HW # 5 Solutions

1

The LaGrangian of the problem is given by —

$$\mathcal{L} = x_1^2 + 4x_2^2 - \lambda(x_1 + x_2 - 4).$$

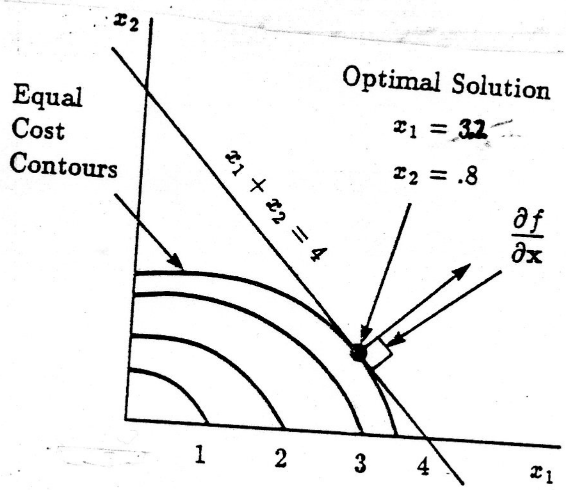
Hence, the necessary conditions of optimality are given by —

$$\frac{\partial \mathcal{L}}{\partial x_1} = 0 = 2x_1 - \lambda$$

$$\frac{\partial \mathcal{L}}{\partial x_2} = 0 = 8x_2 - \lambda$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 = -x_1 - x_2 + 4.$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -1 \\ 0 & 8 & -1 \\ -1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -4 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0.8 \\ 6.4 \end{bmatrix}$$



The optimal value is:

$f(\underline{x}) = x_1^2 + 4x_2^2$

$= 3.2^2 + 4(.8)^2 = 12.8$

2

First, we restate the inequality constraints to be in the form: $g_i \le 0$ —

$$\begin{aligned} g_1(\mathbf{x}) &= 4 - x_1 - x_2 \le 0 \\ g_2(\mathbf{x}) &= x_1 - 3 \le 0 \\ g_3(\mathbf{x}) &= x_2 - 5 \le 0. \end{aligned}$$

The Langrangian of the problem is —

$$\begin{aligned} \mathcal{L} = {} & x_1^2 + x_2^2 + \beta_1(4 - x_1 - x_2) \\ & + \beta_2(x_1 - 3) + \beta_3(x_2 - 5). \end{aligned}$$

Our guess is that the solution will be on the line defined by —

$$x_1 + x_2 - 4 = 0.$$