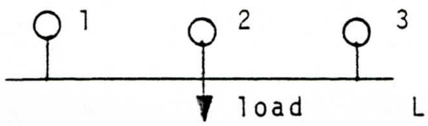
Problem 9.1

Given a single area with three generating units



Load Base: 1000 MVA

Unit	Rating	Speed Droop R (per unit on unit base)
1	100 MVA	.01
2	500 MVA	.015
3	500 MVA	.015

The units are initially loaded:

$P_1 = 80$ MW

$P_2 = 300$ MW

$P_3 = 400$ MW

a) Assume D = 0. What is the new generation on each unit for 50 MW load increase?

$$\Delta\omega = \frac{-\Delta P}{\sum_{1}^{3} \frac{1}{R} + D}$$

Under the common base of 1000 MVA

$$R_1 = 0.01 \left(\frac{1000}{100}\right) = 0.1 \text{ pu}$$

$$R_2 = 0.015 \left(\frac{1000}{500}\right) = 0.03 \text{ pu}$$

$$R_3 = 0.015 \left(\frac{1000}{500}\right) = 0.03 \text{ pu}$$

$$D = 0$$

$$\Delta P = \frac{50}{1000} = 0.05 \text{ pu}$$

$$\Delta\omega = \frac{0.05}{\frac{1}{0.1} + \frac{1}{0.03} + \frac{1}{0.03}} = -652.17 \times 10^{-6} \text{pu}$$

$\omega = \omega o + \Delta\omega$

$\omega = 60 - 652.17 \times 10^{-6} (60)$

$= 59.96$ Hz

Change in unit generation:

$\Delta P_1 = -\frac{\Delta\omega}{R_1} = 0.00652$ pu $= \quad 6.52$ MW

$\Delta P_2 = -\frac{\Delta\omega}{R_1} = 21.739 \times 10^{-3}$ pu $= 21.74$ MW

$\Delta P_3 = -\frac{\Delta\omega}{R_1} = 21.739 \times 10^{-3}$ pu $= \underline{21.74 \text{ MW}}$

50 MW

New Generation:

$P_1 = 80 + 6.52 = \quad 86.52$ MW

$P_2 = 300 + 21.74 = 321.74$ MW

$P_3 = 400 + 21.74 = 421.74$ MW

b) Repeat for D = 1 pu (on load base)

$$\Delta\omega = \frac{-0.05}{\frac{1}{0.1} + \frac{1}{0.03} + \frac{1}{0.03} + 1} = -643.78 \times 10^{-6}$$

$\omega = \omega o + \Delta\omega$

$= 60 - 643.78 \times 10^{-6}(60)$

$= 59.9614$ Hz

Changes in units generation:

$\Delta P_1 = -\frac{\Delta\omega}{R_1} = 6.4378 \times 10^{-3} = \quad 6.44$ MW

$\Delta P_2 = -\frac{\Delta\omega}{R_2} = 21.46 \times 10^{-3} = 21.46$ MW

$\Delta P_3 = -\frac{\Delta\omega}{R_3} = 21.46 \times 10^{-3} = \underline{21.46 \text{ MW}}$

49.36 MW

$D\Delta\omega = \underline{\quad .64 \text{ MW}}$

50 MW

Problem 9.1, continued

New Generation

$$P_1 = 80 + 6.44 = \underline{\underline{86.44 \text{ MW}}}$$

$$P_2 = 300 + 21.46 = \underline{\underline{321.46 \text{ MW}}}$$

$$P_3 = 400 + 21.46 = \underline{\underline{421.46 \text{ MW}}}$$

Problem 9.2

Using the values of R and D in each area, for example 9B, resolve for the 100 MW load change in area 1 under the following conditions:

Area 1 Base MVA = 2000 MVA

Area 2 Base MVA = 500 MVA

Then solve for a load change of 100 MW occurring in area 2 with R's and D's as in Example 9B and Base MVA for each area as above.

Area 1	Area 2
Base 2000 MVA	Base 500 MVA
R = 0.01 pu	R = 0.02 pu
D = 0.8 pu	D = 1.0 pu

Using 2000 MVA as common base:

$R_1 = 0.01 \qquad R_2 = 0.02 \times \frac{2000}{500} = 0.08 \text{ pu}$

$D = 0.8 \qquad D_2 = 1.0 \times \frac{500}{2000} = 0.25 \text{ pu}$

$\Delta P_L = \frac{100}{2000} = 0.05 \text{ pu}$

$$\Delta\omega = \frac{-0.05}{\frac{1}{0.01} + \frac{1}{0.08} + 0.8 + 0.25} = -440.33 \times 10^{-6}$$

$f = 60 - 440.33 \times 10^{-6}(60) = 59.974 \text{ Hz}$

$\Delta P_{tie} = \Delta\omega(\frac{1}{R_2} + D_2) = 440.33 \times 10^{-6}(\frac{1}{0.08} + 0.25) = -5.614 \times 10^{-3}$

$\Delta P_{tie} = -11.2285 \text{ MW}$

$\Delta P_{mech\ 1} = \frac{\Delta\omega}{R_1} = 44.033 \times 10^{-3} \text{pu} = 88.067 \text{ MW}$

$\Delta P_{mech\ 2} = \frac{\Delta\omega}{R_2} = 5.504 \times 10^{-3} \text{pu} = 11.008 \text{ MW}$

Load

$-D_1\Delta\omega = .7045 \text{ MW}$

$-D_2\Delta\omega = .2202 \text{ MW}$

$\Delta P_L = \Delta P_{mech\ 1} + \Delta P_{mech\ 2} + (D_1+D_2)\ \Delta\omega$

b) Solve for a load change of 100 MW occurring in area 2 for the same MVA base as the first part:

$$\Delta\omega = 440.335 \times 10^{-6} \text{pu}$$

$$f = 59.9734 \text{ Hz}$$

$$\Delta P_{tie} = \Delta\omega\left(\frac{1}{R_1} + D_1\right) = 440.335 \times 10^{-6}\left(\frac{1}{0.01} + 0.8\right) = 0.04439 \text{ pu}$$

$$\Delta P_{tie} = 88.78 \text{ MW}$$

All the other values of $\Delta P_{mech\ 1}$, $\Delta P_{mech\ 2}$, $D_1\Delta\omega$ and $D_2\Delta\omega$ are the same as the first part of this problem.

Problem 9.3

$$\Delta\omega_1 = \left(\frac{1}{m_1 S + D_1}\right)(-\Delta P_L - \Delta P_{tie})$$

$$\Delta\omega_2 = \left(\frac{1}{m_2 S + D_2}\right)(\Delta P_{tie})$$

$$\Delta P_{tie} = (T/S)(\Delta\omega_1 - \Delta\omega_2)$$

after much algebra:

$$\Delta\omega_1 = \frac{(m_2 S^2 + D_2 S + T)(-\Delta P_L)}{(m_1 m_2)S^3 + (m_1 D_2 + m_2 D_1)S^2 + (m_1 T + D_1 D_2 + m_2 T)s + (D_1 + D_2)T}$$

An interesting result may be seen if we divide by T:

$$\Delta\omega_1 = \frac{\left(\frac{m_2 S^2 + D_2 S}{T} + 1\right)\ (-\Delta P_L)}{\frac{m_1 m_2 S^3}{T} + \frac{(m_1 D_2 + m_2 D_1)S^2}{T} + \left(m_1 + m_2 + \frac{D_1 D_2}{T}\right)S + (P_1 + P_2)}$$

If we now let T go to infinity we get:

$$\Delta\omega_1 = \frac{-\Delta P_L}{(m_1 + m_2)S + (D_1 + D_2)}$$

Which says that when machines are "tightly coupled" (i.e. $T \to \infty$), the equivalent model has $m = m_1 + m_2$ and $D = D_1 + D_2$. (see Figure 9.6).

(b) with $\Delta P_L(s) = \frac{\Delta P_L}{s}$, then

$$\Delta\omega_1 = \frac{(M_2 s^2 + D_2 s + T)\left(-\frac{\Delta P_L}{s}\right)}{(M_1 M_2)s^3 + (M_1 D_2 + M_2 D_1)s^2 + (M_1 T + D_1 D_2 + M_2 T)s + (D_1 + D_2)T}$$

Then the steady state frequency is

$$\Delta\omega_{1\infty} = \lim_{t\to\infty} \Delta\omega_1(t) = \lim_{s\to 0} s\Delta\omega_1(s) = \frac{-\Delta P_L T}{(D_1 + D_2)T} = \frac{-\Delta P_L}{D_1 + D_2}$$

Therefore, using the given data, we have that

$$\Delta\omega_{1\infty} = \frac{-0.2}{1 + 0.75} = \underline{\underline{-0.1143}}$$

(c) From part (a), we know $\Delta\omega_1 = \frac{1}{M_1 s + D_1}\left[-\Delta P_L(s) - \Delta P_{tie}(s)\right]$

$$\Rightarrow \frac{\Delta P_{tie}(s)}{M_1 s + D_1} = \frac{-\Delta P_L(s)}{M_1 s + D_1} - \Delta\omega_1.$$ Substituting the expression from (a) for $\Delta\omega_1$ leads to:

$$\frac{\Delta P_{tie}(s)}{M_1 s + D_1} = \frac{-\Delta P_L(s)}{M_1 s + D_1} - \frac{(M_2 s^2 + D_2 s + T)(-\Delta P_L(s))}{M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1)s^2 + (M_1 T + D_1 D_2 + M_2 T)s + (D_1 + D_2)T}$$

$$\frac{\Delta P_{tie}(s)}{\Delta P_L(s)} = -1 + \frac{(M_1 s + D_1)(M_2 s^2 + D_2 s + T)}{\Delta(s)} = \frac{-\Delta(s) + (M_1 s + D_1)(M_2 s^2 + D_2 s + [illegible])}{\Delta(s)}$$

$$= \frac{M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1)s^2 + (M_1 T + D_1 D_2 + M_2 T)s + (D_1 + D_2)T + M_1 M_2 s^3 + M_1 D_2 s^2 + M_1 T s + D_1 M_2 s^2 + D_1 D_2 s + D_1 [illegible]}{\Delta(s)}$$

$$= \frac{2M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1 + M_1 D_2 + M_2 D_1)s^2 + (M_1 T + D_1 D_2 + M_2 T + M_1 T + D_1 D_2)s + (2D_1 + D_2)}{\Delta(s)}$$

$$= \frac{2M_1 M_2 s^3 + 2(M_1 D_2 + M_2 D_1)s^2 + (2M_1 T + M_2 T + 2D_1 D_2)s + (2D_1 + D_2)T}{\Delta(s)}$$

where $\Delta(s) = M_1 M_2 s^3 + (M_1 D_2 + M_2 D_1)s^2 + (M_1 T + D_1 D_2 + M_2 T)s + (D_1 + D_2)T$ is the characteristic equation for the transfer function.

The frequency of oscillation is given by the roots of the characteristic equation. Using the data from the problem statement, this is

$$\Delta(s) = (3.5)(4)s^3 + (3.5+0.75+4.1)s^2 + (3.5\cdot 7.54 + 1\cdot 0.75 + 4\cdot 7.54)s + (1.$$

$$= 14s^3 + 6.625s^2 + 57.3s + 13.195$$

Using the "roots" command in Matlab provides the roots of the above characteristic equation:

$$\lambda_1, \lambda_2 = -0.1199 \pm j2.0056$$

$$\lambda_3 = -0.2335$$

So the frequency of oscillation is <u>2.0056 rad/sec</u> or <u>0.3192 Hz</u>

To see what happens as the stiffness increases ($T \to \infty$ is the same as $X \to 0$ when X is the tie line reactance) to the frequency of oscillation, we do the same thing as was done in part (a), divide $\Delta(s)$ by T and then take $T \to \infty$. Since $\Delta(s)$ here is the same as $\Delta(s)$ in part (a), we obtain the same result, i.e., $\Delta(s) = (M_1 + M_2)s + (D_1 + D_2)$.

This just says that a two machine system connected by a tie-line looks like a one-machine system when the reactance of the tie line goes to zero.