Fuel Scheduling (Chapter 6 of W&W)

1.0 Introduction

In economic dispatch we assumed the only limitations were on the output of the generator: $P_{min} \leq P_g \leq P_{max}$. This assumed that we could set P_{gen} to any value we desired within the range, *at any time*, to achieve optimality.

In the UC problem, we went a step further in assuming we could even remove a unit at any time if that would lower cost.

In both of these problems, we were assuming that the fuel supply would always exactly match our need, that is, we could turn on the fuel when we wanted it or turn it off when we did not want it, without regards to how much or how little fuel we might be using.

This may not always be the case. It may be possible that one or more power plants are *energy-constrained* in some fashion. This means that the integral of the plant's power output over time will need to be either less than a certain value or greater than a certain value. This does not constrain power at any particular moment but it does constrain the time-integrated power (energy) over an interval of time. Since energy can be quantified in terms of amounts of fuel (natural gas, coal, oil), constraints on energy in a certain time interval are equivalent to constraints on fuel use in that time interval. This is why W&W-Ch 6 is called "Gen w/Limited Energy Supply."

It is possible to have lower bounds on fuel usage or upper bounds on fuel usage. In hydro systems, such bounds are dictated by water levels in reservoirs supplying hydroelectric power plants. Hydroelectric facilities are made more complex, however, because many reservoir systems have coupled reservoirs such that the energy constraints on one hydro plant are coupled with the energy requirements of the downstream and upstream hydro plants. We address hydro scheduling under the hydro-thermal coordination problem described in Chapter 7.

The problem of incorporating energy constraints for thermal units is referred to as the fuel scheduling (FS) problem.

In the FS problem, bounds on fuel usage are dictated by the *contracts* the power plant owner makes with the fuel supplier. Although this problem has been of interest for many years, reference [1] articulates it in the current context of LMP electricity markets:

> "In this context, it is known that a thermal plant that generates only when spot prices are above its operative cost can meet its financial contract obligations with a low effective operating cost: the plant does not operate in base load, being shut down during the months when spot prices are low (and benefit from spot market purchases at very low costs). In other words, **operational flexibility** is a very attractive characteristic for thermal plants in the hydro-based systems.
>
> However, **this operational flexibility, along with a low diversification of fuel markets, is in opposition with the needs of fuel producers** which have high fixed costs due to capital expenditures in developing production and transportation infrastructure. As a consequence, fuel supply agreements are heavily structured over contracts including **take-or-pay (ToP)** clauses. These are just financial agreements to reduce the volatility of the fuel producer's revenues and (usually) are not associated to consumption obligation. The ToP clauses impose an anticipated purchase of a minimum amount of fuel (on a daily, monthly and/or yearly basis), independently of its consumption. Often, the amount of fuel bought but not consumed is virtually "stored" (under the form of credits) for a pre-set period. During this period at anytime, the fuel can be recovered by the plant. This is known as **make-up clause**."

Operational flexibility is very attractive for thermal power plants in any LMP market, for the same reason as stated here – to take advantage of LMP price variability.

Fuel producers do not like up and down use of their fuel. This motivates ToP contacts.

ToP may or may not have "make-up clauses" which store what is not used. ToP contracts without make-up clauses are common.

There are main types of fuel for thermal power plants, with % of US electricity supply:

- Coal (51% in 2008, 45% in 2010, 42% in 2011)
- Natural gas (17% in 2008, 23% in 2010. 25% in 2011)
- Petroleum (less than 1% throughout)

Although petroleum is not used at a significant level to supply power plants on a percentage basis of national energy (<1%), there are some areas where it is a bit higher (e.g., in New England, 2008, oil-fired units comprised 24.9% of total capacity and 2% of energy production [2]).

It is clear from the above quote that suppliers of these fuels can operate more efficiently if they can obtain take-or-pay (ToP) contracts, effectively reducing their uncertainty.

A ToP contract is where the buyer pays for a minimum amount of fuel whether it is taken or not. The price paid under non-delivery may be equal to or less than the price paid for delivery.

The ToP contract enables the supplier to plan more precisely in regards to fuel production.

Most contracts also include a ceiling on how much fuel can be used in a certain period.

2.0 Minimizing costs for fleet with an energy constrained unit

We first consider that an owner of a fleet of N power plants, none of which are energy constrained, wants to minimize their costs over a time period. The plants are base-loaded and so guaranteed to be running. Notationally, we follow W&W (section 6.2) according to:

$F_{ij}(P_{ij})$: fuel cost rate for unit i during interval j.
P_{ij}: power output for unit i during interval j.
P_{Rj}: total demand in period j.
n_j: number of hours in j^{th} period.

We desire to solve the following optimization problem:

$$\begin{aligned} &\min \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) \\ &\text{subject to} \\ &P_{Rj} - \sum_{i=1}^{N} P_{ij} = 0, \qquad j = 1,\ldots, j_{\max} \end{aligned} \tag{1}$$

where the objective function is the total *cost* (not cost rate) over all periods, and the equality constraint is the requirement that in any one period, the supply must equal demand. We are not accounting for generator power production limits at this point.

Now let's consider if there is one machine (or a group of machines) for which the owner has engaged in a ToP contract with a ceiling. Mathematically, the simplest case is when the minimum "take" equals the ceiling. This means that over the j_{max} time periods, the unit must use exactly a particular amount of fuel. Let's call this amount of fuel q_{TOT}.

q_{TOT} will have units of RE ("raw energy") and will be different for each fuel type:

Coal:	RE=tons
Gas:	RE=ft^3
Oil:	RE=bbl (barrel=42 gallons)

Let the unit under the ToP contract be unit T=N+1 (so that we have N units with unlimited fuel contracts).

Define q_{Tj} as the fuel input rate for unit T in the j^{th} time period. Whereas units of q_{TOT} are RE, the units of q_{Tj} are RE/hr).

Note that q_{Tj} is a function of P_{Tj}, i.e., $q_{Tj}=q_T(P_{Tj})$. in RE/hr.

How to get this function $q_{Tj}=q_T(P_{Tj})$?

Define K_f as the energy content of fuel, in MBTU/RE. Typical values for K_f are:

Fuel	**K_f (MBTU/RE)**
Coal – anthracite	25.64 MBTU/tons
Coal - bituminous	23.25 MBTU/tons
Coal - lignite	22.72 MBTU/tons
Natural gas	0.001 MBTU/ft^3 (standard pressure)
Petroleum	5.88 MBTU/bbl

Another way to think about this is that the below quantities of fuel provide 1 MBTU:

Fuel	**$1/K_f$ (RE/MBTU)**	**Other units**
Coal – anthracite	0.039 tons/MBTU	78lbs/MBTU
Coal - bituminous	0.043 tons/MBTU	86lbs/MBTU
Coal - lignite	0.044 tons/MBTU	88lbs/MBTU
Natural gas	980 ft^3/MBTU (standard pressure)	10ft×10ft×10ft/MBTU
Petroleum	0.17 bbl/MBTU	7 gallons/MBTU

Recall that H, the heat rate, is MBTU/MWhr and that it is a function of the power generation level, i.e., $H=H(P_T)$. Then the fuel per MWhr will be $H(P_T)/K_f$, and multiplying this by the power generation level P_T results in the fuel rate, i.e.,

$$q_{Tj} = q_T(P_{Tj}) = \frac{H(P_{Tj})}{K_f} P_{Tj} \qquad (2)$$

Back to our "simplest case" where the minimum "take" equals the ceiling, the summation of fuel use by unit T over all time intervals must equal the fuel requirement q_{TOT}, that is

$$\sum_{j=1}^{j_{\max}} n_j q_{Tj} = q_{TOT} \qquad (3)$$

Our new problem, then, will be exactly as our old problem as posed in (1), with three exceptions.

1. Add the constraint (3).
2. Add fuel costs of our energy-constrained unit to the objective function.
3. Include in the power balance equation generation corresponding to the energy-constrained unit, for each time period.

Therefore, the problem statement for the constrained energy problem becomes:

$$\min \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) + \sum_{j=1}^{j_{\max}} n_j F_{Tj}(P_{Tj})$$

subject to

$$P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0, \qquad j = 1,\ldots, j_{\max} \tag{4}$$

$$\sum_{j=1}^{j_{\max}} n_j q_{Tj} = q_{TOT}$$

The above problem statement can be simplified, however, by recognizing that the energy-constrained unit must utilize exactly q_{TOT} of fuel, and since the cost of generation is dominated by the fuel costs, the last term in the objective function will be a constant. Since constants in the objective function do not affect the decision to be made (i.e., the solution, as indicated by the decision variables), then we may just remove that term, resulting in the following revised problem statement:

$$\min \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij})$$

subject to

$$P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0, \qquad j = 1,\ldots, j_{\max} \tag{5}$$

$$\sum_{j=1}^{j_{\max}} n_j q_{Tj} = q_{TOT}$$

Let's make the following definitions, corresponding to the two equality constraints of (5):

$$\psi_j(P_{ij}) = P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} = 0, \qquad j = 1,\ldots, j_{\max}$$
$$\phi(P_{Tj}) = \sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} = 0 \tag{6}$$

Then, the problem statement can be written more concisely as:

$$\min \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij})$$
$$\text{subject to}$$
$$\psi_j(P_{ij}) = 0, \qquad j = 1,\ldots, j_{\max} \tag{7}$$
$$\phi(P_{Tj}) = 0$$

The Lagrangian becomes:

Since F_{ij} is in \$/hr, when multiplied by hr gives units for the Lagrangian of \$. This means λ_j must be in \$/MW and γ must be \$/(fuel-units), where "fuel-units" are ft^3 (for natural gas) or ton (for coal).

$$\begin{aligned}\mathcal{L}(P_{ij}, P_{Tj}, \lambda_j, \gamma) &= \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) \\ &+ \sum_{j=1}^{j_{\max}} \lambda_j \left(P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) \\ &+ \gamma \left(\sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} \right) \\ &= \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) + \sum_{j=1}^{j_{\max}} \lambda_j \varphi_j(P_{ij}) + \gamma \phi(P_{Tj})\end{aligned} \tag{8}$$

Now we can write the optimality conditions.
The first optimality condition we will consider is

$$\frac{\partial \mathcal{L}}{\partial P_{ij}} = \frac{\partial}{\partial P_{ij}} \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij})$$

$$+ \frac{\partial}{\partial P_{ij}} \sum_{j=1}^{j_{\max}} \lambda_j \left(P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) \tag{9a}$$

$$+ \frac{\partial}{\partial P_{ij}} \gamma \left(\sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} \right) = 0$$

Performing the differentiation, we obtain

$$\frac{\partial \mathcal{L}}{\partial P_{ij}} = n_j \frac{\partial F_{ij}(P_{ij})}{\partial P_{ij}} - \lambda_j = 0 \tag{9b}$$

Now consider the optimality condition

$$\frac{\partial \mathcal{L}}{\partial P_{Tj}} = \frac{\partial}{\partial P_{Tj}} \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij})$$

$$+ \frac{\partial}{\partial P_{Tj}} \sum_{j=1}^{j_{\max}} \lambda_j \left(P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) \tag{10a}$$

$$+ \frac{\partial}{\partial P_{Tj}} \gamma \left(\sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} \right) = 0$$

Performing the differentiation, we obtain

$$\frac{\partial \mathcal{L}}{\partial P_{Tj}} = -\lambda_j + \gamma\, n_j \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}} = 0 \tag{10a}$$

Finally, the optimality conditions taking derivatives with respect to the Lagrange multipliers simply give us those constraints back again.

In summary, the optimality conditions result in the following equations:

$$\begin{array}{l} \text{Time Interval 1}: \dfrac{\partial \mathcal{L}}{\partial P_{i1}} = n_1 \dfrac{\partial F_{i1}(P_{i1})}{\partial P_{i1}} - \lambda_1 = 0, \qquad i = 1,\ldots,N \\ \vdots \\ \text{Time Interval } j_{\max}: \dfrac{\partial \mathcal{L}}{\partial P_{ij_{\max}}} = n_{j_{\max}} \dfrac{\partial F_{i1}(P_{ij_{\max}})}{\partial P_{ij_{\max}}} - \lambda_{j_{\max}} = 0, \quad i = 1,\ldots,N \end{array} \tag{11}$$

$$\begin{array}{l} \text{Time Interval 1}: \dfrac{\partial \mathcal{L}}{\partial P_{T1}} = -\lambda_1 + \gamma\, n_1 \dfrac{\partial q_T(P_{T1})}{\partial P_{T1}} = 0 \\ \vdots \\ \text{Time Interval } j_{\max}: \dfrac{\partial \mathcal{L}}{\partial P_{Tj_{\max}}} = -\lambda_{j_{\max}} + \gamma\, n_{j_{\max}} \dfrac{\partial q_T(P_{Tj_{\max}})}{\partial P_{Tj_{\max}}} = 0 \end{array} \tag{12}$$

Notice that if γ is specified, then the equations of (11), (12), and (13) from each time period are independent and each time period can be solved by lambda-search.

$$\begin{array}{l} \text{Time Interval 1}: \dfrac{\partial \mathcal{L}}{\partial \lambda_1} = P_{R1} - \sum_{i=1}^{N} P_{i1} - P_{T1} = 0 \\ \vdots \\ \text{Time Interval } j_{\max}: \dfrac{\partial \mathcal{L}}{\partial \lambda_{j_{\max}}} = P_{Rj_{\max}} - \sum_{i=1}^{N} P_{ij_{\max}} - P_{Tj_{\max}} = 0 \end{array} \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial \gamma} = \sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} = 0 \tag{14}$$

We will next consider two different ways of how to solve these equations.

3.0 Fuel scheduling solution by gamma search

The approach here will use an inner and an outer search mechanism.

- The inner search mechanism will be a standard lambda iteration *for each time period* to provide us with all generation levels for each time period. This search will be conditioned on an assumption regarding constrained fuel, represented by a particular value of gamma (γ). Each complete solution of the inner search will result in a total fuel usage corresponding to the chosen value of gamma.

- The outer search mechanism will be a search on gamma to find the certain value which results in the fuel usage being just equal to the desired value q_{TOT}.

This approach is illustrated in Fig. 6.3 of W&W. We will show it, but first we need to recall how to perform the lambda-search....

Implementation of the inner search mechanism requires expressing the power output for each machine in each time interval as a function of lambda for that time interval. From (11), we can write for time interval k:

$$\text{Time Interval k}: \frac{\partial \mathcal{L}}{\partial P_{ik}} = n_k \frac{\partial F_{ik}(P_{ik})}{\partial P_{ik}} - \lambda_k = 0, \qquad i = 1,\ldots,N \tag{15}$$

If we assume $F_{ik}(P_{ik})$ is quadratic, that is

$$F_{ik}(P_{ik}) = a_i + b_i P_{ik} + c_i P_{ik}^2 \qquad i = 1,\ldots,N \tag{16}$$

then

$$\frac{\partial F_{ik}(P_{ik})}{\partial P_{ik}} = b_i + 2c_i P_{ik} \qquad i = 1,\ldots,N \tag{17}$$

Substitution of (17) into (15) results in

$$\frac{\partial \mathcal{L}}{\partial P_{ik}} = n_k \left(b_i + 2c_i P_{ik}\right) - \lambda_k = 0, \qquad i = 1,\ldots,N \tag{18}$$

Solving (18) for P_{ik} results in

$$P_{ik} = \frac{\lambda_k - n_k b_i}{2c_i n_k} \qquad i = 1,\ldots,N \tag{18}$$

This will give us the generation levels for all of the non-energy-constrained units but not for the energy-constrained unit.

To obtain the generation level for the energy-constrained unit, we apply (12) to time interval k.

$$\frac{\partial \mathcal{L}}{\partial P_{Tk}} = -\lambda_k + \gamma\, n_k \frac{\partial q_T(P_{Tk})}{\partial P_{Tk}} = 0 \tag{19}$$

We need an expression for $q_T(P_{Tk})$, but this is just equation (2), repeated here for convenience:

$$q_{Tj} = q_T(P_{Tj}) = \frac{H(P_{Tj})}{K_f} P_{Tj} \tag{2}$$

where K_f is the energy content of fuel, in MBTU/RE (as we have seen). Recall the cost-rate curve (see notes on cost curves) will be

$$C_{Tj} = C_T(P_{Tj}) = KH(P_{Tj})P_{Tj} \tag{20}$$

where K is the price of the input fuel in \$/MBTU. Comparing (2) and (20), we see that the expression for q_{Tj} and C_{Tj} differ only by the constant K_f/K. The point of this is that the form of q_{Tj} will be the same as the form of C_{Tj}. Since C_{Tj} is just the cost-rate function for a unit, it will in general have a quadratic form. Thus, we may represent q_{Tj} likewise, i.e., for time period k,

$$q_{Tk}(P_{Tk}) = a_T + b_T P_{Tk} + c_T P_{Tk}^2 \tag{21}$$

One should be aware, however, that the coefficients in (21) differ from the coefficients of unit T's cost-rate function by K_f/K.

Differentiating (21) with respect to P_{Tk} results in

$$\frac{\partial q_{Tk}(P_{Tk})}{\partial P_{Tk}} = b_T + 2c_T P_{Tk} \tag{22}$$

Substitution of (22) into (19) results in

$$\frac{\partial \mathcal{L}}{\partial P_{Tk}} = -\lambda_k + \gamma\, n_k \left(b_T + 2c_T P_{Tk}\right) = 0 \tag{23}$$

Solving (23) for P_{Tk} results in

$$P_{Tk} = \frac{\lambda_k - \gamma\, n_k b_T}{2c_T \gamma\, n_k} \tag{24}$$

In summary, we have the following equations to express generation levels at each unit in each time period:

$$P_{ik} = \frac{\lambda_k - n_k b_i}{2c_i n_k} \qquad\qquad i = 1,\dots,N \tag{18}$$

$$P_{Tk} = \frac{\lambda_k - \gamma\, n_k b_T}{2c_T \gamma\, n_k} \tag{24}$$

This will allow us to perform lambda iteration. Note (24) is a function of gamma.

One thing remains, however. It will be useful to know how to update lambda following each iteration.

To obtain a lambda update formula, first recall the stopping criterion for lambda iteration is whether the generation meets the load. This is expressed by equation (13), which, for interval k, is

$$\frac{\partial \mathcal{L}}{\partial \lambda_{\text{k}}} = P_{R\text{k}} - \sum_{i=1}^{N} P_{i\text{k}} - P_{T\text{k}} = 0 \tag{25}$$

Solving for P_{Rk}, we obtain

$$P_{R\text{k}} = \sum_{i=1}^{N} P_{i\text{k}} + P_{T\text{k}} \tag{26}$$

Now substitute (18) and (24) into (26) to obtain

$$P_{R\text{k}} = \sum_{i=1}^{N} \frac{\lambda_k \text{ - } n_k b_i}{2c_i n_k} + \frac{\lambda_k - \gamma\, n_k b_T}{2c_T \gamma\, n_k} \tag{27}$$

Then we can differentiate (27) with respect to λ_k to obtain

$$\frac{\partial P_{R\text{k}}}{\partial \lambda_k} = \sum_{i=1}^{N} \frac{1}{2c_i n_k} + \frac{1}{2c_T \gamma\, n_k} \tag{28}$$

We may approximate (28) as

$$\frac{\Delta P_{R\text{k}}}{\Delta \lambda_k} = \sum_{i=1}^{N} \frac{1}{2c_i n_k} + \frac{1}{2c_T \gamma\, n_k} \tag{29}$$

Solving (29) for $\Delta\lambda_k$, we obtain

$$\Delta \lambda_k = \frac{\Delta P_{R\text{k}}}{\sum_{i=1}^{N} \frac{1}{2c_i n_k} + \frac{1}{2c_T \gamma\, n_k}} \tag{30}$$

The Lambda iteration method is illustrated in Fig. 1. Observe that the flow chart of Fig. 1 assumes that gamma is known; also the stopping criterion is based on the difference between demand & gen:

$$\Delta P_{Rk} = P_{Rk} - \sum_{i=1}^{N} P_{ik} - P_{Tk}$$

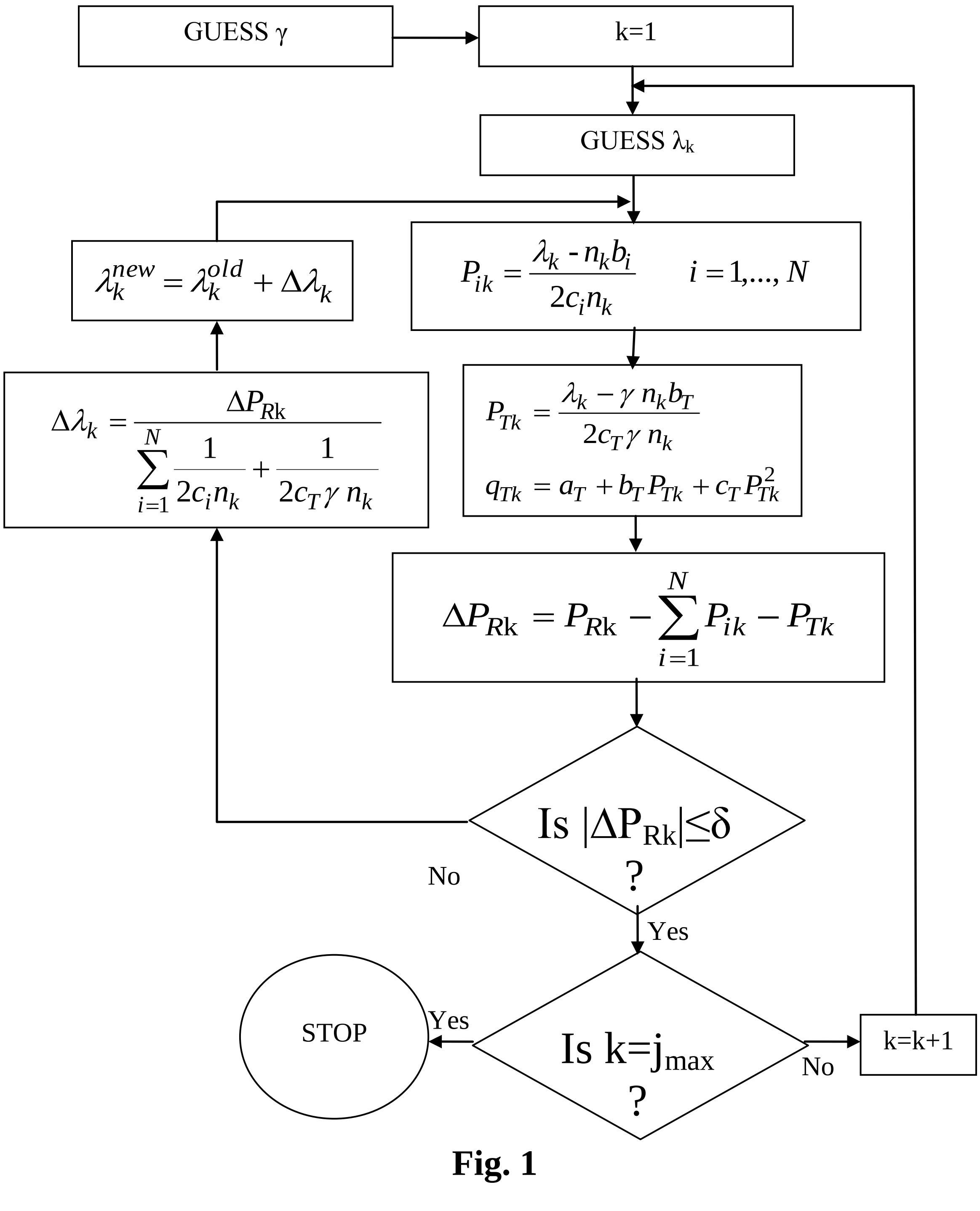


Fig. 1

So Fig. 1 illustrates the "inner loop" of the fuel scheduling solution. What we need to do now is to identify how to adjust gamma. To do that, let's begin by recalling where gamma came from.

We recall the Lagrangian, in (8), repeated here for convenience:

$$\begin{aligned}
\mathcal{L}(P_{ij}, P_{Tj}, \lambda_j, \gamma) \\
&= \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) \\
&+ \sum_{j=1}^{j_{\max}} \lambda_j \left(P_{Rj} - \sum_{i=1}^{N} P_{ij} - P_{Tj} \right) \\
&+ \gamma \left(\sum_{j=1}^{j_{\max}} n_j q_T(P_{Tj}) - q_{TOT} \right) \\
&= \sum_{j=1}^{j_{\max}} n_j \sum_{i=1}^{N} F_{ij}(P_{ij}) + \sum_{j=1}^{j_{\max}} \lambda_j \varphi_j(P_{ij}) + \gamma\phi(P_{Tj})
\end{aligned} \tag{8}$$

Here we see that that γ is the Lagrange multiplier on the fuel constraint.

To get a little better feel for γ, recall the fuel constraint (3):

$$\sum_{j=1}^{j_{\max}} n_j q_{Tj} = q_{TOT} \tag{3}$$

Recall (21)

$$q_{Tk}(P_{Tk}) = a_T + b_T P_{Tk} + c_T P_{Tk}^2 \tag{21}$$

Substitution of (21) into (3) results in

$$q_{TOT} = \sum_{j=1}^{j_{\max}} n_j \left(a_T + b_T P_{Tj} + c_T P_{Tj}^2 \right) \tag{31}$$

What we are after here is the dependence of q_{TOT} on γ. To get this, recall (24):

$$P_{Tk} = \frac{\lambda_k - \gamma\, n_k b_T}{2c_T \gamma\, n_k} \tag{24}$$

I will not go through the details here, but rather just articulate the procedure, which is to substitute (24) into (31) and then differentiate to obtain $\partial q_{TOT}/\partial\gamma$. Then, making the approximation

$$\frac{\partial q_{TOT}}{\partial \gamma} = \frac{\Delta q_{TOT}}{\Delta \gamma}$$

we may derive that

$$\Delta\gamma = \frac{-\Delta q_{TOT}}{\sum_{j=1}^{j_{\max}} \frac{\lambda_j^2}{2c_T n_j \gamma^3}} \tag{32}$$

I will include my hand-written derivation of (32) in the appendix to these notes.

We make three comments at this point.

1. From (32), we observe that if $\gamma>0$, then $\Delta\gamma$ is always opposite sign to Δq_T. That is,

   Increase γ to decrease fuel usage of Unit T.

   Decrease γ to increase fuel usage of Unit T.
2. Analysis of units (see Lagrangian) indicates γ is in \$/RE (recall "RE" is "Raw Energy Units." This indicates that γ is *like* a fuel price. This is consistent with comment #1 – thinking of selling fuel, if we raise the price, the demand decreases and if we lower the price, the demand increases.
3. We should recognize, however, that γ is not the fuel price. The fuel price, also with units of \$/MBTU, is given by K – see (20).

So what is γ?

Thinking in terms of optimization, as a Lagrange multiplier, we know that γ represents the change in the objective function of increasing the right-hand-side of the corresponding constraint by 1 unit.

Said in terms of this problem, γ is the additional cost of requiring the use of one additional RE unit of fuel at unit T during the next j_{max} time periods.

An interesting point made (pg. 174) and proved (Appendix of Chapter 7) is that the additional unit of fuel could be required in *any* time interval. Said another way, γ is constant over time.

The chapter 7 appendix also shows, however, that γ will vary in a particular time period if both the below are true.

- The problem includes constraints on how much fuel can be used in a particular time period (in addition to the total amount of fuel to be used over all time periods, which is q_{TOT}). This can happen for coal-fired plants due to the limitations of coal stored on site.
- The constraint for the particular time period is binding.

Using (32), we are now in a position to draw the entire fuel-scheduling flow chart, as given in Fig. 2a.

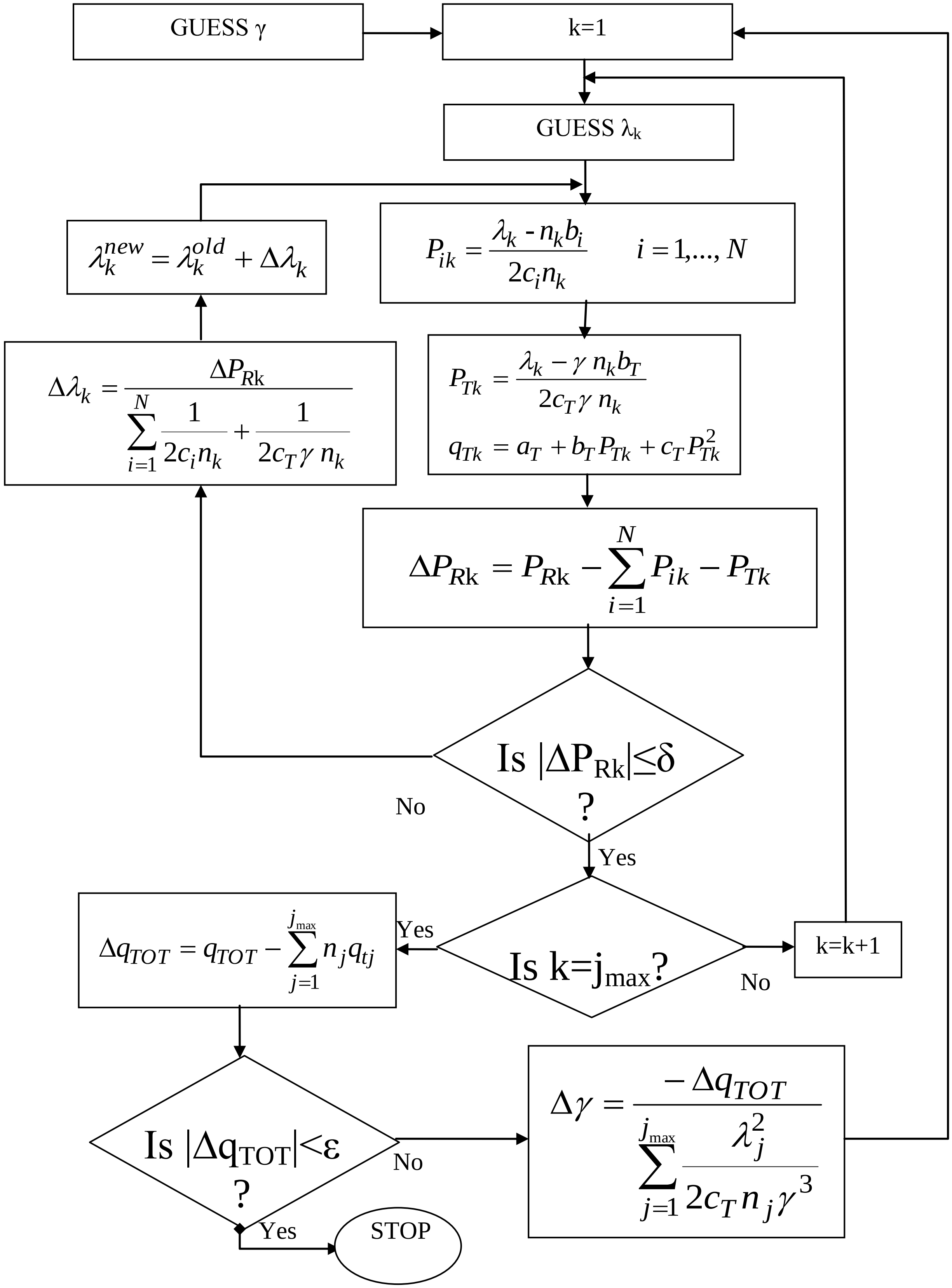

GUESS γ
k=1
GUESS λ_k
$\lambda_k^{new} = \lambda_k^{old} + \Delta\lambda_k$
$P_{ik} = \frac{\lambda_k - n_k b_i}{2c_i n_k} \quad i = 1,\ldots, N$
$\Delta\lambda_k = \frac{\Delta P_{Rk}}{\sum_{i=1}^{N} \frac{1}{2c_i n_k} + \frac{1}{2c_T \gamma n_k}}$
$P_{Tk} = \frac{\lambda_k - \gamma n_k b_T}{2c_T \gamma n_k}$
$q_{Tk} = a_T + b_T P_{Tk} + c_T P_{Tk}^2$
$\Delta P_{Rk} = P_{Rk} - \sum_{i=1}^{N} P_{ik} - P_{Tk}$
Is $|\Delta P_{Rk}| \le \delta$?
No
Yes
$\Delta q_{TOT} = q_{TOT} - \sum_{j=1}^{j_{max}} n_j q_{tj}$
Yes
Is $k = j_{max}$?
No
k=k+1
Is $|\Delta q_{TOT}| < \varepsilon$?
No
$\Delta\gamma = \frac{-\Delta q_{TOT}}{\sum_{j=1}^{j_{max}} \frac{\lambda_j^2}{2c_T n_j \gamma^3}}$
Yes
STOP

Fig. 2a

4.0 Composite generator production cost function (W&W, 6.3)

One problem with the solution procedure introduced in Section 3.0 is that a lambda iteration must be done for every time interval. If there are a large number of units (100), each lambda iteration can take some time, and j_{max} of these can be very computational.

We suggest an alternate procedure in this section. The idea is to obtain a composite cost curve for the non-energy-constrained units, and then the lambda iteration is only for two units rather than N.

Recall that when we introduced the unit commitment problem, we generated a composite cost curve for 4 units.We did this analytically by setting incremental cost expressions equal, using the power balance equation, and solving for each unit generation level. This was messy for four units and would be intractable for 100 units.

An alternative procedure is to develop the following table, a set of numerical data for all non-fuel constrained units, i=1,…,N.

λ	$P_S=\Sigma P_{gi}$	$F_S=\Sigma F_i(P_{gi})$
λ_{min}		
…		
λ_{max}		

where

$$\lambda_{\min} = \min\left[\left.\frac{dF_i}{dP_{gi}}\right|_{Pgi=Pgi,\min}, i=1,\ldots,N \right], \qquad \lambda_{\max} = \max\left[\left.\frac{dF_i}{dP_{gi}}\right|_{Pgi=Pgi,\max}, i=1,\ldots,N \right]$$

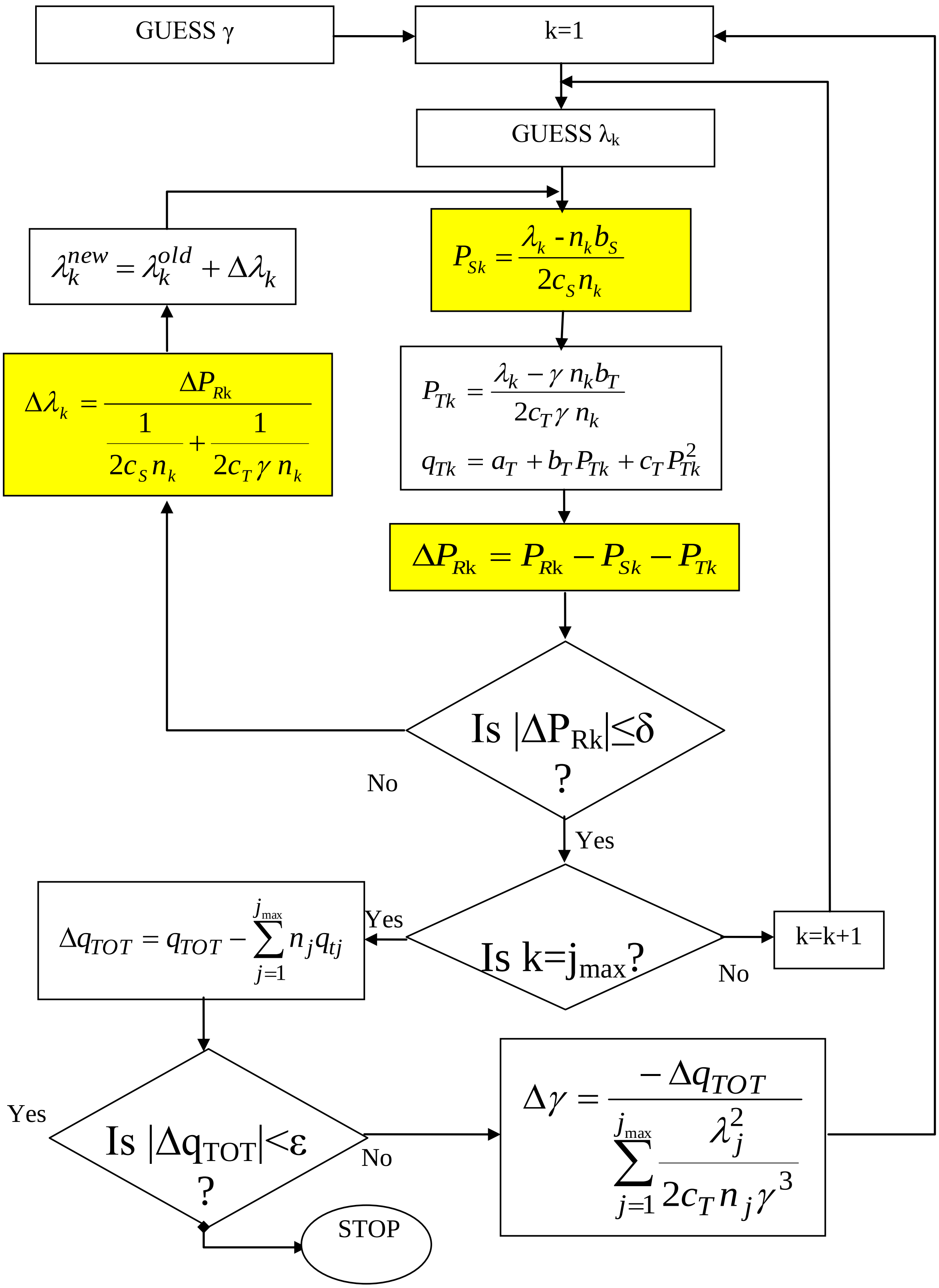
and each P_{gi} in the table is found from $\lambda=dF_i(P_{gi})/dP_{gi}$.

If a unit hits a limit, its output P_{gi} and cost $F_i(P_{gi})$ are held constant.

Note that the above values of λ are used simply to get the composite cost function and are not the same as the λ_k values in the algorithm (which are multiplied by n_k per eq. (9b)).

A curve-fitting approach can then be used to obtain the composite cost-rate function $F_S(P_S)$. Once this is done, then the algorithm of

Fig. 2a is applied, except that there is only 1 non-energy constrained unit, as shown in Fig. 2b (yellow boxes indicated changes).



5.0 Fuel scheduling gradient solution for optimality

Recall the optimality conditions from the Lagrangian. Repeating (9b),

$$\frac{\partial \mathcal{L}}{\partial P_{ij}} = n_j \frac{\partial F_{ij}(P_{ij})}{\partial P_{ij}} - \lambda_j = 0 \tag{9b}$$

and (10a),

$$\frac{\partial \mathcal{L}}{\partial P_{Tj}} = -\lambda_j + \gamma n_j \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}} = 0 \tag{10a}$$

Solving for λ_j in each of these equations, we obtain

$$\lambda_j = n_j \frac{\partial F_{ij}(P_{ij})}{\partial P_{ij}} \tag{33}$$

$$\lambda_j = \gamma n_j \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}} \tag{34}$$

Equating (33) and (34), we obtain

$$\lambda_j = \gamma n_j \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}} = n_j \frac{\partial F_{ij}(P_{ij})}{\partial P_{ij}} \tag{35}$$

Solving for γ results in

$$\gamma = \frac{\dfrac{\partial F_{ij}(P_{ij})}{\partial P_{ij}}}{\dfrac{\partial q_T(P_{Tj})}{\partial P_{Tj}}} \tag{36}$$

Observe the numerator and denominator of (36):

> Numerator is the Incremental Fuel Cost ($/MW-hr) for non-fuel constrained units during interval j.
>
> Denominator is Incremental Fuel Rate (RE/MW-hr) for the constrained unit during interval j.

Our above development shows that, for optimality, this ratio must be constant for all time intervals $j=1,\ldots,j_{max}$. This is consistent with our previous observation that γ should be constant over time. We can formulate an algorithm based on this fact, as illustrated in Fig. 3, adapted from Fig. 6.7a in your text, but we need a feasible schedule.

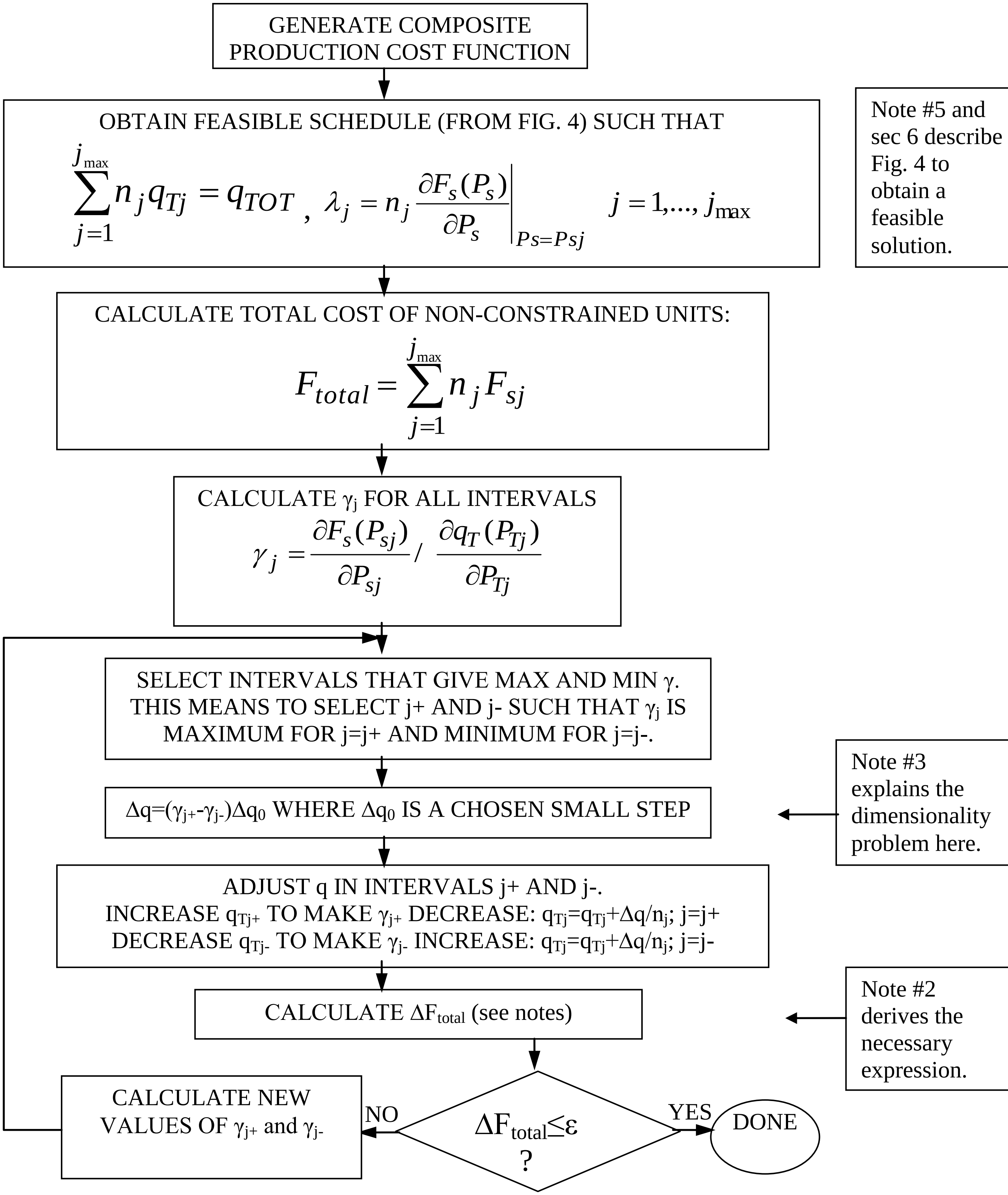


Fig. 3: Gradient solution (note #1 tells why it is "gradient")

We make four comments about the method illustrated in Fig. 3.

1. Your W&W text indicates on pg. 183 that the method "may be called gradient methods because q_{Tj} is treated as a vector and the γ_j values indicate the gradient of the objective function with respect to q_{Tj}." You can observe that this is the case from (36), using the composite cost-curve, i.e.,

$$\gamma_j = \frac{\left.\dfrac{\partial F_s(P_s)}{\partial P_s}\right|_{Psj}}{\left.\dfrac{\partial q_T(P_{Tj})}{\partial P_{Tj}}\right|_{P_{Tj}}} \approx \frac{\dfrac{\Delta F_s(P_{sj})}{\Delta P_{sj}}}{\dfrac{\Delta q_T(P_{Tj})}{\Delta P_{Tj}}} \tag{37}$$

and noting that it must be the case that $\Delta P_{Sj}=-\Delta P_{Tj}$, so that

$$\gamma_j = \frac{\dfrac{\Delta F_s(P_{sj})}{\Delta P_{sj}}}{\dfrac{\Delta q_T(P_{Tj})}{-\Delta P_{sj}}} = \frac{-\Delta F_s(P_{sj})}{\Delta q_T(P_{Tj})} \tag{38}$$

showing that γ_j may be interpreted as a sensitivity of the change in objective function to a change in the amount of fuel used in time interval j. If we think of all of $\underline{q}_T$ as a vector, e.g.,

$$\underline{q}_T = \begin{bmatrix} q_{T1} \\ \vdots \\ q_{Tj\max} \end{bmatrix}$$

then we may write

$$\underline{\gamma} = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_{j\max} \end{bmatrix} = -\nabla_{\underline{q}_T} F_s = -\begin{bmatrix} \dfrac{\partial F_s}{\partial q_{T1}} \\ \vdots \\ \dfrac{\partial F_s}{\partial q_{Tj\max}} \end{bmatrix}$$

as the gradient to F_s.

2. The next-to-last step in the algorithm of Fig. 3 indicates "CALCULATE ΔF_{total} (see notes)." This represents the *change in total costs* (and not the change in total cost rates) corresponding to the adjustments in fuel usage made in the previous step of the algorithm ("ADJUST q IN INTERVALS j+ and j-"). This calculation is done based on the following:

$$\gamma_{j+} - \gamma_{j-} = \frac{\dfrac{\partial F_s(P_{sj+})}{\partial P_{sj+}}}{\dfrac{\partial q_T(P_{Tj+})}{\partial P_{Tj+}}} - \frac{\dfrac{\partial F_s(P_{sj-})}{\partial P_{sj-}}}{\dfrac{\partial q_T(P_{Tj-})}{\partial P_{Tj-}}} \approx \frac{\dfrac{\Delta F_s(P_{sj+})}{\Delta P_{sj+}}}{\dfrac{\Delta q_T(P_{Tj+})}{\Delta P_{Tj+}}} - \frac{\dfrac{\Delta F_s(P_{sj-})}{\Delta P_{sj-}}}{\dfrac{\Delta q_T(P_{Tj-})}{\Delta P_{Tj-}}}$$

But $\Delta P_{Sj+}=-\Delta P_{Tj+}$, and $\Delta P_{Sj-}=-\Delta P_{Tj+}$. Making appropriate substitution results in

$$\gamma_{j+} - \gamma_{j-} \approx \frac{\dfrac{\Delta F_s(P_{sj+})}{\Delta P_{sj+}}}{\dfrac{\Delta q_T(P_{Tj+})}{-\Delta P_{sj+}}} - \frac{\dfrac{\Delta F_s(P_{sj-})}{\Delta P_{sj-}}}{\dfrac{\Delta q_T(P_{Tj-})}{-\Delta P_{sj-}}} = \frac{-\Delta F_s(P_{sj+})}{\Delta q_T(P_{Tj+})} + \frac{\Delta F_s(P_{sj-})}{\Delta q_T(P_{Tj-})}$$

We simplify the notation here as follows:

$$\gamma_{j+} - \gamma_{j-} \approx \frac{-\Delta F_{sj+}}{\Delta q_{Tj+}} + \frac{\Delta F_{sj-}}{\Delta q_{Tj-}} \tag{39}$$

Now recall that γ_{j+} is too high and γ_{j-} is too low, so we need to

- decrease γ_{j+} and increase γ_{j-}, which we do by
- increasing q_{Tj+} and decreasing q_{Tj-}.

The fuel increase in j+ must equal the fuel decrease in j-, therefore, and recalling that q_T is the fuel rate, we have that the fuel increase Δq is given by the following value (chosen to be the same sign as $n_{j+}\Delta q_{Tj+}$, which must be positive consistent with the bullets above which indicate we must increase q_{tj+}.)

$$\Delta q = n_{j+}\Delta q_{Tj+} = -n_{j-}\Delta q_{Tj-} \tag{40}$$

Therefore,

$$\Delta q_{Tj+} = \frac{\Delta q}{n_{j+}}, \qquad \Delta q_{Tj-} = \frac{-\Delta q}{n_{j-}} \tag{41}$$

Substituting (41) into (39) results in

$$\gamma_{j+} - \gamma_{j-} \approx \frac{-\Delta F_{sj+}}{\frac{\Delta q}{n_{j+}}} + \frac{\Delta F_{sj-}}{\frac{-\Delta q}{n_{j-}}}$$

$$= \frac{-\Delta F_{sj+} n_{j+}}{\Delta q} - \frac{\Delta F_{sj-} n_{j-}}{\Delta q} \tag{42}$$

$$= \frac{-1}{\Delta q}\left[\Delta F_{sj+} n_{j+} + \Delta F_{sj-} n_{j-}\right]$$

Bringing the -Δq over to the other side, we get:

$$\Delta F_{sj+} n_{j+} + \Delta F_{sj-} n_{j-} = -\Delta q\left(\gamma_{j+} - \gamma_{j-}\right) \tag{43}$$

and you can recognize the left-hand-side as ΔF_{total} that is required by the next-to-last step in the algorithm of Fig. 3.

Comment: If the algorithm is to *converge*, that is, if the F_{total} gets smaller with each iteration, then the left-hand-side of (43) must be negative. We see this must be the case by inspecting the right-hand-side of (43) since Δq was chosen positive (see (40)) and since $\gamma_{j+} > \gamma_{j-}$ by definition.

Comment: If all time intervals are chosen of equal duration, i.e., if $n_{j+} = n_{j-}$, then (43) becomes

$$\Delta F_{sj+} + \Delta F_{sj-} = -\frac{\Delta q}{n_{j+}}\left(\gamma_{j+} - \gamma_{j-}\right) \tag{44}$$

3. The flow chart step "$\Delta q = (\gamma_{j+} - \gamma_{j-})\Delta q_0$ WHERE Δq_0 IS A CHOSEN SMALL STEP" is not dimensionally correct as it stands, because gamma has units of $/RE, and when multiplied by RE, gives $, consistent with the above discussion regarding (43). You can assume, however, that the relation is really $\Delta q = [(\gamma_{j+} - \gamma_{j-})/1][\Delta q_0]$, where the "1" has the same units as γ. Then we observe that if Δq_0 has units of RE, then so will Δq. Basically, this relation is just telling us that if we want to correct two intervals j- and j+ for their fuel (or water) usage, we should choose an amount of fuel (or water) to shift that is proportional to the difference between the two interval's gamma values.

4. Observe that stopping criterion is to check to see if F_{total} changes significantly.
5. The second step in the algorithm of Fig. 3 indicates that it assumes a *feasible* (but not necessarily *optimal*) schedule in that
    - the fuel use requirement is met and
    - the generation dispatch of each time interval is “locally optimal” meaning that it would be the optimal dispatch if we considered only that time interval.

    The problem at hand is: from where do we obtain a feasible schedule? This is the topic of the next section.

6.0 Fuel scheduling gradient solution for feasibility

This approach is illustrated in Fig. 4.

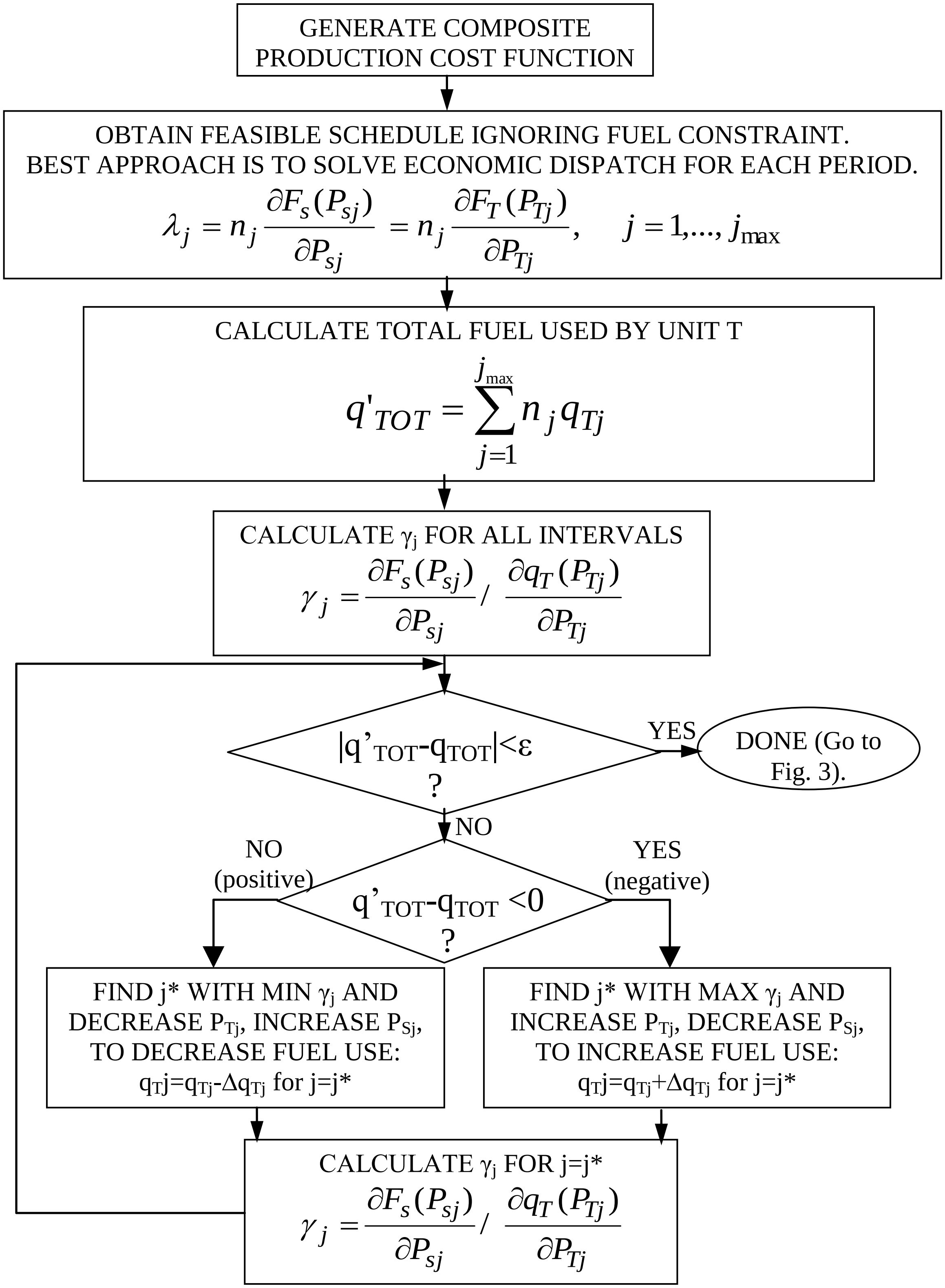

GENERATE COMPOSITE PRODUCTION COST FUNCTION
OBTAIN FEASIBLE SCHEDULE IGNORING FUEL CONSTRAINT. BEST APPROACH IS TO SOLVE ECONOMIC DISPATCH FOR EACH PERIOD.
$\lambda_j = n_j \frac{\partial F_s(P_{sj})}{\partial P_{sj}} = n_j \frac{\partial F_T(P_{Tj})}{\partial P_{Tj}}, \quad j = 1,\ldots, j_{max}$
CALCULATE TOTAL FUEL USED BY UNIT T
$q'_{TOT} = \sum_{j=1}^{j_{max}} n_j q_{Tj}$
CALCULATE γ_j FOR ALL INTERVALS
$\gamma_j = \frac{\partial F_s(P_{sj})}{\partial P_{sj}} / \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}}$
$|q'_{TOT}-q_{TOT}|<\varepsilon$?
YES
DONE (Go to Fig. 3).
NO
$q'_{TOT}-q_{TOT} <0$?
NO (positive)
YES (negative)
FIND j* WITH MIN γ_j AND DECREASE P_{Tj}, INCREASE P_{Sj}, TO DECREASE FUEL USE: $q_Tj=q_{Tj}-\Delta q_{Tj}$ for j=j*
FIND j* WITH MAX γ_j AND INCREASE P_{Tj}, DECREASE P_{Sj}, TO INCREASE FUEL USE: $q_Tj=q_{Tj}+\Delta q_{Tj}$ for j=j*
CALCULATE γ_j FOR j=j*
$\gamma_j = \frac{\partial F_s(P_{sj})}{\partial P_{sj}} / \frac{\partial q_T(P_{Tj})}{\partial P_{Tj}}$

Fig. 4

Two important observations may be made from Fig. 4:

1. The second block from the top indicates that the algorithm begins with a feasible schedule for the problem without the fuel constraint. The best way to obtain this is from an economic dispatch for each period.
2. The third block obtains the fuel used by the particular chosen schedule. This is computed by (31), repeated here for convenience:

$$q_{TOT} = \sum_{j=1}^{j_{\max}} n_j \left(a_T + b_T P_{Tj} + c_T P_{Tj}^2 \right) \qquad (31)$$

Your W&W text provides an example fuel scheduling problem, which is solved by gamma search (Example 6B, pg. 180-181) and by the gradient approach (Example 6C, pg. 184-185). Please review these two examples and know how to work them.

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[1] R. Chabar, M. Pereira, S. Granville, L. Barroso, and N. Iliadis, "Optimization of Fuel Contracts Management and Maintenance Scheduling for Thermal Plants under Price Uncertainty," Proc. of the Power Systems Conference and Exposition, Oct. 29-Nov 1, 2006.

[2] H. Chao, "Integration of Natural Gas and Electricity in new England and the rest of US," presentation at the 2008 APEX Conference in Sydney, Australia, October 13-14, 2008.