

HW # 5 SOLUTIONS

$$1. \text{ (a) } F_1 = 180 + 6.72P_1 + .002P_1^2$$

$$F_2 = 743.5 + 6.426P_2 + .00826P_2^2$$

$$F_3 = 360 + 6.75P_3 + .00225P_3^2$$

$$\frac{\partial F_1}{\partial P_{g1}} = 6.72 + .004P_1 = \lambda$$

$$\frac{\partial F_2}{\partial P_{g2}} = 6.426 + .01652P_2 = \lambda$$

$$\frac{\partial F_3}{\partial P_{g3}} = 6.75 + .0045P_3 = \lambda$$

$$P_{g1} + P_{g2} + P_{g3} = 450$$

$$\begin{bmatrix} .004 & 0 & 0 & -1 \\ 0 & .01652 & 0 & -1 \\ 0 & 0 & .0045 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -6.72 \\ -6.426 \\ -6.75 \\ 450 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \end{bmatrix} = \begin{bmatrix} 205.94 \\ 67.66 \\ 176.39 \\ 7.54 \end{bmatrix}$$

Note all values are within limits.

$$\text{(b) } P_{g1} = \frac{\lambda - 6.72}{.004} \quad \Rightarrow \quad \frac{\partial P_T}{\partial \lambda} = \frac{1}{.004} + \frac{1}{.01652} + \frac{1}{.0045} = 532.75$$

$$P_{g2} = \frac{\lambda - 6.426}{.01652}$$

$$P_{g3} = \frac{\lambda - 6.75}{.0045} \quad \Rightarrow \quad \Delta \lambda = \frac{\Delta P_T}{532.75}$$

Try $\lambda = 10$

$$P_{g1} = \frac{10 - 6.72}{.004} = 820$$

$$P_{g2} = \frac{10 - 6.426}{.01652} = 216.34$$

$$P_{g3} = \frac{10 - 6.75}{.0045} = 722.22$$

$$1, 758.6 \Rightarrow \Delta P_T = 900 - 1758.6 = -858.6$$

$$\Rightarrow \Delta \lambda = \frac{-858.6}{532.75} = -1.61$$