Marginal Loss Modeling in LMP Calculation

Eugene Litvinov, *Member, IEEE*, Tongxin Zheng, *Member, IEEE*, Gary Rosenwald, and Payman Shamsollahi, *Senior Member, IEEE*

***Abstract*—This paper discusses the pricing of marginal transmission network losses in the locational marginal pricing approach recently deployed in the ISO New England (ISO-NE) standard market design (SMD) project implemented by ALSTOM's T&D Energy Automation and Information (EAI) Business. The traditional loss model is studied and a new model is proposed. The new model achieves more defendable and predictable market-clearing results by introducing loss distribution factors to explicitly balance the consumed losses in the lossless dc power system model. The distributed market slack reference is also introduced and discussed. The LMP components produced by the two models are studied and compared under changes in slack reference. Numerical examples are presented to further compare the two models.**

***Index Terms*—Electricity market, marginal pricing, network losses, optimization, power systems.**

NOMENCLATURE

c	Generation bid price vector.
D	Loss distribution factor vector.
e	Vector whose elements are 1.
L	Market demand vector.
LF_W	Loss sensitivity vector, whose elements are calculated with respect to the distributed slack reference represented as vector W.
Loss	System losses variable.
offset$_W$	System loss linearization offset, which is dependent on the slack weight W.
P	Market energy supply output vector.
$P_{\min}$	Minimum generator output limit.
$P_{\max}$	Maximum generator output limit.
$T_{\max}$	Line flow limit vector.
T_W	Constraint sensitivity matrix, whose elements are calculated with respect to the same distributed slack weight W.
W	Distributed slack reference weight vector.
λ	Shadow price for the energy balance equation.
τ	Shadow price for the system losses equation.
μ	Shadow prices vector for the transmission constraints.
$\gamma_{\max}$	Shadow prices for the maximum generator output constraints.
$\gamma_{\min}$	Shadow prices for the minimum generator output constraints.

Manuscript received August 12, 2003.

E. Litvinov and T. Zheng are with ISO New England, Inc., Holyoke, MA 01040 USA (e-mail: elitvinov@iso-ne.com; tzheng@iso-ne.com).

G. Rosenwald and P. Shamsollahi are with ALSTOM EAI Corp., Bellevue, WA 98004 USA (e-mail: gary.rosenwald@esca.com; payman.shamsollahi@esca.com).

Digital Object Identifier 10.1109/TPWRS.2004.825894

I. INTRODUCTION

A congestion management system (CMS) is a major part of the current competitive electricity markets. A 2002 Notice of Public Ruling (NOPR) from Federal Energy Regulatory Commission (FERC) [1] proposes location-based marginal pricing (LMP) together with firm transmission rights (FTR) as a mechanism to build efficient energy markets. LMP is part of the standard market design (SMD) promoted by FERC and is a fundamental principle in the majority of electricity markets [2]–[4].

LMP at a location is defined as a cost of supplying an increment of load at this location. This price reflects not only the cost of producing energy, but also its delivery. Losses and/or transmission network congestion may make delivery of energy from the least expensive resource to a different location impossible or uneconomic. The resulting "out of merit" dispatch to accommodate the system constraints using more expensive energy will, in turn, cause price separation at the various network locations, so it is very important to include the effect of congestion and losses in both economic dispatch and price calculation.

All CMS implementations in the current markets utilize linear-programming (LP) techniques in the market-clearing algorithms. LP requires linear network models that are normally based on the dc idealization of power-flow equations [2]–[5], [9]. The dc model has no transmission losses by definition, which makes it very challenging to correctly model the marginal effect of losses for economic dispatch and electricity pricing.

Marginal loss modeling affects dispatch and LMPs in the system. It is shown in the new approach described below that consistent modeling of the marginal losses cannot only preserve major LMP properties with dc model idealization, but also promote a transparent and liquid electricity market by producing more defendable and predictable results.

II. LMP COMPONENTS

One of the important SMD features is the ability to hedge congestion costs through the use of FTRs. In a system without losses, the value of an FTR can be defined as the difference between the LMP at the sink of the FTR and the LMP at the source [2]. However, in a system with losses, the LMPs at two locations can be different even in the noncongested case. This could be easily seen from the following expression for the LMP at node i [4], [6], [7]

$$LMP_i = \lambda - LF_{W:i}\lambda + \sum_{k=1}^{m} t_{w:i,k}\mu_k, \qquad (1)$$

where $t_{w:i,k}$ is the constraint k's power flow sensitivity to the injection at node i with respect to the slack reference W, and

m is the number of constraints. Even in the case of no congestion, when $\mu = 0$, the prices would be different at different locations due to the variation in the loss sensitivity factors. This makes it impossible to calculate the value of FTRs simply by subtracting the sink LMP from the source LMP for FTRs that hedge only congestion costs. The LMP in (1) could be split into three components [3], [4]: reference energy component LMP_i^{Energy}, loss component LMP_i^{Loss}, and congestion component $LMP_i^{\text{Congestion}}$. In the above, although the reference energy component is called "energy component," in general, it does not represent the price for energy without loss and congestion. Because loss factors and all shift factors are equal to zero at the slack reference (i.e., reference bus), the energy component is equal to the LMP at the reference. The energy component is the same for all locations in the system. In the model with losses, the value of a FTR must be measured by the difference in only the congestion components of LMP. In a lossless system, the loss component of LMP in (1) is zero at all locations and the difference in LMPs defining the value of FTRs is the same as the difference in the congestion components.

With the introduction of transmission losses, the LMP must be decomposed. Although the LMP is not dependent on the slack reference, the split into LMP components is dependent on the selection of the slack reference. This dependency indicates that importance should not be placed on the absolute value of each component by itself. However, the differences in congestion components between locations are not dependent on the slack reference and are more meaningful. In the ISO-NE and NYISO market, the only reason for splitting LMP into components is the need to calculate congestion revenue and the values of FTRs. The energy and loss components should be grouped together as one component—the delivered energy component. There is no need to split delivered energy component into energy and losses. In fact, the ISO-NE settlements process never uses them separately. Neither congestion nor loss revenue is dependent on the location of the slack reference.

III. Traditional Model

In the traditional model [4], [6], [7], losses are added to the energy balance equation, however, the location of losses to be balanced is not explicitly indicated. This model is presented in the problem LP1.

LP1:

$$\begin{aligned} \text{Min} \quad & c^T P \\ \text{S.T.} \quad & e^T(P-L) = \text{Loss}, && (2.1)\\ & \text{Loss} = LF_W^T(P-L) + \text{offset}_W, && (2.2)\\ & T_W(P-L) \le T_{\max}, && (2.3)\\ & P_{\min} \le P \le P_{\max}, && (2.4) \end{aligned}$$

where the values of loss and constraint sensitivities are calculated with respect to the distributed slack [7], [8] whose slack weights are represented by the vector W. (This includes the case of a single reference bus, or nondistributed slack, if all weights but one are equal to 0.) The Lagrangian function for LP1 is

$$\begin{aligned} \psi_1 = & c^T P - \lambda_1(e^T(P-L) - \text{Loss}) \\ & - \tau_1\left(\text{Loss} - LF_W^T(P-L) - \text{offset}_W\right) \\ & - \mu_1^T(T_W(P-L) - T_{\max}) + \gamma_{\max}^T(P - P_{\max}) \\ & + \gamma_{\min}^T(-P + P_{\min}). \end{aligned}$$

The subscript 1 represents the LP1 problem. The Kuhn–Tucker (KT) condition for the loss variable is

$$\frac{\partial \psi_1}{\partial \text{Loss}} = \lambda_1 - \tau_1 = 0. \quad (3)$$

Thus, the locational marginal price (LMP) is defined as

$$\begin{aligned} LMP = \frac{\partial \psi_1}{\partial L} &= \lambda_1 e - \tau_1 LF_W + T_W^T \mu_1 \\ &= \lambda_1 e - \lambda_1 LF_W + T_W^T \mu_1. \end{aligned} \quad (4)$$

The LMPs can be decomposed into the following three components:

$$LMP^{\text{Energy}} = \lambda_1 e, \quad (5.1)$$
$$LMP^{\text{Loss}} = -\lambda_1 LF_W, \quad (5.2)$$
$$LMP^{\text{Congestion}} = T_W^T \mu. \quad (5.3)$$

In the ISO-NEs settlement, the delivered energy and congestion are settled separately. Under LP1, the revenues collected would be

$$\begin{aligned} \text{Rev}^{\text{Energy\&Loss}} &= -(LMP^{\text{Energy}} + LMP^{\text{Loss}})^T \\ &\quad \cdot (P-L) \\ &= \lambda_1 \cdot (\text{Loss}_{\text{marg}} - \text{Loss}), && (6.1)\\ \text{Rev}^{\text{Congestion}} &= -(LMP^{\text{Congestion}})^T \cdot (P-L) \\ &= -\mu_1^T T_{\max}, && (6.2) \end{aligned}$$

where $Loss_{marg} = LF_W^T \cdot (P-L)$ is defined as marginal losses.

In the following, we will study the effect of changing slack weights under the LP1 formulation. Assuming the slack weights are changed to W_1, the following relationship holds [see the Appendix, (A.13) and (A.22)]:

$$T_{W_1} = T_W - T_W W_1 e^T, \quad (7)$$
$$LF_{W_1} = \frac{(LF_W - W_1^T LF_W e)}{(1 - W_1^T LF_W)}. \quad (8)$$

In addition, the following relationship can be derived from the Taylor's series expansion of the system losses equation:

$$\text{offset}_{W_1} = \frac{\text{offset}_W}{(1 - W_1^T LF_W)}. \quad (9)$$

The new LP problem (LP1_W_1) based on the slack weight W_1 is formulated below.

LP1_W_1

$$\begin{aligned} \text{Min} \quad & c^T P \\ \text{S.T.} \quad & e^T(P-L) = \text{Loss}, && (10.1)\\ & \text{Loss} = LF_{W_1}^T(P-L) + \text{offset}_{W_1}, && (10.2)\\ & T_{W_1}(P-L) \le T_{\max}, && (10.3)\\ & P_{\min} \le P \le P_{\max}. && (10.4) \end{aligned}$$

Substituting (10.2) by (10.1), (8), and (9), (10.2) becomes

$$\begin{aligned} \text{Loss} &= \frac{(LF_W - W_1^T LF_W e)^T}{(1 - W_1^T LF_W)}(P-L) \\ &\quad + \frac{\text{offset}_W}{(1 - W_1^T LF_W)} \Rightarrow \\ (1 - W_1^T LF_W)\text{Loss} &= LF_W^T(P-L) \\ &\quad - (W_1^T LF_W)\text{Loss} + \text{offset}_W \Rightarrow \\ \text{Loss} &= LF_W^T(P-L) + \text{offset}_W. \end{aligned}$$

The above shows that (2.2) and (10.2) are equivalent, which proves that the change of slack reference weights W does not affect the loss (2.2).

Substituting (10.3) by (7) and (10.1), (10.3) becomes

$$(T_W - T_W W_1 e^T)(P-L) \le T_{\max} \Rightarrow$$
$$T_W(P-L) - T_W W_1 e^T (P-L) \le T_{\max} \Rightarrow$$
$$T_W(P-L) - T_W W_1 \cdot \text{Loss} \le T_{\max}.$$

Due to the fact that there exits at least one vector W_1 such that $T_W W_1$ can be a nonzero vector, (10.3) is not equivalent to (2.3). LP1 and LP1_W_1 are different linear programming problems and, therefore, give different market-clearing results. This also indicates that the dual problem of LP1_W_1 is also different from that of LP1. In conclusion, the traditional model specified in LP1 is dependent on the selection of slack weights W. The LMPs, the congestion, and loss revenue will be different when the slack weights W change.

IV. Proposed Model

As seen from Section III, the drawback of the traditional model is that the losses are balanced at the slack reference implicitly through energy balance equation. As we know, the losses are consumed by the transmission lines. However, in the dc power-flow model, they must be balanced at the bus level. Thus, the location of balancing system losses should have impact on the line flows, which cannot be explicitly observed from the formulation of security constraints in (2.3). In the traditional model, when the slack weights change, the loss balancing location is changed implicitly. This results in different power flow, and different LMPs due to different megawatt (MW) injections.

The study of dc power flow model shows that the power-flow solution is unique once the injection at each bus is determined, which implies that the power-flow solution and the LMP should be invariant to the selection of slack reference once the method of balancing losses is determined. Based on the above assumption, a model is proposed below to produce prices that are not dependent on the selection of the slack reference. In order to balance the system losses explicitly, the loss distribution factors are introduced in the proposed model, and are represented by a vector D. In this way, each bus in the system may balance a share of total system losses. The optimal choice of D is a subject of separate research and is not being discussed in this paper. The loss distribution factors are normalized to 1

$$e^T D = 1. \tag{11}$$

The proposed model is represented as LP2 below.

LP2

$$\begin{aligned}
\text{Min} \quad & c^T P \\
\text{S.T.} \quad & e^T(P-L) = \text{Loss}, && (12.1)\\
& \text{Loss} = LF_W^T(P-L) + \text{offset}_W, && (12.2)\\
& T_W(P-L-D\cdot\text{Loss}) \le T_{\max}, && (12.3)\\
& P_{\min} \le P \le P_{\max}. && (12.4)
\end{aligned}$$

The Lagrangian function is

$$\begin{aligned}
\psi_2 = {} & c^T P - \lambda_2(e^T(P-L) - \text{Loss}) \\
& - \tau_2\left(\text{Loss} - LF_W^T(P-L) - \text{offset}_W\right) \\
& - \mu_2^T(T_W(P-L-D\cdot\text{Loss}) - T_{\max}) \\
& + \gamma_{\max}^T(P-P_{\max}) + \gamma_{\min}^T(-P+P_{\min}),
\end{aligned}$$

where subscript 2 represents the LP2 problem. The KT conditions for the loss variable is

$$\frac{\partial \psi_2}{\partial \text{Loss}} = \lambda_2 - \tau_2 + D^T T_W^T \mu_2 = 0. \tag{13}$$

The LMP is defined as

$$LMP = \frac{\partial \psi_2}{\partial L} = \lambda_2 e - \tau_2 LF_W + T_W^T \mu_2. \tag{14}$$

Combining (13) and (14) to eliminate λ_2, the LMP components can be defined as

$$\begin{aligned}
LMP^{\text{Energy}} &= \tau_2 e, && (15.1)\\
LMP^{\text{Loss}} &= -\tau_2 LF_W, && (15.2)\\
LMP^{\text{Congestion}} &= T_W^T \mu_2 - (T_W D e^T)^T \mu_2. && (15.3)
\end{aligned}$$

The revenues collected by the ISO are

$$\begin{aligned}
Rev^{\text{Energy\&Loss}} &= \tau_2(\text{Loss}_{\text{marg}} - \text{Loss}), && (16.1)\\
Rev^{\text{Congestion}} &= -(LMP^{\text{Congestion}})^T \cdot (P-L) \\
&= -\mu_2^T T_{\max}. && (16.2)
\end{aligned}$$

Equation (16.2) shows that the congestion charge is determined by the product of the constraint shadow price and the constraint limit. We will show later in (18) that the constraint shadow price and, thus, the total congestion charge, is independent of the slack reference.

To study the effect of changing slack weight W, a similar LP is constructed as below for slack reference W_1.

LP2_W_1

$$\begin{aligned}
\text{Min} \quad & c^T P \\
\text{S.T.} \quad & e^T(P-L) = \text{Loss}, && (17.1)\\
& \text{Loss} = LF_{W_1}^T(P-L) + \text{offset}_{W_1}, && (17.2)\\
& T_{W_1}(P-L-D\cdot\text{Loss}) \le T_{\max}, && (17.3)\\
& P_{\min} \le P \le P_{\max}. && (17.4)
\end{aligned}$$

The equivalence of (5.2) and (10.2), shown in Section III, also shows that (17.2) is equivalent to (12.2). Now we will prove that (17.3) is equivalent to (12.3). Substituting (7) and (17.1) into (17.3), we have

$$\begin{aligned}
& (T_W - T_W W_1 e^T)(P-L-D\cdot\text{Loss}) \le T_{\max} \Rightarrow \\
& T_W(P-L-D\cdot\text{Loss}) \\
& - T_W W_1 e^T(P-L-D\cdot\text{Loss}) \le T_{\max} \Rightarrow \\
& T_W(P-L-D\cdot\text{Loss}) \\
& - T_W W_1(\text{Loss} - e^T D\cdot\text{Loss}) \le T_{\max} \Rightarrow \\
& T_W(P-L-D\cdot\text{Loss}) \le T_{\max}.
\end{aligned}$$

Since the equivalence between (12.3) and (17.3) [and, thus, the equivalence of each equation in (12.1)–(12.4) and (17.1)–(17.4)] indicates that LP2_W_1 is equivalent to LP2, the primary solutions for the two LP problems must be the same. The objective functions for the two LPs are also the same. Let f_W^* be the optimal value of the objective function of the LP2 and $f_{W_1}^*$ be the optimal value of the objective function of LP2_W_1. The shadow price for any transmission constraint k can be calculated as follows:

$$\mu_{2,k} = \frac{f_W^*(t_{\max,k}+\Delta) - f_W^*(t_{\max,k})}{\Delta},$$
$$\mu_{2,k}|_{W_1} = \frac{f_{W_1}^*(t_{\max,k}+\Delta) - f_{W_1}^*(t_{\max,k})}{\Delta}.$$

Since $t_{\max,k}$ are both increased by the same amount of Δ, the values of objective functions for both LP2 and LP2_W_1 are equal. That is

$$f_{W_1}^*(t_{\max,k}+\Delta) = f_W^*(t_{\max,k}+\Delta) \Rightarrow f_{W_1}^*(t_{\max,k}) = f_W^*(t_{\max,k}).$$

Thus, we have

$$\mu_{2,k}|_{W_1} = \mu_{2,k}. \tag{18}$$

This verifies that the μ_2 does not change with W. Similarly, we can calculate the shadow price of the loss equation as

$$\tau_2 = \frac{f_W^*(\text{offset}_W+\Delta) - f_W^*(\text{offset}_W)}{\Delta},$$
$$\tau_2|_{W_1} = \frac{f_{W_1}^*(\text{offset}_{W_1}+\Delta_1) - f_{W_1}^*(\text{offset}_{W_1})}{\Delta_1}.$$

According to (9), if offset_W is increased by Δ, offset_{W_1} has to be increased by Δ_1 such that the LP2 and LP2_W_1 can be equivalent, where $\Delta_1 = \Delta/(1-W_1^T LF_W)$. Thus, we have

$$\begin{aligned}\tau_2|_{W_1} &= \frac{f_{W_1}^*(\text{offset}_{W_1}+\Delta_1) - f_{W_1}^*(\text{offset}_{W_1})}{\Delta_1}\\ &= \frac{f_W^*(\text{offset}_W+\Delta) - f_W^*(\text{offset}_W)}{\frac{\Delta}{(1-W_1^T LF_W)}}\\ &= (1-W_1^T LF_W)\cdot\tau_2.\end{aligned} \tag{19}$$

The LMP components under the new slack weights W_1 are

$$\begin{aligned}LMP^{\text{Energy}}|_{W_1} &= \tau_2|_{W_1}\cdot e\\ &= (1-W_1^T LF_W)\cdot\tau_2\cdot e\\ &= (1-W_1^T LF_W)\cdot LMP^{\text{Energy}},\end{aligned} \tag{20}$$

$$\begin{aligned}LMP^{\text{Loss}}|_{W_1} &= -\tau_2|_{W_1}\cdot LF_{W_1}\\ &= -(1-W_1^T LF_W)\tau_2\\ &\quad\times\frac{(LF_W - W_1^T LF_W e)}{(1-W_1^T LF_W)}\\ &= LMP^{\text{Loss}} + W_1^T LF_W\\ &\quad\cdot LMP^{\text{Energy}},\end{aligned} \tag{21}$$

$$\begin{aligned}LMP^{\text{Congestion}}|_{W_1} &= \left(T_{W_1}^T - (T_{W_1} De^T)^T\right)\mu_2|_{W_1}\\ &= \Big(T_W - T_W W_1 e^T\\ &\quad - \left(T_W - T_W W_1 e^T\right) De^T\Big)^T \mu_2\\ &= \Big(T_W - T_W W_1 e^T\\ &\quad - \left(T_W De^T - T_W W_1 e^T\right)\Big)^T\mu_2\\ &= \left(T_W - T_W De^T\right)^T\mu_2\\ &= LMP^{\text{Congestion}}.\end{aligned} \tag{22}$$

The sum of the LMP loss and energy components is

$$\begin{aligned}&LMP^{\text{Energy}}|_{W_1} + LMP^{\text{Loss}}|_{W_1}\\ &= (1-W_1^T LF_W)\cdot LMP^{\text{Energy}} + LMP^{\text{Loss}}\\ &\quad + W_1^T LF_W\cdot LMP^{\text{Energy}}\\ &= LMP^{\text{Energy}} + LMP^{\text{Loss}}.\end{aligned} \tag{23}$$

From (22) and (23), we have

$$LMP|_{W_1} = LMP|_W. \tag{24}$$

From the above study, we can draw the following conclusions for the proposed model. The change in the selection of slack weights W:

1) does not influence the primary solution of the proposed model, including the dispatch of units;
2) does not affect the value of LMP;
3) does not change the LMP congestion component and congestion revenue;
4) does change the LMP energy and loss components;
5) does not change the sum of LMP energy and loss components, and the sum of the energy and loss revenues are kept the same.

These advantages of the proposed method are important in the calculation of predictable and defendable prices in electricity markets.

V. Distributed Slack as a Market Reference

In the traditional model, losses are usually balanced at a single slack bus. The loss sensitivities and constraint sensitivities are usually calculated based on that slack bus. Due to the concentration of all system losses at one network bus, the dc power-flow model could produce inaccurate power flows in the transmission lines compared to those of the ac power-flow model. The introduction of loss distribution factors D in LP2 provides more flexibility in balancing system losses than the LP1 model. The most significant feature of the proposed model is that the choice of buses for balancing losses is explicitly indicated. There are several ways of selecting loss distribution: one common methodology is to use the real-time or historical load ratios [2], [3].

In the LP1 model, the LMPs and the sum of the energy and loss components change with the change of the slack reference and the associated change in the loss and constraint sensitivities. Because loss and constraint sensitivities for the slack reference are zero, the energy component is the price at the slack reference.

As shown above, LMPs in the LP2 model are not dependent on the selection of the reference for a given loss distribution. However, it is often perceived by market participants that the best place to be located on the network is at the reference bus because both loss and congestion components are zero. The fact is that the amount of money charged or credited at a given location is the same no matter where the slack reference is located. It is more convenient to establish a "fair" location of the market reference that is not subject to abrupt changes. A load weighted distributed slack is proposed to serve as the market reference to keep reference at the system load center and avoid arguments about the location of the reference bus. Equations (7) and (8) provide means to convert sensitivities calculated at different times and by different applications (e.g., network applications and market-clearing applications). This ability creates additional convenience by removing restrictions on how and where loss and shift factors are calculated.

As discussed in Section IV, the energy component in LP2 is the shadow price of system losses equation and is dependent on

TABLE I
RESULTS OF LP1 WITH SLACK REFERENCE AT A

Bus Name	Bus Gen	Bus Load	Bus Loss	Loss Factor	Shift Factor	LMP	LMP Energy	LMP Loss	LMP Congestion
A	210.00	0.00	23.19	0.0000	0.0000	23.16	23.16	0.00	0.00
B	0.00	300.00	0.00	-0.0627	-0.1509	28.50	23.16	1.45	3.89
C	331.61	300.00	0.00	-0.0627	-0.2090	30.00	23.16	1.45	5.39
D	0.00	400.00	0.00	-0.0621	-0.3685	34.10	23.16	1.44	9.50
E	481.58	0.00	0.00	0.0117	0.1120	20.00	23.16	-0.27	-2.89
Total	1023.19	1000.00	23.19						

The shadow price for the energy balance equation is 23.16.
The shadow price for the system losses equation is 23.16.
The shadow price for the flow contraint on line ED is -25.78.

the slack weight W. In general, under the LP2 formulation, the energy component will not be the price at the slack reference unless the slack weight W is chosen to be the same as the loss distribution factor D. When $D = W$, the loss and constraint sensitivities at the market reference are 0 [see (A.12) and (A.20) in the Appendix]

$$T_D D = 0 \tag{25}$$

$$D^T LF_D = 0. \tag{26}$$

The congestion component will be

$$LMP^{\text{Congestion}} = T_D^T \mu_2 - (T_D D e^T)^T \mu_2 = T_D^T \mu_2. \tag{27}$$

And the loss distribution weighted LMP equals to the price at the market reference

$$\begin{aligned} D^T \cdot LMP &= D^T \cdot \left(\tau_2 e - \tau_2 \cdot LF_D + T_D^T \mu_2\right) \\ &= \tau_2 D^T e - \tau_2 D^T LF_D + (T_D D)^T \mu_2 \\ &= \tau_2 \cdot 1 - \tau_2 \cdot 0 + (0)^T \mu_2 \\ &= \tau_2. \end{aligned} \tag{28}$$

The LMP energy component in LP2 is the distributed market reference price, which is the weighted average of LMPs at each bus balancing a portion of the losses. This is more of a convenience than of a necessity.

VI. NUMERICAL EXAMPLE

In this section, a five-bus example [2] is presented to compare the results from the two models. Fig. 1 is a diagram of the PJM five-bus system. The flow limit on line E-D is 240 MW. The line impedances shown in the diagram are in the per unit values. In the following cases, the loads at busses B, C, and D are 300, 300, and 400 MW, respectively. The bid prices for generator Alta, Park City, Solitude, Sundance, and Brighton are \$14, \$15, \$30, \$40, and \$20 /MWh. The economic maximum of those generators is 110, 100, 500, 200, and 600 MW, respectively. In the following examples, line E-D is the limiting element.

In order to estimate the losses in the lossless network, a base case power flow is solved. The loss sensitivities for all of the buses are calculated with bus A as the slack reference. Several cases are constructed below to illustrate the differences between

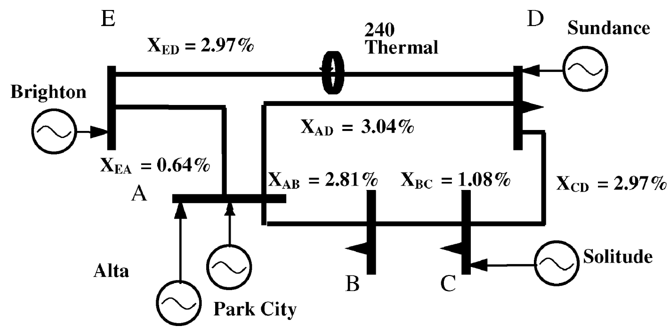

Fig. 1. Five-bus system diagram.

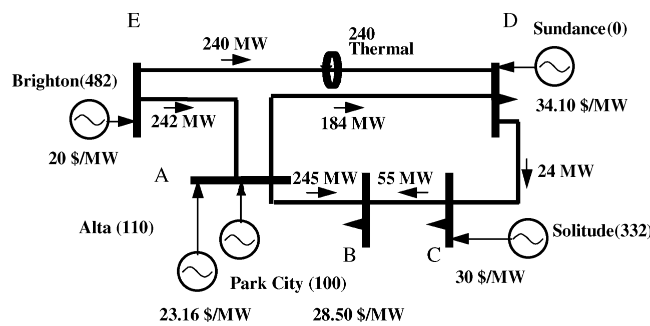

Fig. 2. Power-flow results of case 1 (note that 23 MW of losses are withdrawn at bus A).

models of LP1 and LP2. The loss offset is –24.11 MW, which is calculated against the reference bus A.

A. Case 1—LP1 With Slack Reference at A

In this case, the LP1 model is applied. The loss factors are calculated from the base power flow with the slack at bus A. The solution is summarized in Table I. The marginal units are Solitude and Brighton. The losses are balanced at the slack bus A from the nodal energy balance equation.

The power flows and bus prices are shown in Fig. 2.

B. Case 2—LP1 With Slack Reference at E

The LP1 model is considered again, but the slack bus is moved to bus E. The results are summarized in Table II. The line flows are shown in Fig. 3.

Cases 1 and 2 have different power flows and LMPs, which show that the LP1 model produces different solutions if the

TABLE II
RESULTS OF LP1 WITH SLACK REFERENCE AT E

Bus Name	Bus Gen	Bus Load	Bus Loss	Loss Factor	Shift Factor	LMP	LMP Energy	LMP Loss	LMP Congestion
A	210.00	0.00	0.00	-0.0118	-0.1120	23.20	20.00	0.24	2.96
B	0.00	300.00	0.00	-0.0753	-0.2629	28.46	20.00	1.51	6.96
C	323.52	300.00	0.00	-0.0753	-0.3210	30.00	20.00	1.51	8.49
D	0.00	400.00	0.00	-0.0747	-0.4805	34.21	20.00	1.49	12.72
E	490.28	0.00	23.80	0.0000	0.0000	20.00	20.00	0.00	0.00
Total	1023.80	1000.00	23.80						

The shadow price for the energy balance equation is 20.
The shadow price for the system losses equation is 20.
The shadow price for the flow contraint on line ED is -26.46

TABLE III
RESULTS OF LP2 WITH DISTRIBUTED SLACK REFERENCE AT B, C, AND D

Bus Name	Bus Gen	Bus Load	Bus Loss	D	W	Loss Factor	Shift Factor	LMP	LMP Energy	LMP Loss	LMP Congestion
A	210.00	0.00	0.00	0.00	0.00	0.0588	0.2554	23.07	31.12	-1.83	-6.22
B	0.00	300.00	6.57	0.30	0.30	-0.0002	0.1045	28.58	31.12	0.01	-2.55
C	348.59	300.00	6.57	0.30	0.30	-0.0002	0.0464	30.00	31.12	0.01	-1.13
D	0.00	400.00	8.76	0.40	0.40	0.0003	-0.1131	33.87	31.12	-0.01	2.76
E	463.31	0.00	0.00	0.00	0.00	0.0698	0.3674	20.00	31.12	-2.17	-8.95
Total	1021.91	1000.00	21.91								

The shadow price for the energy balance equation is 31.12.
The shadow price for the system losses equation is 31.12.
The shadow price for the flow contraint on line ED is -24.36.

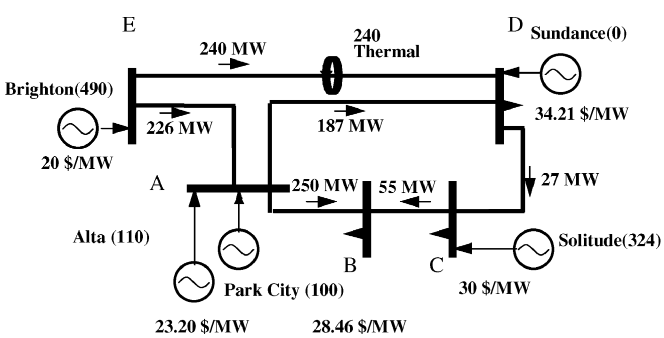

Fig. 3. Power-flow results of case 2 (note that 23.8 MW of losses are withdrawn at bus E).

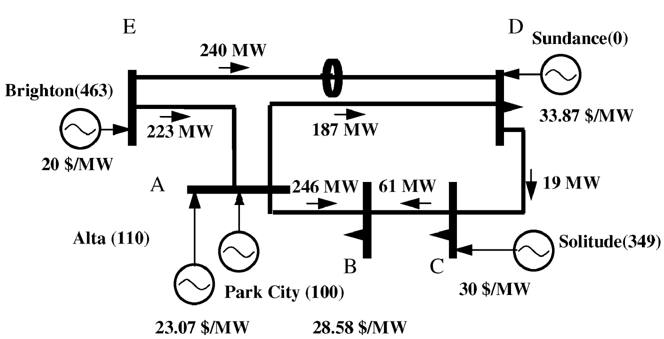

Fig. 4. Power-flow results of case 3 (note that 7, 7, and 9 MW of losses are withdrawn at busses B, C, and D, respectively).

slack bus of both loss and constraint sensitivities is changed. The system losses are balanced at the slack bus.

C. Case 3—LP2 With Distributed Slack Reference at B, C, and D

In this case, a distributed slack reference is adopted, and LP2 model is used. The slack weights and loss distribution factors are 0.3, 0.3, and 0.4 for busses B, C, and D, respectively. The results are shown in Table III.

The loss and constraint sensitivities are all described with respect to the same slack weights W. The power flows are shown in Fig. 4.

As the loss distribution is the same as the slack weight, the energy component is the weighted sum of LMPs at busses B, C, and D.

D. Case 4—LP2 With Slack Reference at A

In this case, the loss distribution factors are kept the same as in case 3, but the slack weights are changed. The results are shown in Table IV.

The primary solution, LMPs, and LMP congestion components in case 4 are the same as those in case 3, even though the loss and constraint sensitivities are relative to slack bus A. The energy and the loss components are different from those of case 3. However, the sum of energy and loss components in case 4 is the same at the same location as in case 3. Since the loss distribution factors are different from the slack weights $(D \neq W)$, the energy component defined in (15.1) is not equal to the weighted average of the LMPs at busses B, C, and D.

Case 1 is a special form of the LP2 model in which both the loss distribution factors and the slack reference are at bus A.

TABLE IV
RESULTS OF LP2 WITH SLACK REFERENCE AT A

Bus Name	Bus Gen	Bus Load	Bus Loss	D	W	Loss Factor	Shift Factor	LMP	LMP Energy	LMP Loss	LMP Congestion
A	210.00	0.00	0.00	0.00	1.00	0.0000	0.0000	23.07	29.29	0.00	-6.22
B	0.00	300.00	6.57	0.30	0.00	-0.0627	-0.1509	28.58	29.29	1.84	-2.55
C	348.59	300.00	6.57	0.30	0.00	-0.0627	-0.2090	30.00	29.29	1.84	-1.13
D	0.00	400.00	8.76	0.40	0.00	-0.0621	-0.3685	33.87	29.29	1.82	2.76
E	463.31	0.00	0.00	0.00	0.00	0.0117	0.1120	20.00	29.29	-0.34	-8.95
Total	1021.91	1000.00	21.91								

The shadow price for the energy balance equation is 23.07.
The shadow price for the system losses equation is 29.29.
The shadow price for the flow contraint on line ED is -24.36.

A comparison of the results from case 1 and case 4 shows the change in the LP2 optimization due to the change in the loss distribution reference. As would be expected from changing the location of the loss withdrawals in the network model, the difference in case 1 and case 4 indicates that the change of loss distribution factors will result in different optimization problems.

VII. CONCLUSION

This paper presents a marginal loss pricing model developed jointly by the ISO New England and ALSTOM EAI. In the proposed model, the losses are balanced explicitly at selected locations in the system. The introduction of the loss distribution factors provides more flexibility in balancing the losses in the lossless dc power-flow model to achieve more realistic power flow and more defendable and predictable market-clearing results. The proposed model is invariant to the selection of the slack bus for losses and constraint sensitivities once the loss distribution factors are fixed. This gives stability to the market prices if network topology changes necessitate a change in reference and gives more freedom for the software components that produce the loss or constraint sensitivities, since they do not have to be synchronized with the market reference bus. The energy component of this proposed model is the distributed market reference price, which could serve as an index of the system-wide energy cost. However, the model does produce different LMPs when the loss distribution is changed, as the change in the location of loss balancing energy produces different power flow.

APPENDIX

A. Loss Sensitivity Calculation

In the following, the loss factors are calculated relative to the distributed slack reference. Let w_i be the weight for each bus i and $\sum_{i=0}^{n} w_i = 1$. The power-flow equations are as follows:

$$P - W_n p_{\text{slack}} = f_p(V, \theta, v_0, \theta_0), \tag{A.1}$$

$$Q - W_n q_{\text{slack}} = f_q(V, \theta, v_0, \theta_0), \tag{A.2}$$

$$P_o - w_0 p_{\text{slack}} = f_{po}(V, \theta, v_0, \theta_0), \tag{A.3}$$

$$Q_o - w_0 q_{\text{slack}} = f_{qo}(V, \theta, v_0, \theta_0). \tag{A.4}$$

The system losses are

$$P_{\text{loss}} = \sum_{i=0}^{n} (p_i) - p_{\text{slack}}, \tag{A.5}$$

where

- P and Q active and reactive power injection vectors that do not include bus zero;
- V and θ voltage and phase angle vectors that do not include bus zero;
- P_0 and Q_0 active and reactive power injection at bus zero;
- v_0 and θ_0 voltage and phase angle at bus zero;
- W_n vector of slack weights for bus 1 to N.

Since the voltage and the angle at the reference bus zero are constant, we have the following:

$$\begin{bmatrix} dP - W_n \cdot dp_{\text{slack}} \\ dQ - W_n \cdot dq_{\text{slack}} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_p}{\partial \theta} & \frac{\partial f_p}{\partial V} \\ \frac{\partial f_q}{\partial \theta} & \frac{\partial f_q}{\partial V} \end{bmatrix} \begin{bmatrix} d\theta \\ dV \end{bmatrix} = J \begin{bmatrix} d\theta \\ dV \end{bmatrix}, \tag{A.6}$$

$$\begin{bmatrix} dP_o - w_0 \cdot dp_{\text{slack}} \\ dQ_o - w_0 \cdot dq_{\text{slack}} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_{po}}{\partial \theta} & \frac{\partial f_{po}}{\partial V} \\ \frac{\partial f_{qo}}{\partial \theta} & \frac{\partial f_{qo}}{\partial V} \end{bmatrix} \begin{bmatrix} d\theta \\ dV \end{bmatrix}, \tag{A.7}$$

where J is the Jacobian matrix. When the fast-decoupled method is used [$(\partial f_{p,i}/\partial V) = 0$ and $(\partial f_{q,i}/\partial \theta) = 0$, for each bus i], we will have the following relationship:

$$\begin{bmatrix} dP_o - w_0 dp_{\text{slack}} \\ dQ_o - w_0 dq_{\text{slack}} \end{bmatrix} = \begin{bmatrix} \frac{\partial f_{po}}{\partial \theta} & \frac{\partial f_{po}}{\partial V} \\ \frac{\partial f_{qo}}{\partial \theta} & \frac{\partial f_{qo}}{\partial V} \end{bmatrix} J^{-1} \begin{bmatrix} dP - W_n dp_{\text{slack}} \\ dQ - W_n dq_{slack} \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{n} (kp_i \cdot (dp_i - w_i dp_{\text{slack}})) \\ \sum_{i=1}^{n} (lq_i \cdot (dq_i - w_i dq_{\text{slack}})) \end{bmatrix}, \tag{A.8}$$

where kp_i and lq_i are the coefficients associated with bus i as a result of the matrix multiplication. From (A.8), we have

$$dp_{\text{slack}} = \frac{\sum_{i=1}^{n} (kp_i \cdot dp_i) - dP_o}{\sum_{i=1}^{n} (kp_i \cdot w_i) - w_0}. \tag{A.9}$$

The loss sensitivity can be calculated from (A.5) and (A.9) as

$$dP_{\text{loss}} = \sum_{i=0}^{n}(dp_i) - dp_{\text{slack}} = \sum_{i=0}^{n}\left(1 - \frac{kp_i}{\sum_{i=1}^{n}(kp_i \cdot w_i) - w_0}\right)dp_i, \quad \text{(A.10)}$$

where we define $kp_0 = -1$. Thus, the loss sensitivity at each bus i is

$$LF_{W:i} = \frac{\partial P_{\text{loss}}}{\partial P_i} = 1 - \frac{kp_i}{\sum_{i=0}^{n}(kp_i \cdot w_i)}. \quad \text{(A.11)}$$

In addition, we have the following property:

$$W^T \cdot LF_W = \sum_{i=0}^{n} w_i\left(1 - \frac{kp_i}{\sum_{i=0}^{n}(kp_i \cdot w_i)}\right) = \sum_{i=0}^{n} w_i - \frac{\sum_{i=0}^{n}(kp_i \cdot w_i)}{\sum_{i=0}^{n}(kp_i \cdot w_i)} = 0. \quad \text{(A.12)}$$

When the slack weights are changed to W_1, the loss factor for bus i is changed to

$$\begin{aligned} LF_{W_1:i} &= 1 - \frac{kp_i}{\sum_{k=0}^{n}(kp_k \cdot w_{1,k})} \\ &= 1 - \frac{(1 - LF_{w,i}) \cdot \sum_{j=0}^{n}(kp_j \cdot w_j)}{\sum_{k=0}^{n}\left((1 - LF_{w,k})\sum_{j=0}^{n}(kp_j \cdot w_j) \cdot w_{1,k}\right)} \\ &= 1 - \frac{(1 - LF_{w,i}) \cdot \sum_{j=0}^{n}(kp_j \cdot w_j)}{\sum_{j=0}^{n}(kp_j \cdot w_j) \cdot \sum_{k=0}^{n}(w_{1,k} - LF_{w,k} \cdot w_{1,k})} \\ &= 1 - \frac{(1 - LF_{w,i})}{1 - \sum_{k=0}^{n}(LF_{w,k} \cdot w_{1,k})} \\ &= \frac{LF_{w,i} - W_1^T LF_W}{1 - W_1^T LF_W}. \end{aligned} \quad \text{(A.13)}$$

B. Constraint Sensitivity Calculation

To calculate the constraint sensitivity for a line with the distributed slack, dc power flow model is adopted. The nodal power balance equations are

$$B\theta = P - W_n p_{\text{slack}}, \quad \text{(A.14)}$$

$$p_{\text{slack}} = \sum_{i=0}^{n} p_i, \quad \text{(A.15)}$$

where B is the bus admittance matrix, the angle reference bus is bus 0, and P is the vector of injections modeled in the dc power flow including losses allocated according to distribution D. Let us assume the power flow on the line (i,j) is

$$P_{i,j} = y_{i,j} C_{i,j}^T \theta \quad \text{(A.16)}$$

where $C_{i,j}$ is the incidence matrix for line (i,j) and $y_{i,j}$ is the admittance of line (i,j).

Substituting (A.14) and (A.15) into (A.16), we will have the following:

$$\begin{aligned} dP_{i,j} &= y_{i,j} C_{i,j}^T d\theta \\ &= y_{i,j} C_{i,j}^T B^{-1}(dP - W_n dp_{\text{slack}}) \\ &= \sum_{k=1}^{n} sp_k dp_k - \left(\sum_{k=1}^{n} sp_k w_k\right)\sum_{k=0}^{n} dp_k \\ &= \sum_{k=1}^{n}\left(sp_k - \sum_{j=1}^{n} sp_j w_j\right)dp_k \\ &\quad - \left(\sum_{k=1}^{n} sp_k w_k\right)dp_0, \end{aligned} \quad \text{(A.17)}$$

where sp_k is the kth element of $y_{i,j} C_{i,j}^T B^{-1}$.

Let $sp_0 = 0$. Then (A.17) can be rewritten as

$$dP_{i,j} = \sum_{k=0}^{n}\left(sp_k - \sum_{j=0}^{n} sp_j w_j\right)dp_k. \quad \text{(A.18)}$$

Thus, the sensitivity of bus k to constraint (i,j) is

$$t_{w,k} = \frac{dP_{i,j}}{dp_k} = sp_k - \sum_{j=0}^{n} sp_j w_j. \quad \text{(A.19)}$$

In addition, we have the following property:

$$\begin{aligned} \sum_{k=0}^{n} w_k t_{w,k} &= \sum_{k=0}^{n} w_k sp_k - \sum_{k=0}^{n}\left(w_k \sum_{j=0}^{n} sp_j w_j\right) \\ &= \sum_{k=0}^{n} w_k sp_k - \sum_{k=0}^{n}(w_k)\sum_{j=0}^{n} sp_j w_j \\ &= 0. \end{aligned} \quad \text{(A.20)}$$

When the slack weights are changed to W_1, the constraint sensitivity of bus k is changed to

$$\begin{aligned} t_{W_1,k} &= sp_k - \sum_{j=0}^{n} sp_j w_{1j} \\ &= t_{W,k} + \sum_{j=0}^{n} sp_j w_j - \sum_{j=0}^{n}\left(t_{W,k} + \sum_{l=0}^{n} sp_l w_l\right)w_{1j} \\ &= t_{W,k} + \sum_{j=0}^{n} sp_j w_j - \sum_{j=0}^{n}(t_{W,k})\,w_{1j} \\ &\quad - \sum_{j=0}^{n}\left(\sum_{l=0}^{n} sp_l w_l\right)w_{1j} \\ &= t_{W,k} - \sum_{j=0}^{n}(t_{W,k})\,w_{1j}. \end{aligned} \quad \text{(A.21)}$$

In the case of multiple constraints, (A.21) becomes

$$T_{W_1} = T_W - T_W W_1 e^T. \tag{A.22}$$

References

[1] (2002) FERC NOPR. [Online]. Available: http://www.ferc.gov/Electric/rto/Mrkt-Strct-comments/nopr/Web-NOPR.pdf

[2] PJM Interconnection LLC, "Locational Marginal Pricing". [Online]. Available: http://www.pjm.com/training/downloads/ferc_pres.pdf

[3] Market Rule 1 [Online]. Available: http://www.iso-ne.com/smd/market_rule_1_and_NEPOOL_manuals/Market_Rule_1/

[4] FERC Electric Tariff—Attachment B [Online]. Available: http://www.nyiso.com/services/oatt.html

[5] T. Alvey, D. Goodwin, X. Ma, D. Streiffert, and D. Sun, "A security-constrained bid-clearing system for the New Zealand wholesale electricity market," *IEEE Trans. Power Syst.*, vol. 13, pp. 340–346, May 1998.

[6] M. Shahidehpour, H. Yamin, and Z. Li, *Market Operations in Electric Power Systems*. New York: Wiley, 2002.

[7] M. Ilic, F. Galiana, and L. Fink, *Power Systems Restructuring: Engineering and Economics*. Norwell, MA: Kluwer, 1998.

[8] Y.-H. Moon *et al.*, "Slack-bus independent penalty factor for regional spot pricing under deregulation," *Int. J. Elect. Power Energy Syst.*, vol. 24, no. 10, pp. 821–826, Dec. 2002.

[9] A. J. Wood and B. F. Wollenberg, *Power Generation, Operation and Control*, 2nd ed. New York: Wiley, 1996.

Eugene Litvinov (M'91) received the B.S. and M.S. degrees from the Technical University, Kiev, Ukraine, and the Ph.D. degree from Urals Polytechnic Institute, Sverdlovsk, Russia.

Currently, he is a Director of Technology at the ISO New England, Holyoke, MA. His main interests include power system market-clearing models, system security, computer applications to power systems, and information technology.

Tongxin Zheng (M'02) received the Ph.D. degree in electrical engineering from Clemson University, Clemson, SC, in 1999.

Currently, he is a Senior Analyst at the ISO New England. His main interests are power system optimization and electricity market design.

Gary Rosenwald received the B.S. and Ph.D. degrees in electrical engineering from the University of Washington, Seattle.

Currently, he is with ALSTOM EAI Corp., Bellevue, WA, where his focus is on development and implementation of deregulated electricity market solutions.

Payman Shamsollahi (S'96–M'97–SM'02) received the Ph.D. degree from the University of Calgary, Calgary, AB, Canada, in 1997.

Currently, he is a Senior Engineer with ALSTOM EAI Corp., Bellevue, WA. His research interests include deregulation applications, artificial intelligence, and power system control.