Final exam question: Use DP to solve Problem 5.2a. Bring to the final exam, pre-worked, as it will be problem #1.

Some comments about problem 5.2-a.

1. The problem gives "no-load costs" which I call no-load energy rate (NLER, MBTU/h) and incremental heat rate (IHR, BTU/kWhr) and fuel cost (FC, \$/MBTU). You can obtain the incremental cost (IC, \$/MWh), and the no-load costs (NLC, \$) according to:

$$IC(\text{Unit j}) = \frac{IHR(\text{Unit j}) * FC}{1000}$$
$$NLC(\text{Unit j}) = NLER(\text{Unit j}) * FC$$

2. The cost rate function for a unit is denoted in W&W as F (see p. 146) and given by

$$F(\text{Unit j}) = IC(\text{Unit j}) * P(\text{Unit j}) + NLC(\text{Unit j})$$

   and the production cost function is then given by

$$P_{\cos t}(\text{Unit j}) = F(\text{Unit j}) * T$$

   where T is the duration for which unit j generates power level P.

3. Our DP formula (Bellman's equation) is

$$F_{\cos t}(k,n) = \min_{m}\left\{F_{\cos t}(k-1,m) + S_{\cos t}(k-1,m:k,n)\right\}$$

   However, you need to be clear that in the UC problem, S_{cost}(k-1,m:k,n) includes only the start-up costs (depending on whether it is in cold reserve or hot). But the "transition" from state m in period k-1 to state n in period k also involves production costs. Let's call these production costs P_{cost}. We could model them as part of the transition costs from state m in period k-1 to state n in period k, in which case, notationally, we would write

$$P_{cost}(k-1,m:k,n).$$

   On the other hand, this cost is incurred for *every* transition into state k at period n. Therefore we could also write it as a sort of "nodal" cost, $P_{cost}(k,n)$, and this is what W&W do. Therefore, Bellman's equation as used in W&W for forward DP is:

$$F_{\cos t}(k,n) = \min_{m}\{P_{\cos t}(k,n) + S_{\cos t}(k-1,m:k,n) + F_{\cos t}(k-1,m)\}$$

and this is what I suggest you to use for this problem. Note in the above that $P_{cost}(k,n)$ is the sum of all the individual unit costs, i.e.,

$$P_{\cos t}(k,n) = \sum_{j \text{ committed in (k,n)}} P_{\cos t}(\text{Unit j})$$

4. The problem requires that you construct a priority list. This you will do simply by ordering the units based on increasing incremental cost. Observe that this will define your states according to the below:

   State 1: One unit up.
   State 2: Two units up.
   State 3: Three units up.
   State 4: Four units up.
   State 5: Five units up.
5. You will not need to consider separate states for cooling or banking – there is just a single start-up cost for each unit. Therefore you can account for the given start-up cost in the arc corresponding to the state transition where a unit is started.
6. So a first cut DP state diagram is to draw the above five states for each period of interest (there are 8 hours but only 5 periods: 0, 2, 4, 6, 8 hours correspond to k=0,1,2,3,4). Then you will need to draw the appropriate arcs and label them with appropriate costs.
7. You should reduce the states to consider for each time period to be only those for which the load can be met.
8. The problem indicates X=3 and N=3. You need to understand what is X and N.
   - Parameter X: In the step where we reach out and select possible states in period k, we will only allow ourselves to consider X states in period k. That is, we will only construct arcs from a state in period k-1 to no more than X states in period k.

- Parameter N: In the step where we have constructed the arcs and are solving the minimization problem to identify the least-cost path to arrive at node n in period k, we look back to period k-1 and add the node cost to each arc value and then take the minimum. We will only allow ourselves to look back at N different states in period k-1.

Parameters X and N are illustrated in Fig. 5.5 of W&W, pg. 144.