State Estimation 2

1.0 Introduction

In these notes, we explore two very practical and related issues in regards to state estimation:

- use of pseudo-measurements
- network observability

2.0 Exact Pseudo-measurments

It is important to keep in mind that the objective of state estimation is to obtain a computer model that accurately represents the current conditions in the power system. So if we can think of ways to improve the model using something other than actual measurements, we should feel free to do that.

Psuedo-measurements are not measurements but are used in the state-estimation algorithm as if they were. If we can know with certainty that a particular pseudo-measurement is accurate, we should use it as it will increase the accuracy of our state estimate.

The most common "exact" pseudo-measurement is the bus injection at a substation that has no generation and serves no load. Figure 1 below illustrates.

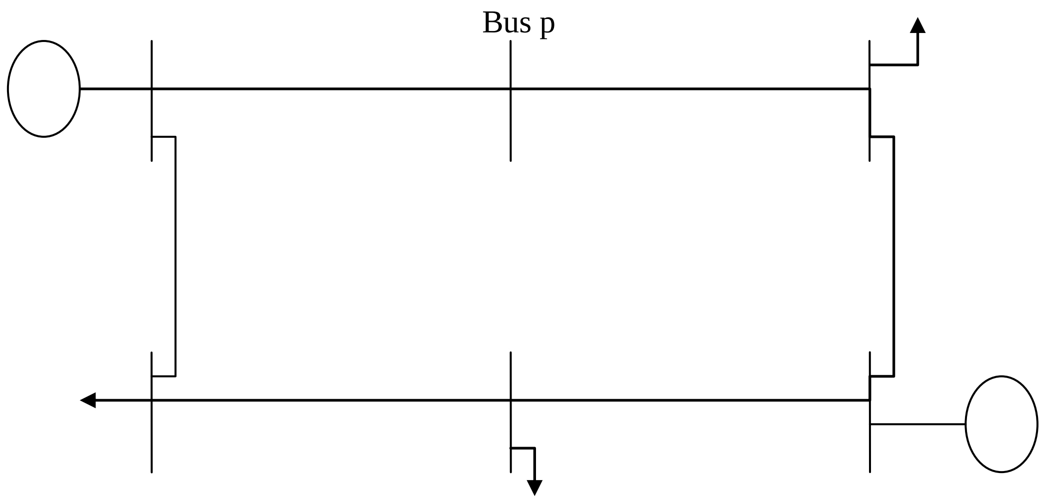

Fig. 1

In Fig. 1, bus p has no generation or load. We therefore know the real and reactive power injection of this bus with precision; it is 0. And so we can add two more measurements to the measurements that we actually have:

$$z_i = h_i(\underline{x}) + \eta_i \quad (1)$$

$$z_{i+1} = h_{i+1}(\underline{x}) + \eta_{i+1} \quad (2)$$

where:

"Measurements"

$$z_i = P_{p,inj} = 0 \tag{3}$$

$$z_{i+1} = Q_{p,inj} = 0 \tag{4}$$

$$h_i(\underline{x}) = P_{p,inj} = \sum_{k=1}^{n} |V_p||V_k|\left(G_{pk}\cos(\theta_p - \theta_k) + B_{pk}\sin(\theta_p - \theta_k)\right) = 0 \tag{5}$$

$$h_{i+1}(\underline{x}) = Q_{p,inj} = \sum_{k=1}^{n} |V_p||V_k|\left(G_{pk}\sin(\theta_p - \theta_k) - B_{pk}\cos(\theta_p - \theta_k)\right) = 0 \tag{6}$$

We recognize the summations of eqs. (5) and (6) as the power flow equations for real and reactive power injection, respectively.

The terms η_i and η_{i+1} are zero-mean Gaussian distributed errors for the pseudo-measurements. We can account for the fact that these pseudo-measurements are exact by letting σ_i and σ_{i+1} be very small (and therefore the corresponding weights, $1/\sigma_i$, and $1/\sigma_{i+1}$, to be very large) The weighted least-square estimation algorithm is then carried out as usual.

3. Observability

Recall our very first example at the beginning of the first set of state estimation notes. It is below.

In the circuit given of Fig. 2, current injections I_1, I_2, and voltage E are unknown. Let $R_1=R_2=R_3=1.0$ Ω. The measurements are:

- meter A_1: $i_{1,2}$=1.0 Ampere
- meter A_2: $i_{3,1}$=-3.2 Ampere
- meter A_3: $i_{2,3}$=0.8 Ampere
- meter V: e=1.1 volt

The problem is to determine the state of the circuit, which in this case is nodal voltages v_1, v_2, and the voltage e across the voltage source.

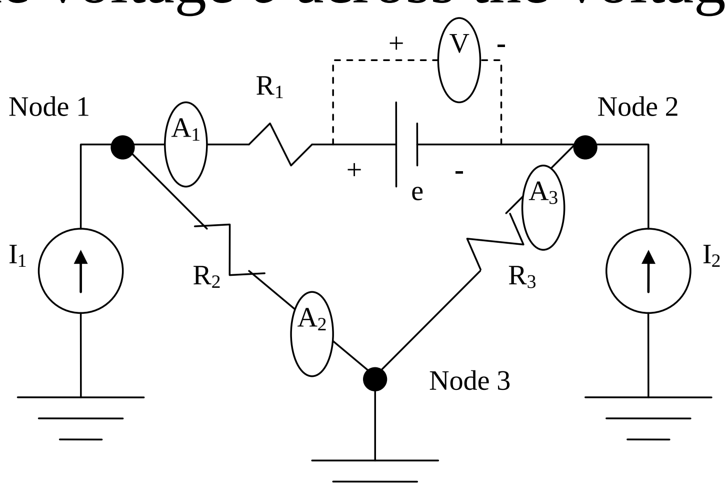

Fig. 2

To determine the state of the circuit (v_1, v_2, and e), we wrote each one of the measurements in terms of the states. We then expressed these four equations in matrix form:

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 1.0 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \qquad (7)$$

Let's denote terms in eq. (7) as A, x, and b, so:

$$\underline{A}\underline{x} = \underline{b} \qquad (8)$$

We solved eq. (8) using:

$$\underline{x} = \left(\underline{A}^T \underline{A}\right)^{-1} \underline{A}^T \underline{b} = \underline{G}^{-1} \underline{A}^T \underline{b} = \underline{A}^I \underline{b} \qquad (9)$$

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \qquad (10)$$

The inverse of the gain matrix is then found from Matlab as

$$\underline{G}^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} \qquad (11)$$

The pseudo-inverse is then

$$\underline{A}^I = \underline{G}^{-1}\underline{A}^T = \frac{1}{4}\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \quad (12)$$

Then we obtained the least squares estimate of the 3 states from the 4 measurements as

$$\underline{x} = \underline{A}^I \underline{b} = \frac{1}{4}\begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1.0 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} = \begin{bmatrix} 3.125 \\ 0.875 \\ 1.175 \end{bmatrix} \quad (13)$$

Question: What would happen to this problem if we lost a measurement? Let's say that we lost A_1, the measurement on the current flowing from bus 1 to bus 2. Let's see what happens.

To solve it, we just remove the first equation (corresponding to, in eq. (7), the first row of the A-matrix and the top element in b-vector).

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \quad (14)$$

Actually, here the matrix is 3×3 and therefore we can solve exactly as:

$$\begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \tag{15}$$

But let's go ahead and use eq. (9) to see what happens.

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{16}$$

The inverse of the gain matrix is then

$$\underline{G}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{17}$$

The pseudo-inverse is then

$$\underline{A}^I = \underline{G}^{-1} \underline{A}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{18}$$

The least squares estimate of the 3 states from the 3 measurements is then

$$\underline{x} = \underline{A}^I \underline{b} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \tag{19}$$

Compared to the solution of eq. (13), our estimate can be assumed to be less accurate (since it is based on fewer measurements), but at least we still did obtain a solution.

Question: What if we lost two measurements? Let's say that we lost A_1, the measurement on the current flowing from bus 1 to bus 2, and A_2, the measurement on the current flowing from bus 1 to bus 3. Let's see what happens.

To solve it, we just remove the first two equations (corresponding to, in eq. (7), the first row of the A-matrix and the top element in b-vector).

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 0.8 \\ 1.1 \end{bmatrix} \tag{20}$$

The matrix is once again non-square, so we must use our least-squares procedure.

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (16)$$

The inverse of the gain matrix is, however, singular – it's determinant is zero (or equivalently, it has a zero eigenvalue). As a result, it can not be inverted. In this case, our process must stop since we need $\underline{G}^{-1}$ to evaluate x, as indicated in eq. (9), repeated below for convenience.

$$\underline{x} = \left(\underline{A}^T \underline{A}\right)^{-1} \underline{A}^T \underline{b} = \underline{G}^{-1} \underline{A}^T \underline{b} = \underline{A}^I \underline{b} \qquad (9)$$

What is the problem here?

The basic problem is that we do not have enough measurements. In this case, the system is said to be <u>*unobservable*</u>. This means that despite the availability of some measurements, it is not possible to provide an estimate of the states with those available measurements.

Example:

Consider the system in Fig. 3. This is the same system considered in our previous set of state estimation notes and in your homework assignment. There are real power measurements taken at P_{12}, P_{13}, and P_{32}. But we assume that the measurements P_{13} and P_{32} fail so that we only have the measurement P_{12}=0.62 pu. As in previous examples, assume all voltages are 1.0 per unit, all measurement devices have σ=0.01, and that the bus 3 angle is reference. The state vector is $\underline{x}=[\theta_1\ \ \theta_2]^T$ as before.

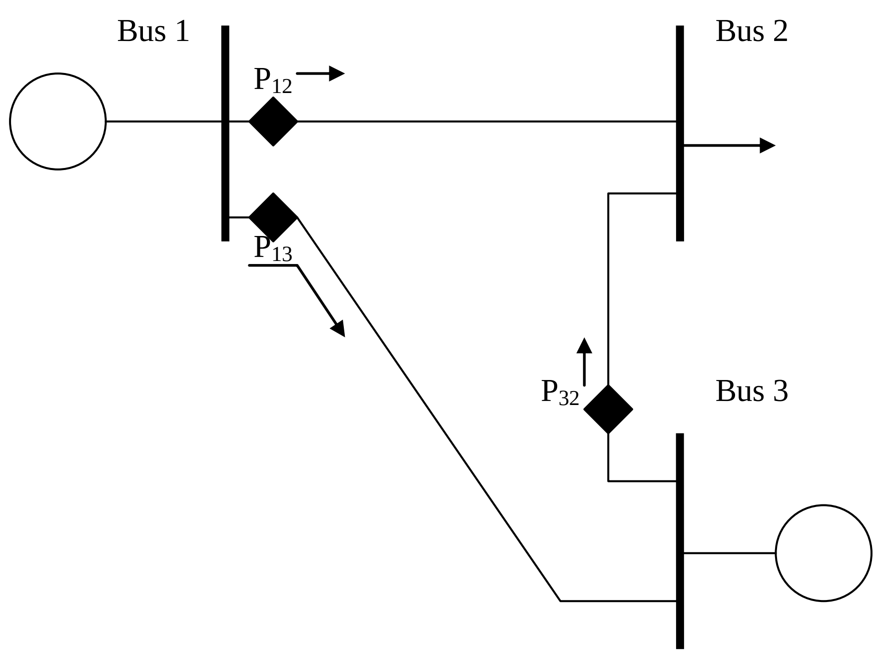

Fig. 3

a) Determine the vector of measurement expressions $\underline{h}(\underline{x})$, the derivative expressions $\underline{H} = \frac{\partial \underline{h}(\underline{x})}{\partial \underline{x}}$, and the weighting matrix $\underline{R}$.

b) Develop $\underline{A} = \underline{H^T}(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x})$, $\underline{b} = \underline{H^T}(\underline{x})\underline{R}^{-1}[(\underline{z} - \underline{h}(\underline{x}))]$ and comment on our ability to solve $\underline{A}\Delta\underline{x}=\underline{b}$ for $\Delta\underline{x}$.

Solution:

(a)

$$h_1(\underline{x}) = P_{12} = |V_1|^2 g_{12} - |V_1||V_2| g_{12}\cos(\theta_1 - \theta_2) - |V_1||V_2| b_{12}\sin(\theta_1 - \theta_2) = 5\sin(\theta_1 - \theta_2)$$

Since $\underline{x}^T=[\theta_1\ \theta_2]^T$, the derivative expressions are

$$H(\underline{x}) = \frac{\partial h_1(\underline{x})}{\partial \underline{x}} = \begin{bmatrix} \frac{\partial h_1(x)}{\partial \theta_1} & \frac{\partial h_1(x)}{\partial \theta_2} \end{bmatrix} = [5\cos(\theta_1 - \theta_2) \quad -5\cos(\theta_1 - \theta_2)]$$

and with $\sigma^2=0.0001$, $R = \lfloor \sigma^2 \rfloor = 0.0001$.

(b)

First get $\underline{A}$.

$$\underline{A} = \underline{H^T}(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x}) = \begin{bmatrix} 5\cos(\theta_1 - \theta_2) \\ -5\cos(\theta_1 - \theta_2) \end{bmatrix} 10000[5\cos(\theta_1 - \theta_2) \quad -5\cos(\theta_1 - \theta_2)]$$

$$= 10000 \begin{bmatrix} 25\cos^2(\theta_1 - \theta_2) & -25\cos^2(\theta_1 - \theta_2) \\ -25\cos^2(\theta_1 - \theta_2) & 25\cos^2(\theta_1 - \theta_2) \end{bmatrix}$$

Now get $\underline{b}$.

$$\underline{b} = \underline{H}^T(\underline{x})\underline{R}^{-1}\left[\left(\underline{z} - \underline{h}(\underline{x})\right)\right] = \begin{bmatrix} 5\cos(\theta_1 - \theta_2) \\ -5\cos(\theta_1 - \theta_2) \end{bmatrix} 10000\left[0.62 - 5\sin\left(\theta_1 - \theta_2\right)\right]$$

$$= 10000\begin{bmatrix} 3.1\cos(\theta_1 - \theta_2) - 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \\ -3.1\cos(\theta_1 - \theta_2) + 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \end{bmatrix}$$

Therefore the equation we need to solve is

$$10000\begin{bmatrix} 25\cos^2(\theta_1 - \theta_2) & -25\cos^2(\theta_1 - \theta_2) \\ -25\cos^2(\theta_1 - \theta_2) & 25\cos^2(\theta_1 - \theta_2) \end{bmatrix}\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{bmatrix}$$

$$= 10000\begin{bmatrix} 3.1\cos(\theta_1 - \theta_2) - 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \\ -3.1\cos(\theta_1 - \theta_2) + 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \end{bmatrix}$$

or

$$\begin{bmatrix} 25\cos^2(\theta_1 - \theta_2) & -25\cos^2(\theta_1 - \theta_2) \\ -25\cos^2(\theta_1 - \theta_2) & 25\cos^2(\theta_1 - \theta_2) \end{bmatrix}\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} 3.1\cos(\theta_1 - \theta_2) - 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \\ -3.1\cos(\theta_1 - \theta_2) + 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \end{bmatrix}$$

The above equation solves according to

$$\begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \end{bmatrix}$$

$$= \begin{bmatrix} 25\cos^2(\theta_1 - \theta_2) & -25\cos^2(\theta_1 - \theta_2) \\ -25\cos^2(\theta_1 - \theta_2) & 25\cos^2(\theta_1 - \theta_2) \end{bmatrix}^{-1}\begin{bmatrix} 3.1\cos(\theta_1 - \theta_2) - 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \\ -3.1\cos(\theta_1 - \theta_2) + 25\cos(\theta_1 - \theta_2)\sin\left(\theta_1 - \theta_2\right) \end{bmatrix}$$

To perform the necessary matrix inversion, we require the matrix determinant, which is:

$$\det\begin{bmatrix} 25\cos^2(\theta_1 - \theta_2) & -25\cos^2(\theta_1 - \theta_2) \\ -25\cos^2(\theta_1 - \theta_2) & 25\cos^2(\theta_1 - \theta_2) \end{bmatrix} = 25\cos^4(\theta_1 - \theta_2) - 25\cos^4(\theta_1 - \theta_2) = 0$$

The matrix is singular and we cannot solve the equation.

In general, we say that the power system is observable if $\underline{A} = \underline{H^T}(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x})$ is non-singular.

Linear algebra text books [1, pg. 46], [2] define <u>matrix rank</u>:

- The maximum number of linearly independent rows in an m×n matrix <u>A</u> is the row rank of <u>A</u>, and the maximum number of linearly independent columns in an m×n matrix <u>A</u> is the column rank of <u>A</u>.
- If <u>A</u> is m×n, then: row rank≤m & col rank≤n.
- For any matrix <u>A</u>: row rank=col rank≡rank.

<u>Observability analysis from H</u>: Assume for a power system state estimation problem that m>n (more measurements than states). The power system is observable if matrix $\underline{A} = \underline{H^T}(\underline{x})\underline{R}^{-1}\underline{H}(\underline{x})$ is nonsingular, and this occurs if <u>H</u> has rank n, where n is the number of columns of <u>H</u>.

Most state estimators will perform an observability analysis. If the network is unobservable, it may be the case that some pockets or islands of the network are still observable. Thus we also need to be able to identify observable parts of the system from unobservable parts of the system. Doing so will will enable us to determine from which part of the network we need to obtain additional measurements.

The topic of observability analysis is well-covered in [3, ch 7].

4. Approximate pseudo-measurements

A key step in state estimation is to test for observability. If the network is not observable, i.e., if we do not have enough independent measurements, then we will not be able to obtain a network model.

When the state estimator detects that the network is unobservable, it can make use of approximate pseudo-measurements. Examples of such approximate pseudo-measurements include:

- Information obtained from plant operators over phone or e-mail.
- Information obtained from previous measurements.
- Information obtained from a load flow calculation.

In using approximate pseudo-measurements, it is generally good practice to pair it with a relatively large variance in the weighting matrix, given that it is in fact "approximate."

5. Quality of the state estimate

Although the state estimator provides the "best" estimate of the states given all available measurements, there remains the rather important question of:

How good is the state estimate?

Clearly, if the state estimate is very poor, we will not want to use it even if it is the "best."

There are two issues related to state estimate quality.

- Bad data: Some data might be completely erroneous, polluted with some kind of gross error beyond the normal meter error. For example, a transducer may be wired incorrectly or is malfunctioning and gives a negative number or a zero or a number that is 10 times what it should be.
- Meter error: We have already discussed that meter error will always be present. We should check to ensure that there is not one or more meters in a crucial location having magnitude large enough to cause the entire state estimate to have unacceptable quality.

Your text provides an overview of these topics in Section 12.6.1 "Detection and Identification of Bad Measurements," which depends on Hypothesis Testing.

[1] W. Brogan, "Modern Control Theory," Prentice-Hall, 1982.
[2] G. Strang, Linear Algebra and its applications," Harcourt-Brace, 1988.
[3] A. Monticelli, "State estimation in electric power systems, a generalized approach," Kluwer, 1999.