HW 3 SOLUTIONS
EE 553, Spring 2010, Dr. McCalley

1. Text Problem 11.1

Problem 11.1

The B_x matrix for this problem is:

$$[B_x] = \begin{bmatrix} 9 & 0 & 0 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 9.17 & -2.5 \\ -4 & 0 & -2.5 & 9.83 \end{bmatrix}$$

Then

$$[X] = \begin{bmatrix} .13791 & 0 & .01644 & .0603 \\ 0 & 0 & 0 & 0 \\ .01644 & 0 & .11913 & .03699 \\ .0603 & 0 & .03699 & .13567 \end{bmatrix}$$

a) Gen. Shift Sensitivity Coefficients for shift from bus 1 to bus 2:

line 1-2 $a_{1-2,1} = 5.(.13791) = .6896$

line 1-4 $a_{1-4,1} = 4.(.13791-.0603) = .3104$

line 2-3 $a_{2-3,1} = 6.67(0-.01644) = -.1097$

line 2-4 $a_{2-4,1} = 3.33(0-.0603) = -.2008$

line 3-4 $a_{3-4,1} = 2.5(.01644-.0603) = -.1097$

Line outage distribution factors

	k=1 line 1-2	k=2 line 1-4	k=3 line 2-3	k=4 line 2-4	k=5 line 3-4
ℓ=1 line 1-2	-	1.0	-.399	-.550	-.4
ℓ=2 line 1-4	1.0	-	.399	.550	.4
ℓ=3 line 2-3	-.353	.353	-	.450	-1.0
ℓ=4 line 2-4	-.647	.647	.599	-	.6
ℓ=5 line 3-4	-.353	.353	-1.0	.450	-

2. Text Problem 11.2

a) Contingency (outage) flow distribution on CKTS A, B, C and D for an outage on CKT A.

$P_{flow\ell} = P_{flow\ell}$ (before outage) + $d_{\ell,k} \cdot P_{flow_k}$ (before outage)

$P_{flowA} = 0$

$P_{flowB} = 200 + 0.9\ (350) = 515$ MW

$P_{flowC} = 175 + 0.06\ (350) = 196$ MW

$P_{flowD} = 275 + 0.04\ (350) = 289$ MW

Contingency (outage) flow distribution on CKTS A, B, C and D for an outage on CKT B.

$P_{flowA} = 350 + 0.8\ (200) = 510$ MW

$P_{flowB} = 0$

$P_{flowC} = 175 + 0.12\ (200) = 199$ MW

$P_{flowD} = 275 + 0.08\ (200) = 291$ MW

Contingency (outage) flow distribution on CKTS A, B, C and D for an outage on CKT C.

$P_{flowA} = 350 + .21\ (175) = 386.75$ MW

$P_{flowB} = 200 + .06\ (175) = 210.50$ MW

$P_{flowC} = 0$

$P_{flowD} = 275 + .73\ (175) = 402.75$ MW *OVERLOAD

Contingency (outage) flow distribution on CKTS A, B, C and D for an outage on CKT D.

$P_{flowA} = 350 + .14\ (275) = 388.50$ MW

$P_{flowB} = 200 + .04\ (275) = 211.0$ MW

$P_{flowC} = 175 + .82\ (275) = 400.50$ MW

$P_{flowD} = 0$

Problem 11.2, continued

b) Can you shift generation from gen. 1 to gen. 3 or from gen. 2 to gen. 3 so that no overloads occur? Since circuit D is overloaded with an outage of circuit C, we need a shift in circuit D's flow of:

$402.75 + \Delta fD = 350$ MW

$\Delta fD = -52.75$

$\Delta fD = (a_{D,i} + d_{D,c} \times a_{c,i})\ \Delta P_i$ ← change in generator i

For i = 1

$(a_{D,1} + d_{D,c} \times ac1) = 0.04 + .73(.06) = 0.0838$

for i = 2

$(a_{D,2} + d_{D,c} \times a_{c2}) = .36 + .73(.54) = 0.7542$

Now we can build a generation "lower list" of:

GEN2

GEN1

$P_{g2}^{0} = 400$ MW

$P_{g1}^{0} = 600$ MW

$$90 \leq P_2 \leq 400$$

$$100 \leq P_1 \leq 600$$

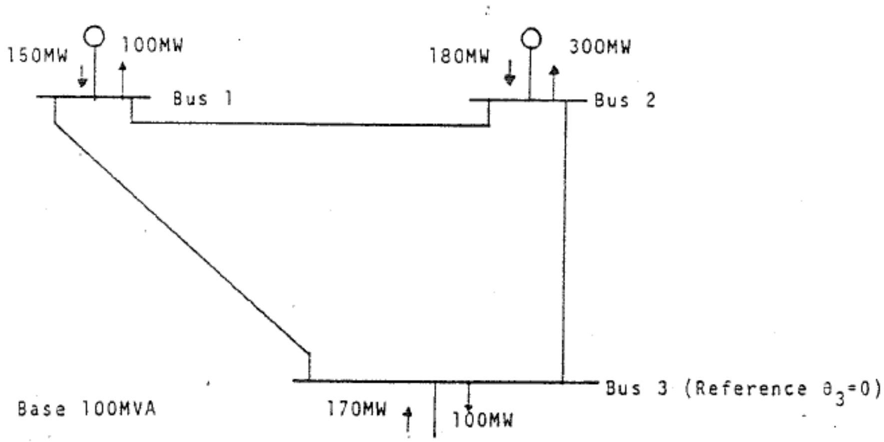
max ΔP_{g2} (lower) = 90 - 400 = -310 MW

max correction for ΔP_{G2} = (-310)(0.7542) = -233.80 MW enough

$\Delta P_{g2} = \frac{-52.75}{0.7542} = -69.94$ MW ← amount generator 2 should be lowered.

Since generator 2 can accomplish the required shift, we need not shift generation on generator 1. Also note that generator 3 would increase output from 300 MW to 369.94 MW which is still well within its limits.

3. Text Problem 11.3



Impedance

$X_{12} = .2$ pu

$X_{13} = .4$ pu

$X_{23} = .25$ pu

The X matrix is:

$$\begin{bmatrix} .2118 & .1177 & 0 \\ .1177 & .1765 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

The base load and generation are:

Bus	Load(MW)	Gen.(MW)	Gen.Min.(MW)	Gen.Max.(MW)
1	100	150	50	250
2	300	180	60	250
3	100	170	60	300

a) Find base power flow on the transmission lines:

$$\left.\begin{aligned} P_1 &= 150 - 100 = 50 \text{ MW} = 0.5 \text{ pu} \\ P_2 &= 180 - 300 = -120 \text{ MW} = -1.2 \text{ pu} \\ P_3 &= 170 - 100 = 70 \text{ MW} = 0.7 \text{ pu} \end{aligned}\right\} \quad \text{net injections}$$

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = \begin{bmatrix} 0.2118 & .1177 & 0 \\ 0.1177 & .1765 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} .5 \\ -1.2 \\ .7 \end{bmatrix} = \begin{bmatrix} -0.0353 \\ -0.1530 \\ 0 \end{bmatrix}$$

$$P_{ij} = \frac{\theta_i - \theta_j}{X_{ij}}$$

Problem 11.3, continued

$P_{12} = \frac{1}{0.2}(-0.03534+0.15295) = 0.588$ pu $\qquad P_{12} = 58.8$ MW

$P_{13} = \frac{1}{0.4}(-0.03534-0) = 0.088$ pu $\qquad P_{31} = 8.8$ MW

$P_{23} = \frac{1}{0.25}(-0.15295-0) = -0.612$ pu $\qquad P_{32} = 61.2$ MW

b) Calculate the generation shift factors for line 1-2

$a_{1-2,1} = \frac{1}{0.2}(X_{11}-X_{21}) = \frac{1}{0.2}(0.2118-0.1177) = 0.4705$

$a_{1-2,2} = \frac{1}{0.2}(X_{12}-X_{22}) = \frac{1}{0.2}(0.1177-0.1765) = -0.294$

Calculate the shift in generation on bus 1 and 2 so as to force the flow on line 1-2 to zero MW.

Max ΔP_{g2}(rise) = 250 - 180 = 70

Max Correction for P_{g2} = -0.294(70) = -20.58 MW

Flow 1-2 after shift gen 2. 58.80 - 20.58 MW = 38.22 MW

Correction with generator 1 = $\frac{-38.22}{0.4705}$ = -81.23 (lower)

New generation unit 1 = 150 - 81.23 = 68.77 MW

Final new generation:

P_{g1} = 68.77 MW $\qquad P_{12} = 0$

P_{g2} = 250 MW

P_{g3} = 181.23 MW

4. The loading on circuit ℓ is f_ℓ^0. It is known that the flow on circuit ℓ is limited by outage of circuit k under a stress condition defined by ΔP_i (e.g., a generation shift between buses I and j, or an increase in load at bus j compensated by an increase in generation at bus i). Determine the SOL for circuit ℓ expressed as a function of f_ℓ^0, f_ℓ^{max}, GSFs, and LODFs.
   Hint 1: The SOL for circuit ℓ occurs under the condition that

$$f_\ell^0 + \Delta f_\ell = f_\ell^{\max}$$

where f_ℓ^{max} is the emergency overload rating for circuit ℓ.

Hint 2: There are two network changes that determine Δf_ℓ.

Hint 3: The stress condition affects both the flow on circuit ℓ and the flow on circuit k.

The GSFs are linear estimates of the ratio: change in flow to change in power injection at a bus. A change of injection at bus i (ΔP_i) results in a change of MW power flow on line ℓ (Δf_ℓ), and the ratio of Δf_ℓ to ΔP_i is the GSF and is given by $a_{\ell i} = \Delta f_\ell / \Delta P_i$. Thus the change in flow on circuit ℓ due to change in injection ΔP_i is $\Delta f_\ell = a_{\ell i} \times \Delta P_i$. We note that ΔP_i necessarily implies an equal and opposite change in injection elsewhere in the network. We designate this other change as ΔP_j, recognizing that $\Delta P_i = -\Delta P_j$.

The LODFs are linear estimates of the ratio: change in flow on circuit ℓ due to outage of circuit k, denoted by Δf_ℓ, to pre-contingency flow on circuit k, denoted by f_k^0. In other words, it provides the fraction of pre-contingency flow on circuit k that appears on circuit ℓ following outage of circuit k, and is given by $d_{\ell,k} = \Delta f_\ell / f_k^0$. It is then clear that the change in flow on circuit ℓ due to the outage of circuit k is given by $\Delta f_\ell = d_{\ell,k} \times f_k^0$.

The SOL for circuit ℓ occurs under the condition that

$$f_\ell^0 + \Delta f_\ell = f_\ell^{\max} \qquad (1)$$

where f_ℓ^{max} is the emergency overload rating for circuit ℓ. Our goal is to identify the value of flow on circuit ℓ, denoted f_ℓ^0, that makes (1) true, under the condition that flows are changed due to a stress defined by ΔP_i (e.g., a generation shift between buses i and j, or an increase in load at bus j compensated by an increase in generation at bus i) and an outage of circuit k.

There are two network changes that determine Δf_ℓ. One is the stress, which is given by

$$\Delta f_\ell^{(1)} = a_{\ell i}\Delta P_i + a_{\ell j}\Delta P_j = (a_{\ell i} - a_{\ell j})\Delta P_i \qquad (2)$$

The other is the outage of circuit k, $\Delta f_\ell^{(2)} = d_{\ell,k} \times f_k^0$. However, the flow on circuit k will be affected by the stress according to $\Delta f_k = (a_{ki} - a_{kj}) \times \Delta P_i$. Therefore,

$$\Delta f_\ell^{(2)} = d_{\ell,k} f_k^0 + d_{\ell,k}(a_{ki} - a_{kj})\Delta P_i \qquad (3)$$

Combining (2) (representing the increase in flow on circuit ℓ due to the stress) and (3) (representing the increase in flow on circuit ℓ due to the outage), the combined change is

$$\begin{aligned}\Delta f_\ell &= \Delta f_\ell^{(1)} + \Delta f_\ell^{(2)} \\ &= (a_{\ell i} - a_{\ell j})\Delta P_i + d_{\ell,k} f_k^0 + d_{\ell,k}(a_{ki} - a_{kj})\Delta P_i\end{aligned} \qquad (4)$$

We desire the amount of stress ΔP_i necessary to increase the flow on circuit ℓ to f_ℓ^{max}, as expressed by (1). To determine this amount of stress, we substitute (4) into (1), resulting in

$$f_\ell^{\max} = f_\ell^0 + \left(a_{\ell i} - a_{\ell j}\right)\Delta P_i + d_{\ell,k} f_k^0 + d_{\ell,k}\left(a_{ki} - a_{kj}\right)\Delta P_i \quad (5)$$

Solving (5) for ΔP_i, we obtain:

$$\Delta P_i = \frac{f_\ell^{\max} - f_\ell^0 - d_{\ell,k} f_k^0}{a_{\ell i} - a_{\ell j} + d_{\ell,k}\left(a_{ki} - a_{kj}\right)} \quad (6)$$

The SOL for circuit ℓ is then the flow on circuit ℓ following the increased stress resulting from ΔP_i, $-\Delta P_j$, which can be computed from (1) and (2), resulting in

$$f_\ell^{SOL} = f_\ell^0 + \left(a_{\ell i} - a_{\ell j}\right)\Delta P_i \quad (7)$$

Substituting (6) into (7) results in

$$f_\ell^{SOL} = f_\ell^0 + \frac{\left(a_{\ell i} - a_{\ell j}\right)\left(f_\ell^{\max} - f_\ell^0 - d_{\ell,k} f_k^0\right)}{a_{\ell i} - a_{\ell j} + d_{\ell,k}\left(a_{ki} - a_{kj}\right)} \quad (8)$$

We emphasize that the SOL computed by (8) is a pre-contingency value and therefore represents a value that an operator can easily monitor. However, the constraint is driven by the post-contingency scenario. Thus, we see that if the operator keeps the flow on circuit ℓ below the value computed by (8), we can be sure that circuit ℓ will not overload if circuit k is outaged.

Equation (8) provides the SOL of circuit ℓ with respect to a particular contingency, which is the outage of circuit k. For a given contingency list, one must compute (8) for each contingency on the list, and then the contingency that gives the lowest value in (8) determines the SOL. This is not computationally intensive, but it can be made even less computational by recognizing that only a few contingencies typically drive a particular SOL.