0 & 0 & 0 & r_Q
   \end{bmatrix}
   \begin{bmatrix}
   i_d \\
   i_q \\
   i_F \\
   i_G \\
   i_D \\
   i_Q
   \end{bmatrix}
   +
   \begin{bmatrix}
   -\omega \lambda_q \\
   \omega \lambda_q \\
   0 \\
   0 \\
   0 \\
   0
   \end{bmatrix}
   -
   \begin{bmatrix}
   \dot{\lambda}_q \\
   \dot{\lambda}_G \\
   \dot{\lambda}_D \\
   \dot{\lambda}_Q
   \end{bmatrix}

   (4.36)

   We assume a step change is applied to the field winding (with the stator winding open or short-circuited, it is the only way we can provide an external forcing function). We want to characterize the time constant of the D-winding. Therefore, we pull out of (4.36) the $v_F$ and $v_D$ equations:

   $v_F = r_F i_F + \dot{\lambda}_F \quad (4.181a)$

   $v_D = 0 = r_D i_D + \dot{\lambda}_D \quad (4.181b)$

2. **Replace fluxes with currents**: Use eq. (4.20) (see “macheqts”) to replace fluxes with currents.
For example, we see that

\[ \lambda_F = \sqrt{\frac{3}{2}} M_F i_d + L_F i_F + M_R i_D \]  
\[ \lambda_D = \sqrt{\frac{3}{2}} M_D i_d + M_R i_F + L_D i_D \]  

3. **Apply conditions**: Apply appropriate open circuit or short circuit conditions to simplify the equation. For example, if we are getting the open circuit time constants, then the stator windings are open circuited, and \( i_d=0 \). This causes \( i_d=i_q=0 \) and (*) and (**) become

\[ \lambda_F = L_F i_F + M_R i_D \]  
\[ \lambda_D = M_R i_F + L_D i_D \]  

Notice that from (4.182b), for a step change applied to the field voltage, CFLT indicates that \( \lambda_D(0^+)=0 \), which implies that

\[ 0 = M_R i_F + L_D i_D \Rightarrow i_F = \frac{-L_D}{M_R} i_D \]  

4. **Manipulate**: Differentiate (4.182a) and (4.182b), respectively,

\[ \dot{\lambda}_F = L_F i_F + M_R \dot{i}_D \]  
\[ \dot{\lambda}_D = M_R i_F + L_D \dot{i}_D \]  

and then substitute into (4.181a) and (4.181b), respectively. This results in

\[ v_F = r_F i_F + L_F \dot{i}_F + M_R \dot{i}_D \]  
\[ 0 = r_D \dot{i}_D + M_R \dot{i}_F + L_D \dot{i}_D \]  

Divide (4.184a) by \( L_F \) and (4.184b) by \( M_R \) to get
\[ \frac{v_F}{L_F} = \frac{r_F}{L_F} i_F + i_F + \frac{M_R}{L_F} i_D \]  
\[ (4.184c) \]

\[ 0 = \frac{r_D}{M_R} i_D + i_F + \frac{L_D}{M_R} i_D \]  
\[ (4.184d) \]

Subtract (4.184c) from (4.184d) to get
\[ \frac{r_D}{M_R} i_D - \frac{r_F}{L_F} i_F + \frac{L_D}{M_R} i_D - \frac{M_R}{L_F} i_D + i_F - i_F = -\frac{v_F}{L_F} \]
\[ \frac{r_D}{M_R} i_D - \frac{r_F}{L_F} i_F + \left( \frac{L_D}{M_R} - \frac{M_R}{L_F} \right) i_D = -\frac{v_F}{L_F} \]

Now replace \( i_F \) with (4.183) to get
\[ \left( \frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right) i_D + \left( \frac{L_D}{M_R} - \frac{M_R}{L_F} \right) i_D = -\frac{v_F}{L_F} \]

Now divide through by the coefficient of the derivative term:
\[ \left( \frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right) i_D + i_D = \left( \frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \frac{-v_F}{L_F} \]

Multiply top and bottom of the first term on the left-hand-side by \( M_R \), and do the same to the right-hand-side, to get
\[ \left( \frac{r_D}{M_R} + \frac{r_F}{L_F} \frac{L_D}{M_R} \right) \left( \frac{L_D}{M_R} - \frac{M_R}{L_F} \right) i_D + \frac{i_D}{L_F} = \frac{-v_F}{L_F} \left( \frac{L_D}{M_R} - \frac{M_R}{L_F} \right) \]

5. **Approximate and apply LaPlace**: Use the following information (in per-unit):
- Damper circuits are very fast, because \( r_D \) and \( r_Q \) are large.
- Field circuits are very slow, because \( r_F \) and \( r_G \) are small.

Reference to Example 4.1 (p. 107) indicates, in per-unit:
\[ r_D=0.0131, \ r_Q=0.054 \]
\[ r_F=0.000742, \ r_G=0.00584 \]
VMAF (pg. 134) make the statement that “usually in pu \( r_D >> r_F \) while \( L_D \) and \( L_F \) are of similar magnitude.” This means \( r_D >> r_F L_D / L_F \), it is also true that \( r_Q >> r_G \) while \( L_Q \) and \( L_G \) are of similar magnitude. Data for the pu inductances from Example 4.2 in VMAF (p. 112) are as follows:

\[
L_F = 1.65 \\
L_D = 1.605 \\
L_G = 1.76 \\
L_Q = 1.526
\]

and so the above becomes

\[
\frac{r_D}{(L_D - M_R^2 / L_F)} i_D + \dot{i}_D = -\frac{M_R v_F / L_F}{(L_D - M_R^2 / L_F)}
\]

Rearranging, we obtain

\[
i_D + \frac{r_D}{(L_D - M_R^2 / L_F)} i_D = -v_F \frac{M_R / L_F}{(L_D - M_R^2 / L_F)}
\]

Now define

\[
K_1 = \frac{r_D}{(L_D - M_R^2 / L_F)} \\
K_2 = -v_F \frac{M_R / L_F}{(L_D - M_R^2 / L_F)}
\]

Then (4.186a) becomes

\[
i_D + K_1 i_D = K_2
\]

(4.186b)

Using LaPlace transforms, we get

\[
sI_D(s) + K_1 I_D(s) = K_2 / s
\]

\[
I_D(s)(s + K_1) = K_2 / s
\]

(4.186c)

\[
I_D(s) = \frac{K_2}{s(s + K_1)}
\]

Taking partial fraction expansion, we have:

\[
I_D(s) = \frac{K_2}{s(s + K_1)} = \frac{K_2 / K_1}{s} - \frac{K_2 / K_1}{s + K_1} = \frac{K_2}{K_1} \left( \frac{1}{s} - \frac{1}{s + K_1} \right)
\]

The inverse LaPlace transform is then

\[
I_D(s) = \frac{K_2}{s(s + K_1)}
\]

PFE:

\[
= \frac{A}{s} + \frac{B}{s + K_1}
\]

\[
A = sI_D(s)|_{s=0} \\
B = (s+K_1)I_D(s)|_{s=-K_1}
\]
\[ i_D(t) = \frac{K_2}{K_1} \left( 1 - e^{-K_1 t} \right) u(t) \] (4.186d)

This shows that if we were to apply a step change in the field voltage \( v_F = V_F u(t) \) per bottom of p. 133, the current in the D-damper winding would rise in accordance with a time constant of \( K_1 \), similar to the function \( y = (1 - e^{-t}) u(t) \) as indicated below.

Replacing \( K_1 \) and \( K_2 \), we obtain

\[
i_D(t) = -v_F \frac{M_R / L_F}{\left( L_D - M_R^2 / L_F \right)} r_D \left( 1 - e^{-\frac{r_D}{L_D - M_R^2 / L_F}} \right) u(t)
\]

\[
= -v_F \frac{M_R / L_F}{r_D} \left( 1 - e^{-\frac{r_D}{L_D - M_R^2 / L_F}} \right) u(t) \quad (4)
\]

Recall

\[
i(t) = \frac{1}{R} e^{-t/(L/R)} \quad (2)
\]

where \( T = L/R \), and so we see that \( I/K_1 \) is the time constant. We define this time constant as the open circuit direct-axis subtransient time constant, i.e.

\[
\tau_{d0}' = \frac{1}{K_1} = \frac{L_D - M_R^2 / L_F}{r_D}
\]

It’s name comes from the fact that

- it is computed when the stator windings are open circuit,
it characterizes the behavior of the D-winding and is therefore a \textit{subtransient} response.

Comments:

a. I have added a HW problem where you need to perform this same development except for the quadrature axis time constant.

b. On p. 134, VMAF writes the following: “When the damper winding is not available or after the decay of the subtransient current, we can show that the field current is affected only by the parameters of the field circuit, i.e.,

\[ r_F i_F + L_F i_F' = V_F u(t) \] (4.188)

The time constant of this transient is the d axis transient open circuit time constant \( \tau'_{d0} \), given by

\[ \tau'_{d0} = \frac{L_F}{r_F} \] (4.189)

Likewise, for round rotor machines (for which we need to model the G-winding), we can obtain the q-axis transient open circuit time constant \( \tau'_{q0} \), given by

\[ \tau'_{q0} = \frac{L_G}{r_G} \] (similar to 4.189)

c. The time constants given using the Greek letter \( \tau \) are in per-unit time, that is, they are related to time constants given using “\( t \)” (in seconds) according to

\[ \tau = t \omega_B \Rightarrow \tau = t \times 377. \]

d. Most texts indicate time constants in seconds. These values must be normalized before using them in relations (4.189) or (similar to 4.189), e.g., see below example 4.2, p. 112.
Data in pu is as follows:

\[ L_q = 1.70 \quad k M_Q = 1.49 \]
\[ L_q = 1.64 \quad r = 0.001096 \]
\[ L_F = 1.65 \quad r_F = 0.000742 \]
\[ L_D = 1.605 \quad r_D = 0.0131 \]
\[ L_Q = 1.526 \quad r_Q = 0.0540 \]
\[ k M_F = M_k = k M_D = 1.55 \quad H = 2.37 \text{ s} \]
\[ \ell_d = \ell_q = 0.15 \]

And Example 4.7 indicates \( x'_q = 0.38 \) pu which is for the same machine.

Recall our development for \( L'_q \) in the last set of notes (SubtransientTransientLT) which resulted in

\[ L'_q = L_q - \frac{L_{AQ}^2}{L_G} \quad \text{(4.180)} \]

Solving (4.180) for \( L_G \) results in

\[ L_G = \frac{L_{AQ}^2}{L_q - L'_q} \]

Recall from the “perunitization” notes, p. 30 that the mutual, \( L_{AQ} \), is the difference between the self and the leakage, i.e.,

\[ L_{qu} - l_{qu} = L_{Qu} - l_{Qu} = L_{Gu} - l_{Gu} = L_{AQ} \]

Substitution of this last expression for \( L_{AQ} \) in the \( L_G \) expression results in

\[ L_G = \frac{(L_q - l_q)^2}{L_q - L'_q} \]

Now we may utilize the above Example 4.2 data to obtain:

\[ L_G = \frac{(L_q - l_q)^2}{L_q - L'_q} = \frac{(1.64 - 0.15)^2}{1.64 - 0.380} = \frac{2.2201}{1.26} = 1.76 \]

This value is given in the pu values computed via Example 4.1, which shows Example 4.2 is for the same machine. VMAF does not provide \( t'_{q0} \) or \( \tau'_{q0} \) anywhere for the machine in these examples, and so I chose a value of 0.8sec that is typical of round-rotor machines based on my experience and review of various sources including, for example, Table 4.7 in VMAF, data in Kundur’s book, and data in some WECC data sets I have. Then, the pu value of this would be

\( \tau'_{q0} = 0.8 \times 377 = 301.6 \).
So then, \( r_G = L_G/\tau'_{q0} = 1.76/301.6 = 0.00583554 \text{pu} \), which again agrees with pu values computed for Example 4.1.

This value makes intuitive sense because it is an order of magnitude larger than \( r_F \) (and so it is faster than the field cct), but an order of magnitude lower than the damper values \( r_D \) and \( r_Q \) (and so it is slower than the damper winding circuits). This value, as computed here, was used in Ex. 4.2 (p. 112).

Application of similar procedures results in the expressions that Kundur calls the “classical expressions” given as follows (the VMAF equation number appears in the box to the right).

Without G-winding (salient pole machine):

\[
\tau'_{d0} = \frac{L_F}{r_F} \quad \text{(D-axis field)} \quad (4.189)
\]

\[
\tau''_{d0} = \frac{L_D - (L_{AD})^2 / L_F}{r_D} \quad \text{(D-axis damper)} \quad (4.187)
\]

\[
\tau''_{q0} = \frac{L_Q}{r_Q} \quad \text{(Q-axis damper)} \quad (4.193)
\]

With G-Winding (round rotor machine):

\[
\tau'_{d0} = \frac{L_F}{r_F} \quad \text{(D-axis field)} \quad (4.189)
\]

\[
\tau''_{d0} = \frac{L_D - (L_{AD})^2 / L_F}{r_D} \quad \text{(D-axis damper)} \quad (4.187)
\]

\[
\tau''_{q0} = \frac{L_Q - L^2_{AQ} / L_G}{r_Q} \quad \text{(Q-axis damper)} \quad (4.192a)
\]

\[
\tau'_{q0} = \frac{L_G}{r_G} \quad \text{(Q-axis field)} \quad (4.192a)
\]

In the last equation, \( L_G \) may be obtained as follows:

\[
L_q = \left[ L_q - \frac{L^2_{AQ}}{L_G} \right] \Rightarrow L_G = \frac{L^2_{AQ}}{L_q - L_q} = \frac{(L_q - L_q)^2}{L_q - L_q}
\]
In the above
OC : Open-circuit
DA : direct-axis
QA : quadrature axis
T : transient
ST : subtransient
TC : time constant

The short circuit time constants are as follows:

Without G-winding (salient pole machine):

SC/DA/T/TC: \( \tau_d' = \tau_{d0}' \frac{L_d'}{L_d} \) (D-axis field) \hspace{1cm} (4.191)

SC/DA/ST/TC: \( \tau_d'' = \tau_{d0}'' \frac{L_d''}{L_d} \) (D-axis damper) \hspace{1cm} (4.190)

SC/QA/ST/TC: \( \tau_q'' = \tau_{q0}'' \frac{L_q''}{L_q} \) (Q-axis damper) \hspace{1cm} (4.192b)

With G-Winding (round-rotor machine):

SC/DA/T/TC: \( \tau_d' = \tau_{d0}' \frac{L_d'}{L_d} \) (D-axis field) \hspace{1cm} (4.191)

SC/DA/ST/TC: \( \tau_d'' = \tau_{d0}'' \frac{L_d''}{L_d} \) (D-axis damper) \hspace{1cm} (4.190)

SC/QA/ST/TC: \( \tau_q'' = \tau_{q0}'' \frac{L_q''}{L_q} \) (Q-axis damper) \hspace{1cm} (4.192b)

SC/QA/ST/TC: \( \tau_q' = \tau_{q0}' \frac{L_q'}{L_q} \) (Q-axis damper) \hspace{1cm} (4.192b)
It is useful at this point to take note of the following from p. 132 of VMAF, where it says,

“Before we examine the $q$ axis inductances, some clarification of the circuits that may exist in the $q$ axis is needed. For a salient pole machine with amortisseur windings, a $q$ axis damper circuit exists, but there is no other actual or effective $q$ axis rotor winding. For such a machine the stator flux linkage after the initial subtransient dies out is determined by essentially the same circuit as that of the steady-state $q$ axis flux linkage. Thus, for a salient pole machine, it is customary to consider the $q$ axis transient inductance to be the same as the $q$ axis synchronous inductance.

The situation for a round-rotor machine is different. Here the solid iron rotor provides multiple paths for circulating eddy currents, which act as equivalent windings during both transient and subtransient periods. Such a machine will have effective $q$ axis rotor circuits that will determine the $q$ axis transient and subtransient inductances. Thus, for such a machine, it is important to recognize that a $q$ axis transient inductance (much smaller in magnitude than $L_q$) exists.

Another time constant used to characterize synchronous machines is the stator time constant, given by

$$\tau_a = \frac{(L'_d + L'_q)/2}{r}$$

Note that the text uses $L_q$ in the above equation instead of $L'_q$ (since $L_q = L'_q$ when the G-winding is not represented).

Table 4.3, pg. 135 in VMAF, provides a comparison of typical numerical range for time constants. Kundur also provides such a table, Table 4.2, pg. 150. Note transient $T >>$ subtransient $T.$
Another way to get the time constants is to use the equivalent circuits.

Then derive the inductances in terms of the LaPlace variable “s” according to

\[ L_d(s) = \frac{\lambda_d(s)}{i_d(s)} \]
\[ L_q(s) = \frac{\lambda_q(s)}{i_q(s)} \]

I will not go through the development here, but you can find it on pp. 140-143 of Kundur’s text.

The denominator of the above expressions is the characteristic equation for the circuit. The roots of this equation are the inverse of the time constants.

This approach makes no approximations, and therefore Kundur refers to the resulting expressions for the parameters as the “accurate expressions.”

The relationship between our nomenclature and that used by Kundur is as follows:

**Kundur ➔ VMAF**

\[ L_{ad} \rightarrow L_{AD} \]
\[ L_{fd} \rightarrow l_F \]
\[ R_{FD} \rightarrow r_F \]
\[ R_{1d} \rightarrow r_D \]
\[ L_{1d} \rightarrow l_D \]
\[ L_i \rightarrow l_d \]

You can review some of the data in appendix C of your text to see if it conforms to our conclusions about “fast” vs. “slow” circuits.
And you can check Kundur, page 153, for some comparative data for both salient pole and round-rotor machines, which I have copied out below.

![Table 4.2](image)

**Table 4.2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Hydraulic Units</th>
<th>Thermal Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synchronous Reactance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_d$</td>
<td>0.6 - 1.5</td>
<td>1.0 - 2.3</td>
</tr>
<tr>
<td>$X_q$</td>
<td>0.4 - 1.0</td>
<td>1.0 - 2.3</td>
</tr>
<tr>
<td>Transient Reactance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X'_d$</td>
<td>0.2 - 0.5</td>
<td>0.15 - 0.4</td>
</tr>
<tr>
<td>$X'_q$</td>
<td></td>
<td>0.3 - 1.0</td>
</tr>
<tr>
<td>Subtransient Reactance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X''_d$</td>
<td>0.15 - 0.35</td>
<td>0.12 - 0.25</td>
</tr>
<tr>
<td>$X''_q$</td>
<td>0.2 - 0.45</td>
<td>0.12 - 0.25</td>
</tr>
<tr>
<td>Transient OC Time Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{d0}$</td>
<td>1.5 - 9.0 s</td>
<td>3.0 - 10.0 s</td>
</tr>
<tr>
<td>$T'_{d0}$</td>
<td></td>
<td>0.5 - 2.0 s</td>
</tr>
<tr>
<td>Subtransient OC Time Constant</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T''_{d0}$</td>
<td>0.01 - 0.05 s</td>
<td>0.02 - 0.05 s</td>
</tr>
<tr>
<td>$T''_{q0}$</td>
<td>0.01 - 0.09 s</td>
<td>0.02 - 0.05 s</td>
</tr>
<tr>
<td>Stator Leakage Inductance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_l$</td>
<td>0.1 - 0.2</td>
<td>0.1 - 0.2</td>
</tr>
<tr>
<td>Stator Resistance</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_a$</td>
<td>0.002 - 0.02</td>
<td>0.0015 - 0.005</td>
</tr>
</tbody>
</table>

**Notes:**
1. Reactance values are in per unit with stator base values equal to the corresponding machine rated values.
2. Time constants are in seconds.

Similar data is in Chapter 4 of Anderson & Fouad, p. 135: Note Table 4.3 comes from Kimbark, see next page of these notes.
And From Kimbark, p. 40 (note time constant data at bottom of table is used in A&F’s table, given on previous page).
Typical Constants of Three-Phase Synchronous Machines
(Adapted from Refs. 15, 34, and 41)

<table>
<thead>
<tr>
<th></th>
<th>Turbo-generators (solid rotor)</th>
<th>Water-Wheel Generators (with dampers)†</th>
<th>Synchronous Condensers</th>
<th>Synchronous Motors (general purpose)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reactances in per unit</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$x_d'$</td>
<td>0.95</td>
<td>1.10</td>
<td>1.45</td>
<td>0.60</td>
</tr>
<tr>
<td>$x_q'$</td>
<td>0.92</td>
<td>1.08</td>
<td>1.42</td>
<td>0.40</td>
</tr>
<tr>
<td>$x_d''$</td>
<td>0.12</td>
<td>0.23</td>
<td>0.28</td>
<td>0.20</td>
</tr>
<tr>
<td>$x_q''$</td>
<td>0.12</td>
<td>0.23</td>
<td>0.28</td>
<td>0.40</td>
</tr>
<tr>
<td>$x_{d'''}$</td>
<td>0.07</td>
<td>0.12</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>$x_{q'''}$</td>
<td>0.10</td>
<td>0.15</td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td>$x_p$</td>
<td>0.07</td>
<td>0.14</td>
<td>0.21</td>
<td>0.17</td>
</tr>
<tr>
<td>$x_5$</td>
<td>0.07</td>
<td>0.12</td>
<td>0.17</td>
<td>0.13</td>
</tr>
<tr>
<td>$z_0^*$</td>
<td>0.01</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Resistances in per unit

|                      | $r_d$(d-c.) | 0.0015 | 0.005 | 0.003 | 0.002 | 0.002 | 0.015 |
|                      | $r$(a-c.)  | 0.003  | 0.008 | 0.003 | 0.015 | 0.004 |      |
|                      | $r_3$      | 0.025  | 0.045 | 0.012 | 0.20  | 0.025 | 0.07  |

Time constants in seconds

|                      | $T_{d}\prime$ | 2.8  | 5.6  | 9.2  | 1.5  | 5.6  | 9.5  | 6.0  | 9.0  | 11.5 |
|                      | $T_{d}\prime$ | 0.4  | 1.1  | 1.8  | 0.5  | 1.8  | 3.3  | 1.2  | 2.0  | 2.8  |
|                      | $T_{d'''} = T_{q''}$ | 0.02 | 0.035 | 0.05 | 0.01 | 0.035 | 0.05 | 0.02 | 0.035 | 0.05 |
|                      | $T_{a}$      | 0.04  | 0.16 | 0.35 | 0.03 | 0.15 | 0.25 | 0.1  | 0.17 | 0.3  |

* $z_0$ varies from about 0.15 to 0.60 of $x_{d'''}$, depending upon winding pitch.
† For water-wheel generators without damper windings,

$$x_{d'''} = 0.85x_{d}', \quad x_{q'''} = x_{q}' = x_{q}, \quad x_2 = (x_{d'} + x_{q})/2,$$

and $x_0$ is as listed.

† For curves showing the normal value of $x_{d'}$ of water-wheel-driven generators as a function of kilovolt-ampere rating and speed, see Ref. 54.