

# AGC (Chapter 9 of W&W)

## 1.0 Introduction

Synchronous generators respond to load-generation imbalances by accelerating or decelerating (changing speeds). For example, when load increases, generation slows down, effectively releasing some of its inertial energy to compensate for the load increase. Likewise, when load decreases, generation speeds up, absorbing the oversupply as increased inertial energy.

Because load is constantly changing, an unregulated synchronous generator has highly variable speed resulting in highly variable system frequency, an unacceptable situation because:

- NERC penalties for poor-performance (CPS)
- Load performance can be frequency-dependent
  - Motor speed (without a speed-drive)
  - Electric clocks
- Steam-turbine blades may lose life or fail under frequencies that vary from design levels.
- Some relays are frequency-dependent:
  - Underfrequency load shedding relays
  - Volts per hertz relays
- Frequency dip may increase for given loss of generation

The fact that frequency changes with the load-generation imbalance gives a good way to regulate the imbalance: use frequency (or frequency deviation) as a regulation signal. A given power system will have many generators, so we must balance load with total generation by appropriately regulating each generator in response to frequency changes.

As a result of how power systems evolved, the load-frequency control problem is even more complex. Initially, there were many isolated interconnections, each one having the problem of balancing its load with its generation. Gradually, in order to enhance reliability, isolated systems interconnected to assist one another in emergency situations (when one area had insufficient generation, another area could provide assistance by increasing generation to send power to the needy area via tie lines).

For many years, each area was called a *control area*, and you will still find this term used quite a lot in the industry. For example, on page 348 of W&W, the term is defined as “An interconnected system within which the load and generation will be controlled as per the rules in Fig. 9.20.” The rules of Fig. 9.20 are discussed in more depth in Section 4.0 below.

The correct terminology now, however, is *balancing authority area*, which is formally defined by the North American Electric Reliability Council (NERC) as [1]:

*Balancing authority area*: The collection of generation, transmission, and loads within the metered boundaries of the Balancing Authority. The Balancing Authority maintains load-resource balance within this area.

This definition requires another definition [1]:

*Balancing authority*: The responsible entity that integrates resource plans ahead of time, maintains load-interchange-generation balance

within a Balancing Authority Area, and supports Interconnection frequency in real time.

Each balancing authority will have its own AGC. The basic functions of AGC are identified in Fig 9.2 of W&W (Fig. 1a below).

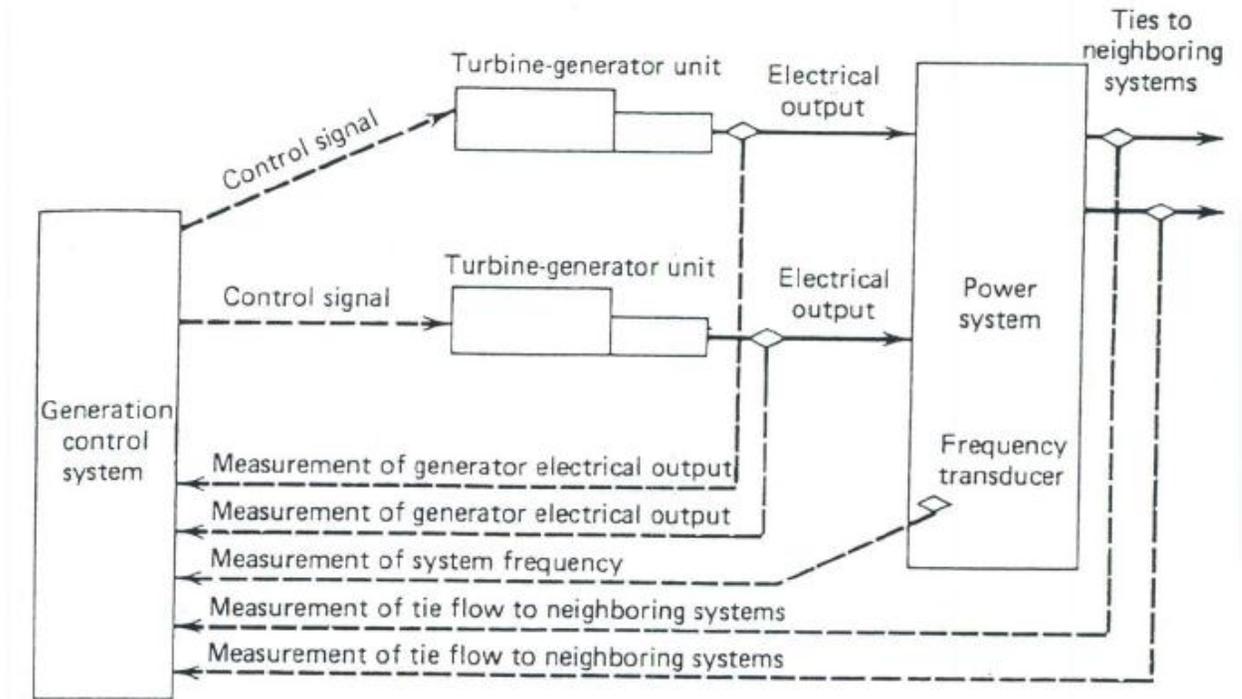


FIG. 9.2 Overview of generation control problem.

Fig. 1a

Figure 9.2 should also provide a “local” loop feeding back a turbine speed signal to the inputs of the generators. An alternative illustration showing this “local” loop is given in Fig. 1b.

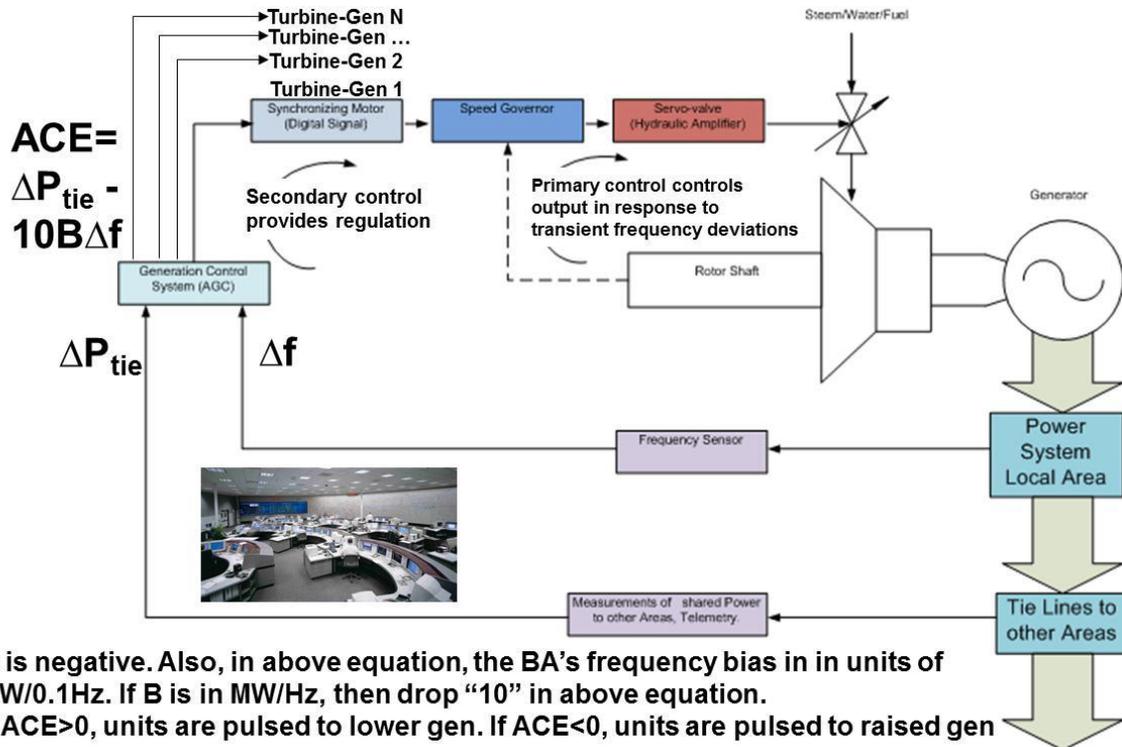


Fig. 1b

## 2.0 Historical View

The problem of measuring frequency and net tie deviation, and then redispatching generation to make appropriate corrections in frequency and net deviation was solved many years ago by engineers at General Electric Company, led by a man named Nathan Cohn. Their solution, which in its basic form is still in place today, is referred to as Automatic Generation Control, or AGC. We will study their solution in this section of the course. Dr. Cohn wrote an excellent book on the subject [2].



As its name implies, AGC is a control function.

There are two main levels of control:

1. Primary control

2. Secondary control

We will study each of these in what follows.

To provide you with some intuition in regards to the main difference between these two control levels, consider a power system that suddenly loses a large generation facility. The post-contingency system response, in terms of frequency measured at various buses in the power system, is shown in Figs. 2b and 2c. This is understood in the context of Figs 2a and 2d.

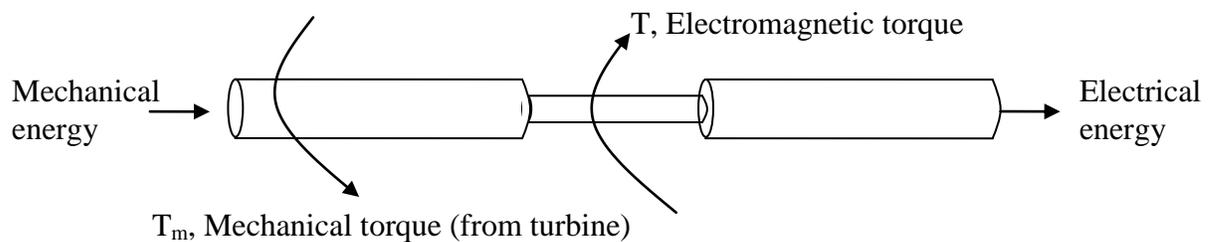


Fig. 2a

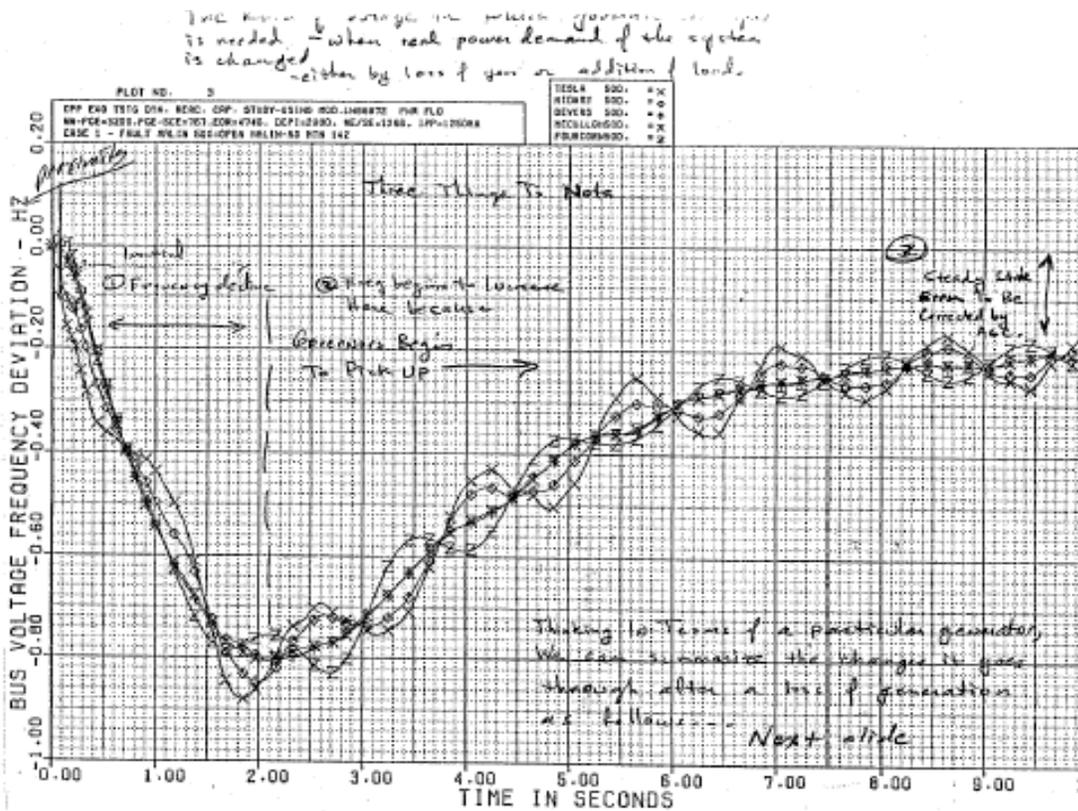


Fig. 2b: transient time frame

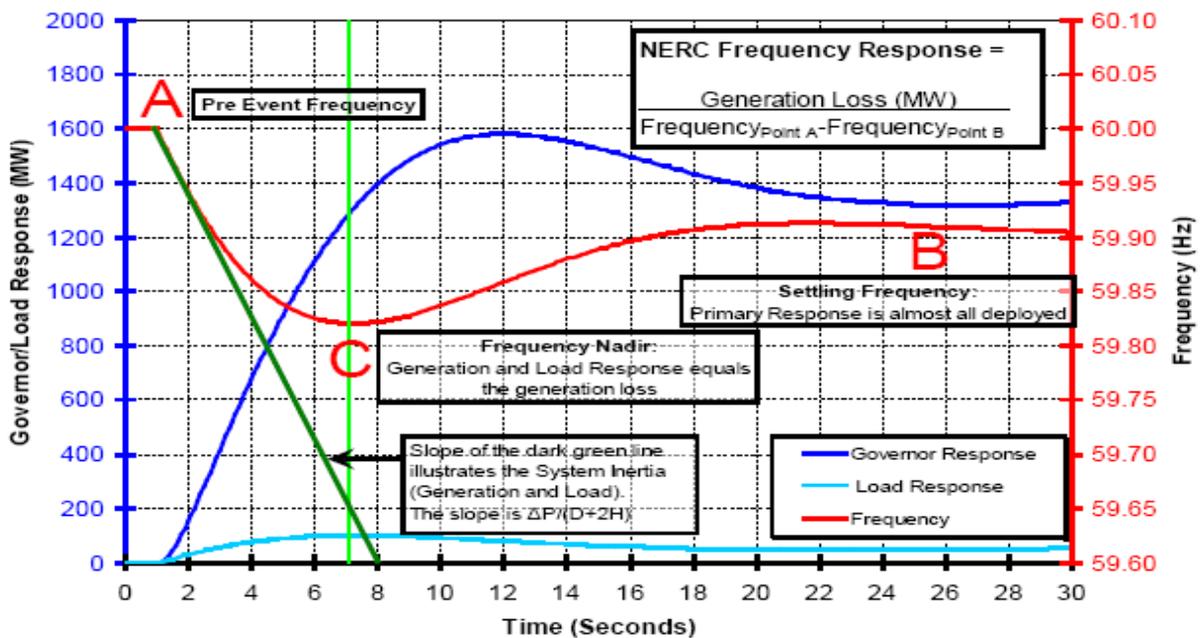


Figure 1 – Frequency Response Basics<sup>2</sup>

Fig. 2c: transient time frame

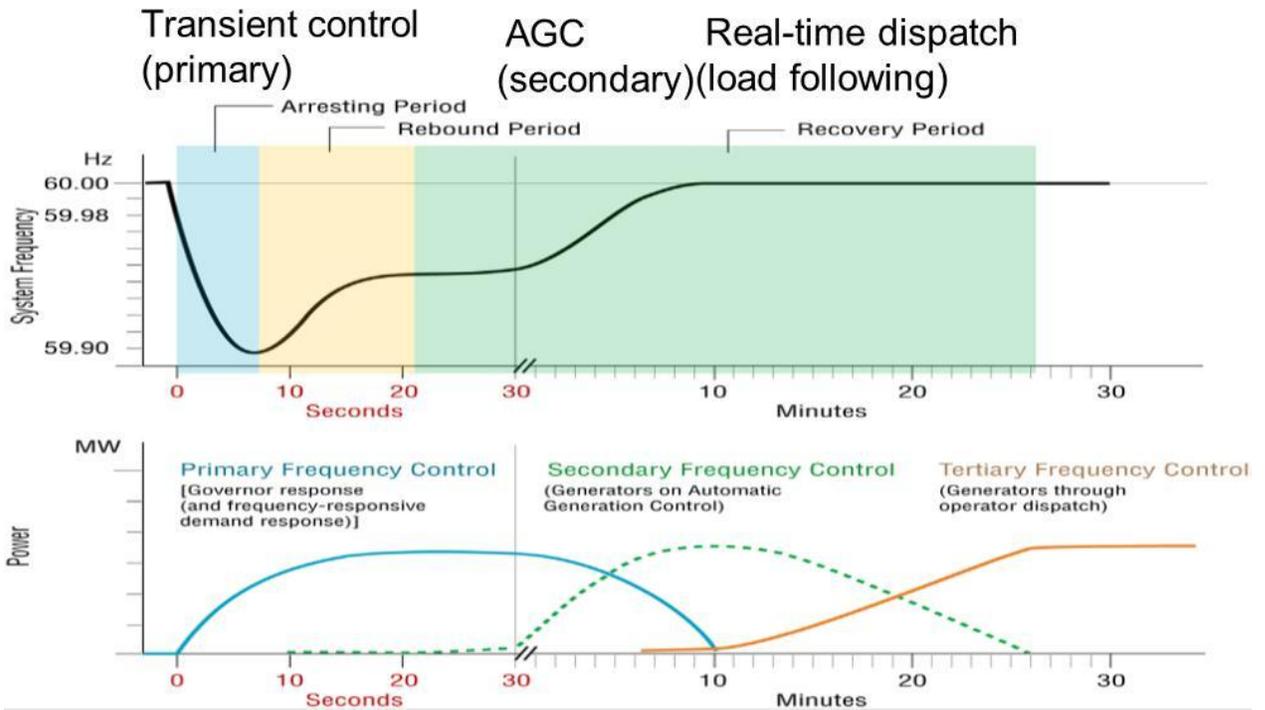


Fig. 2d: Transient & post-transient time frames

The following chart identifies the various time intervals associated with the above figures.

DELINEATION OF TIME PERIODS FOLLOWING CHANGE IN SYSTEM REAL POWER DEMAND  
FOR RESPONSE OF A SINGLE GENERATOR

Time	Most Influential Factor for Determining $\Delta P_{gen}$	Must be the Effect of this Factor on $\Delta P_{gen}$ ?	Describing Equation
$\mu$	Proximity to disturbance of machine	The closer the generator to the disturbance, the more it will compensate for change in system real power demand, $P_{LD}(0^+)$	$\Delta P_{gen1} = \frac{P_{01k}}{\sum_{j=1}^n P_{0jk}} P_{LD}(0^+)$ $P_{0jk} = \frac{V_j^2 Y_k \cos(\theta_j - \theta_k)}{X_{jk}}$
$\dagger \rightarrow 3 \text{ sec}$	Size (Inertia) of machine	The larger the generator, the more it will compensate for change in system real power demand, $P_{LD}(0^+)$ .	$\Delta P_{gen1} = \left[ \frac{H_1}{\sum_{j=1}^n H_j} \right] P_{LD}(0^+)$ <p><math>H_1</math> = Inertia constant of machine 1.</p>
$\vdots \rightarrow 20 \text{ sec}$	Primary Speed Control (Governor)	Generation increases with frequency decline.	$\Delta P_{gen1} = \frac{-\Delta f}{K} \left[ \frac{S_{01}}{100} \right]$ <p><math>K</math> = Steady state droop in p.u.  <math>S_{01}</math> = MVA rating for machine 1.</p>
$\text{min} \rightarrow 10 \text{ min}$	Supplementary Control (AGC)	Control Center Computer reallocates a new set point for each generator to return frequency to 60 Hz and the lines to scheduled transfer.	$\Delta P_{gen1} = \Delta P_{set1}$ $\Delta P_{set1} = \Delta P_{sced1} + K_1 [ACE]$ <p><math>ACE = IA - IS + B\Delta f</math>  <math>K_1</math> = Constant, <math>\text{MW/Hz}</math> for generator 1.  <math>ACE</math> = Area control error.  <math>IA</math> = Actual interchange  <math>IS</math> = Scheduled interchange  <math>B</math> = Frequency bias.</p>

Fig. 2e

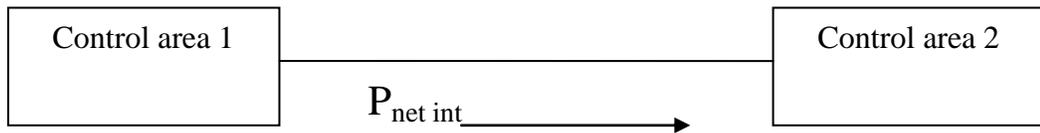
## 4.0 Two areas

As mentioned in Section 1.0 above, on page 348 of W&W, the term *control area* is defined as "An interconnected system within which the load and generation will be controlled as per the

rules in Fig. 9.20.” The rules of Fig. 9.20 are applied to a two-control area system as illustrated in Table 1 and Fig. 3 below.

Table 1

$\Delta\omega$	$\Delta P_{net\ int}$	Load change		Resulting secondary control action
-	-	$\Delta P_{L1}$	+	Increase $P_{gen}$ in system 1
		$\Delta P_{L2}$	0	
+	+	$\Delta P_{L1}$	-	Decrease $P_{gen}$ in system 1
		$\Delta P_{L2}$	0	
-	+	$\Delta P_{L1}$	0	Increase $P_{gen}$ in system 2
		$\Delta P_{L2}$	+	
+	-	$\Delta P_{L1}$	0	Decrease $P_{gen}$ in system 2
		$\Delta P_{L2}$	-	



$\Delta P_{L1}$ =Load change in area 1  
 $\Delta P_{L2}$ =Load change in area 2

Fig. 3

The table should be viewed with the following thoughts in mind:

- The above system may be considered to be comprised of one control area of interest (let’s say #1) on one side of the tie line and a “rest of the world” control area on the other side. The

“rest of the world” control area may be one control area or many.

- All values are steady-state values (not transient) following governor action (primary control) but before AGC action (secondary control).
- Since all values are steady-state values, frequency change  $\Delta\omega$  is the same throughout the interconnection (i.e., the same  $\Delta\omega$  is seen in area 1 as area 2).
- Frequency change  $\Delta\omega$  is positive (above 60 Hz) when load decreases and negative (below 60 Hz) when load increases.
- Change in net interchange, denoted by  $\Delta P_{\text{net int}}$ , is positive for flow increase from area 1 to area 2 and negative for flow decrease from area 1 to area 2.
- The indications in the “load change” column can be understood to be the cause of the governor action (primary control) which results in the frequency and tie line change.

There are two important points that come from studying the above table of “rules”:

1. **AGC corrects both tie line deviations and frequency deviations.**
2. **The tie line and frequency deviations are corrected by AGC in such a way so that each control area compensates for its own load change.**

These are core concepts underlying AGC.

## **5.0 Interchange**

The interconnection of different balancing authority areas create the following complexity:

Given a steady-state frequency deviation (seen throughout an interconnection) and therefore a load-generation imbalance, how does an area know whether the imbalance is caused by its own area load or that of another area load?

To answer the last question, it is necessary to provide some definitions [1]:

### **Summary of terms:**

My term	W&W term	My symbol	W&W symbol
Net actual interchange		$AP_{ij}$	
Net schedule interchange		$SP_{ij}$	
Interchange deviation		$\Delta P_{ij}$	
Actual export	Total actual net interchange	$AP_i$	$P_{net\ int}$
Scheduled export	Scheduled or desired value of interchange	$SP_i$	$P_{net\ int\ sched}$
Net (export) deviation		$\Delta P_i$	$\Delta P_{net\ int}$

Net actual interchange: The algebraic sum of all metered interchange over all interconnections between 2 physically Adjacent Balancing Authority Areas.

Net scheduled interchange: The algebraic sum of all Interchange Schedules across a given path or between 2 Balancing Authorities for a given period or instant in time.

Interchange schedule: An agreed-upon Interchange Transaction size (MW), start & end time, beginning & ending ramp times & rate, & type required for delivery/receipt of power/energy between Source & Sink Balancing Authorities involved in the transaction.

We illustrate net actual interchange & net scheduled interchange in Fig. 4 below.

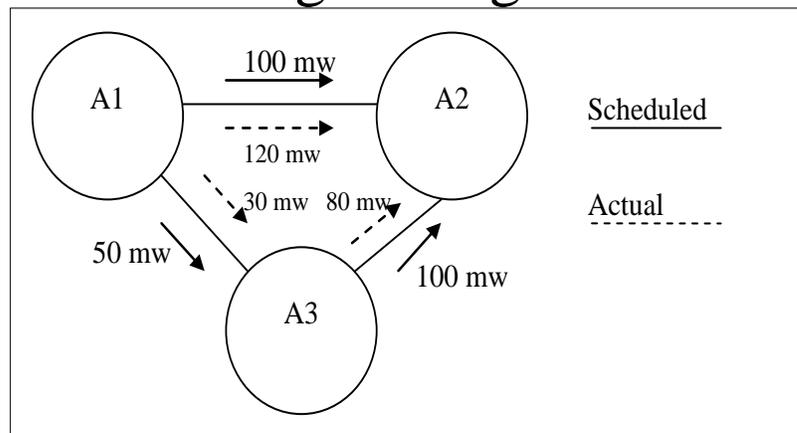


Fig. 4

The word “net” is used with actual and scheduled interchange because there may be more than one interconnection between two areas.

The *net actual interchange* between 2 areas:

- A1 to A2:  $AP_{12}=120$  MW    A2 to A3:  $AP_{23}=-80$  MW
- A2 to A1:  $AP_{21}=-120$  MW    A3 to A2:  $AP_{32}=80$  MW
- A1 to A3:  $AP_{13}=30$  MW
- A3 to A1:  $AP_{31}=-30$  MW

The *net scheduled interchange* between 2 areas:

- A1 to A2:  $SP_{12}=100$  MW    A2 to A3:  $SP_{23}=-100$  MW
- A2 to A1:  $SP_{21}=-100$  MW    A3 to A2:  $SP_{32}=100$  MW
- A1 to A3:  $SP_{13}=50$  MW
- A3 to A1:  $SP_{31}=-50$  MW

The *interchange deviation* between two areas is  
 Net Actual Interchange-Net Scheduled Interchange

We will define this as  $\Delta P_{ij}$ , so:

$$\Delta P_{ij}=AP_{ij}-SP_{ij} \tag{1}$$

In our example:

Area 1:

$$\Delta P_{12}=AP_{12}-SP_{12}=120-100=20 \text{ MW}$$

$$\Delta P_{13}=AP_{13}-SP_{13}=30-50=-20 \text{ MW}$$

Area 2:

$$\Delta P_{21}=AP_{21}-SP_{21}=-120-(-100)=-20 \text{ MW}$$

$$\Delta P_{23}=AP_{23}-SP_{23}=-80-(-100)=20 \text{ MW}$$

### Area 3:

$$\Delta P_{31} = AP_{31} - SP_{31} = -30 - (-50) = 20 \text{ MW}$$

$$\Delta P_{32} = AP_{32} - SP_{32} = 80 - (100) = -20 \text{ MW}$$

Some observations:

1. The net actual interchange may not be what is scheduled due to loop flow. For example, the balancing authorities may schedule 50 MW from A1 to A3 but only 30 MW flows on the A1-A3 tie line. The other 20 MW flows through A2. This is called “loop flow” or “inadvertent flow.”
2. We may also define, for an area  $i$ , an “actual export,” a “scheduled export,” and a “net deviation” (or “net export deviation”) as:

$$\text{Actual Export: } AP_i = \sum_{\substack{j=1 \\ j \neq i}}^n AP_{ij} \quad (2)$$

$$\text{Scheduled Export: } SP_i = \sum_{\substack{j=1 \\ j \neq i}}^n SP_{ij} \quad (3)$$

$$\text{Net Deviation: } \Delta P_i = \sum_{\substack{j=1 \\ j \neq i}}^n \Delta P_{ij} \quad (4)$$

W&W use different nomenclature/terminology, p. 347:

- “Total actual net interchange” denoted as  $P_{\text{net int}}$  to mean “actual export.”
- “Scheduled or desired value of interchange” denoted as  $P_{\text{net int sched}}$  to mean “scheduled export.”
- $\Delta P_{\text{net int}}$  to mean “net deviation”

*Since* an area’s net deviation is the sum of its interchange deviations per eq. (4), and

*Since* each interchange deviation is Net Actual Interchange-Net Scheduled Interchange per eq. (1), we can write

$$\Delta P_i = \sum_{\substack{j=1 \\ j \neq i}}^n \Delta P_{ij} = \sum_{\substack{j=1 \\ j \neq i}}^n (AP_{ij} - SP_{ij}) = \sum_{\substack{j=1 \\ j \neq i}}^n AP_{ij} - \sum_{\substack{j=1 \\ j \neq i}}^n SP_{ij} = AP_i - SP_i \quad (5)$$

This says that the net deviation for an area is just the difference between the actual export and the scheduled export for that area.

**Summary of terms:**

My term	W&W term	My symbol	W&W symbol
Net actual interchange		$AP_{ij}$	
Net schedule interchange		$SP_{ij}$	
Interchange deviation		$\Delta P_{ij}$	
Actual export	Total actual net interchange	$AP_i$	$P_{\text{net int}}$
Scheduled export	Scheduled or desired value of interchange	$SP_i$	$P_{\text{net int sched}}$
Net (export) deviation		$\Delta P_i$	$\Delta P_{\text{net int}}$

### 3. Net deviation is unaffected by loop flow.

For example, in Fig. 5a, the right side has the same net deviation as the left side but shows a new set of actual flows between areas, due to a change in transfer path impedance. But  $\Delta P_i$  doesn't change.

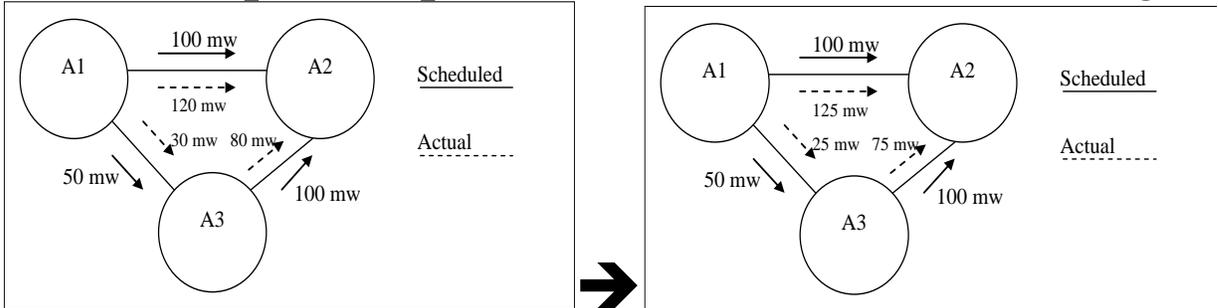


Fig. 5a

What affects net deviation is varying load (& varying gen), as illustrated in Fig. 5b, where the right side has the same scheduled flows as the left side but shows new net deviation for areas A1, A2.

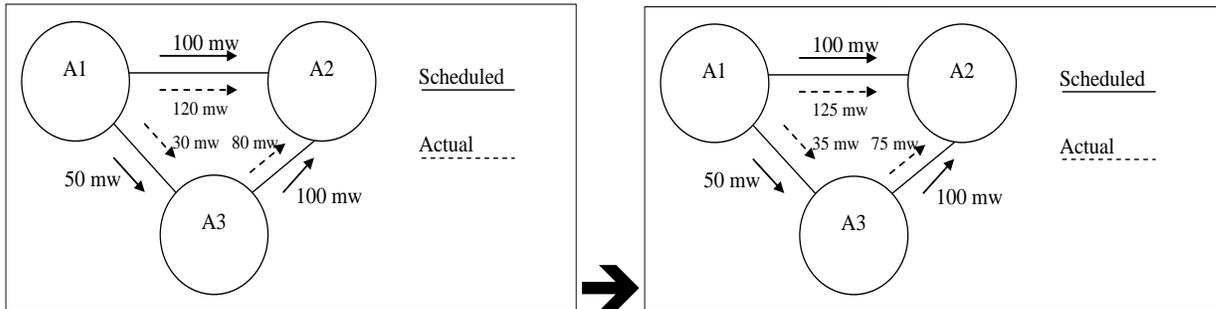


Fig. 5b

We see that the A1 actual export is 160 MW instead of the scheduled export of 150MW,  $\Delta P_1=160-150=10\text{MW}$ .

Likewise, the A3 actual export is 40 MW instead of the scheduled 50 MW,  $\Delta P_3=40-50=-10\text{MW}$ .

The area A2 actual export is still the same as the scheduled export of -200 MW,  $\Delta P_2=-200-(-200)=0\text{MW}$ .

Conclusion: Area A1 has corrected for a load increase in Area A3. So we need to signal Area A1 generators to back down and Area A3 generators to increase.

Overall conclusion: To perform load-frequency control in a power system consisting of multiple balancing authorities, we need to measure two things:

- *Steady-state frequency deviation:* to determine whether there is a generation/load imbalance in the overall interconnected system.

$$\Delta f = f - 60 \quad (6)$$

When  $\Delta f > 0$ , it means the generation in the system exceeds the load and therefore we should reduce generation in the area.

- *Net deviation:* to determine whether the actual exports are the same as the scheduled exports.

$$\Delta P_i = AP_i - SP_i \quad (5)$$

When  $\Delta P_i > 0$ , it means that the actual export exceeds the scheduled export, and so the generation in area i should be reduced.

The measurements of these two things is typically combined in an overall signal called the *area control error*, or ACE.

From the above, our first impulse may be to immediately write down the ACE for area  $i$  as:

$$ACE_i = \Delta P_i + \Delta f \quad (7)$$

Alternatively,

$$ACE_i = \Delta P_i + \Delta \omega \quad (8)$$

But we note 2 problems with eq. (8). First, we are adding 2 quantities that have different units. Anytime you come across a relation that adds 2 or more units having different units, beware.

The second problem is that the magnitudes of the two terms in eq. (8) may differ dramatically. If we are working in MW and Hz (or rad/sec), then we may see  $\Delta P_i$  in the 100's of MW whereas we will see  $\Delta f$  (or  $\Delta \omega$ ) in the hundredths or at most tenths of a Hz. The implication is that the control signal, per eq. (8), will greatly favor the export deviations over the frequency deviations.

Therefore we need to scale one of them. To do so, we define area  $i$  frequency bias as  $B_i$ . It has units of MW/Hz, so that

$$ACE_i = \Delta P_i + B_i \Delta \omega \quad (9)$$

We will look closely at use of this equation for control. Before we do that, however, we need to look at the system that we are trying to control and obtain models for each significant part.

## **6.0 Generator model**

A well-known relation in power system analysis is the swing equation. This equation is derived in EE 554 and relates acceleration of a synchronous machine to imbalance between input mechanical power and output electrical power, according to

$$\frac{2H}{\omega_{0e}} \frac{d^2 \delta_e}{dt^2} = P_m - P_e \quad (10)$$

Or, since  $\omega = d\delta/dt$ , we can write (10) as

$$\frac{2H}{\omega_{0e}} \frac{d\omega_e}{dt} = P_m - P_e \quad (11)$$

where

$\delta_e$  is the machine “torque” (electrical) angle by which the rotor leads the synchronously rotating reference;

$P_m$  is mechanical input power in per-unit

$P_e$  is electrical output power in per-unit

$H$  is the inertia constant in Mjoules/MW=sec given by

$$H = \frac{1}{2} \frac{I \omega_{0m}^2}{S_b} \quad (12)$$

$S_b$  is the machine MVA rating;

$I$  is moment of inertia of all machine masses in  $\text{kg}\cdot\text{m}^2 \times 10^6$ ;

$\omega_{0m}$  is the synchronous rotor speed in mechanical rad/sec

$\omega_{0e}$  is the synchronous rotor speed in electrical rad/sec (will always be 377 in North America).

If you have the EE554 text (Anderson & Fouad), you will find (11) as eq. 2.18 in that text.

Here, to agree with eq. 9.16 in W&W, we need to make three changes.

1. Put frequency in pu:

$$\frac{2H}{\omega_{0e}} \frac{d\omega_e}{dt} = P_m - P_e \quad (13)$$

$$\frac{\omega_{0e} 2H}{\omega_{0e}} \frac{d \frac{\omega_e}{\omega_{0e}}}{dt} = P_m - P_e \quad (14)$$

$$2H \frac{d\omega}{dt} = P_m - P_e \quad (15)$$

where  $\omega = \omega_e / \omega_{0e}$ .

2. Use a different inertia term:

Anderson & Fouad define the angular momentum of the machine as  $M$ ; we denote it here as  $M_{AF}$  where

$$M_{AF} = I\omega_{0m} \quad (16a)$$

From (12) (repeated below, left), we can write

$$H = \frac{1}{2} \frac{I\omega_{0m}^2}{S_b} \rightarrow I\omega_{0m} = \frac{2HS_b}{\omega_{0m}} \quad (16b)$$

Thus we see that

$$M_{AF} = \frac{2HS_b}{\omega_{0m}} \quad (17)$$

and solving for  $2H$ , we obtain:

$$2H = \frac{M_{AF}\omega_{0m}}{S_b} \quad (18)$$

Substituting into (15) (repeated below, left):

$$2H \frac{d\omega}{dt} = P_m - P_e \rightarrow \frac{M_{AF}\omega_{0m}}{S_b} \frac{d\omega}{dt} = P_m - P_e \quad (19)$$

Now where A&F use the angular momentum  $M_{AF}$ , W&W use a per-unitized angular momentum, according to

$$M_{WW} = \frac{M_{AF}\omega_{0m}}{S_b} \quad (20)$$

Comparison to (18) indicates

$$M_{WW} = 2H \quad (21)$$

Therefore, we have

$$M \frac{d\omega}{dt} = P_m - P_e \quad (22)$$

where it is understood from now on that  $M=M_{WW}$ .

W&W indicate on pg. 332 that the units of  $M$  are watts per radian per second per second.

However, it is typically used in a form of per-unit power over per-unit speed per second, as we have derived it above, and here it has units of seconds. In fact, W&W themselves mention this (pg 332) and use it in per-unit form in all of the rest of their work in this text.

### 3. Use deviations:

Express each variable in (22) as sum of an equilibrium value and a deviation, i.e.,

- $P_m = P_{m0} + \Delta P_m$
- $P_e = P_{e0} + \Delta P_e$
- $\omega(t) = \frac{\omega_{e0}}{\omega_{e0}} + \Delta\omega(t) = 1 + \Delta\omega(t)$

Substitution of the above into (22) results in

$$M \frac{d(1 + \Delta\omega(t))}{dt} = P_{m0} + \Delta P_m - P_{e0} - \Delta P_e \quad (23)$$

Simplifying, and noting that at equilibrium,  $P_{m0} = P_{e0}$ , we have

$$M \frac{d\Delta\omega(t)}{dt} = \Delta P_m - \Delta P_e \quad (24)$$

Equation (24) is eq. 9.16 in your text and is what W&W use to represent the dynamics of a synchronous machine.

Because we want to derive block diagrams for our control systems, we will transform all time-domain expressions into the Laplace ( $s$ ) domain.

$$Ms\Delta\omega(s) = \Delta P_m - \Delta P_e \quad (25)$$

where we have assumed  $\Delta\omega(t=0)=0$ .

## **7.0 Load model**

The load supplied by a power system at any given moment consists of many types of elements, including electronic loads, heating, cooking, and lighting loads, and motor loads, with the latter two types of load comprising the larger percentage. Of these two, it is quite typical that heating, cooking, and lighting loads have almost no frequency sensitivity, i.e., their power consumption remains constant for variations away from nominal frequency. More discussion about various load types is in [3].

Motor loads, on the other hand, are different. To see this, we will focus on only the induction motor which tends to comprise the largest percentage of motor loads.

Consider the standard per-phase steady-state model of an induction motor, as given in Fig. 6.

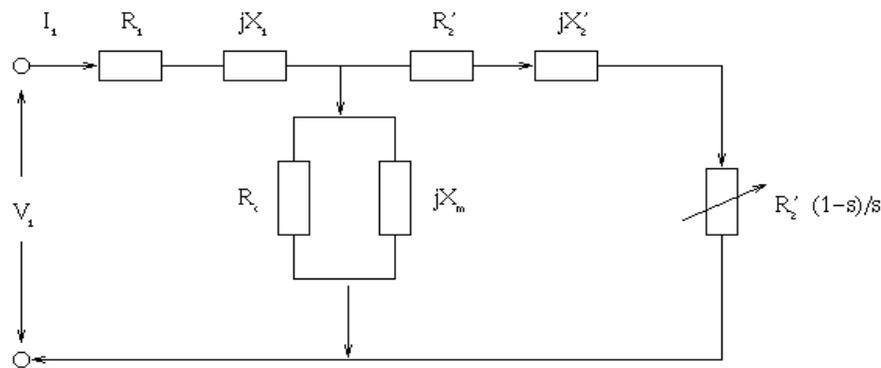


Fig. 6

From this model, the electric power delivered to the motor can be derived as:

$$P = \frac{3V_{th}^2 R_2'}{\left[ \left( R_{th} + \frac{R_2'}{s} \right)^2 + (X_{th} + X_2')^2 \right]} \quad (26)$$

where  $R_{th}+jX_{th}$  and  $V_{th}$  are the Thevenin equivalent impedance and voltage, respectively, looking from the rotor circuit (the junction between  $jX_S$  and  $R_2'$  in Fig. 6) left, back into the stator circuit.

Recall that induction motor slip is given by

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} \quad (27)$$

where  $\omega_s$  is the mechanical synchronous speed (set by the network frequency) and  $\omega_m$  is the mechanical speed of the rotor. Substituting (27) into (26) we obtain

$$P = \frac{3V_{th}^2 R_2'}{\left[ \left( R_{th} + \frac{R_2'}{(\omega_s - \omega_m)} \right)^2 + (X_{th} + X_2')^2 \right]} \quad (28)$$

Question: Let's assume that the voltage  $V_{th}$  and the mechanical speed  $\omega_m$  remain almost constant (the "almost" is because there is some variation in  $\omega_m$  which can be observed by plotting torque vs. speed curves—see notes from EE 559).

What happens to P as  $\omega_s$  decreases?

Example: Assume constant mechanical speed  $\omega_m$  of 180 rad/sec. What happens to

$$s = \frac{(\omega_s - \omega_m)}{\omega_s}$$

for a 4-pole machine when frequency decreases from 60 Hz to 59.9 Hz? Then indicate qualitatively what happens to power drawn by the motor under this same frequency deviation.

Solution: At 60 Hz, the synchronous speed is given and corresponding slip are given by

$$\omega_s = 2\pi f / (p/2) = (2\pi)(60) / (4/2) = 188.5 \text{ rad/sec}$$

$$s = (188.5 - 180) / 188.5 = 0.0451$$

where  $p$  is the number of poles. At 59.9 Hz, it is given by

$$\omega_s = (2\pi)(59.9) / (4/2) = 188.18 \text{ rad/sec}$$

$$s = (188.18 - 180) / 188.18 = 0.0435$$

And so reduction in frequency causes a reduction in slip. From (26), we see that this will cause the term  $\frac{R_2'}{s}$  to increase, which will cause the overall power expression to decrease.

Conclusion: Power drops with frequency.

EPRI [4] provides an interesting figure which compares frequency sensitivity for motors loads with non-motor loads, shown below in Fig. 7.

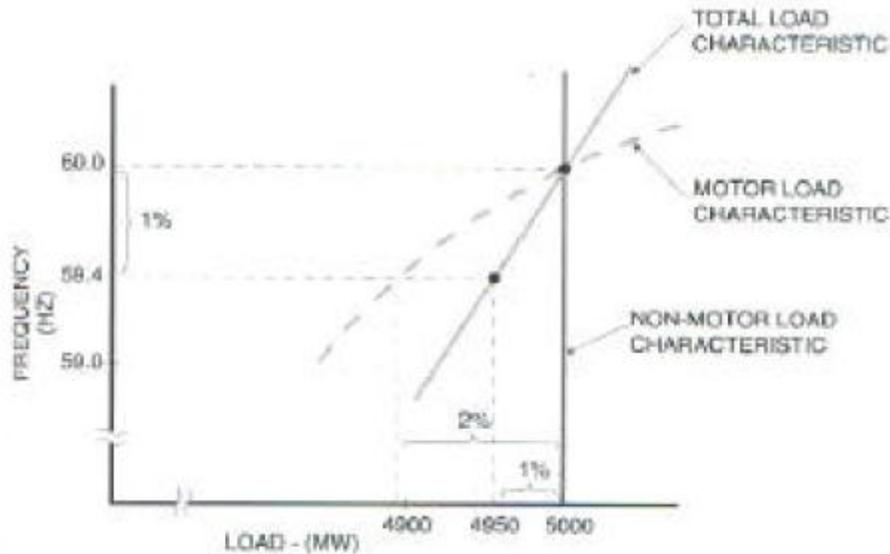


Fig. 7

Figure 7 shows that motor loads reduce about 2% for every 1% drop in frequency. If we assume that non-motor loads are unaffected by frequency, a reasonable composite characteristic might be that total load reduces by 1% for every 1% drop in frequency, as indicated by the “total load characteristic” in Fig. 7.

To account for load sensitivity to frequency deviations, we will define a parameter  $D$  according to

$$D = \frac{\text{pu change in load}}{\text{pu change in frequency}} \quad (29a)$$

from which we may write:

$$\text{pu change in load} = D\Delta\omega \quad (29b)$$

If our system has a 1% decrease in power for every 1% decrease in frequency, then  $D=1$ .

In determining  $D$  based on (29a), the base frequency should be the system's nominal frequency, in North America, 60 Hz. The base load should be the same as the base MVA,  $S_b$ , used to per-unitize power in the swing equation (24), repeated here for convenience.

$$M \frac{d\Delta\omega(t)}{dt} = \Delta P_m - \Delta P_e \quad (24)$$

In (24),  $\Delta P_e$  is the change in electric power out of the synchronous generator.

The change in electric power out of the synchronous generator will be balanced by

- any changes in net electric demand in the network, which we will denote as  $\Delta P_L$  and

- the change in load due to frequency deviation, according to (29b).

Therefore

$$\Delta P_e = \Delta P_L + D\Delta\omega \quad (30)$$

Please note that this “D” differs from the “D” used in stability studies to represent windage & friction.

Substitution of (30) into (24) results in

$$M \frac{d\Delta\omega(t)}{dt} = \Delta P_m - \Delta P_L - D\Delta\omega(t) \quad (31)$$

Taking the Laplace transform of (31) results in

$$Ms\Delta\omega(s) = \Delta P_m(s) - \Delta P_L(s) - D\Delta\omega(s) \quad (32)$$

where again we have assumed  $\Delta\omega(t=0)=0$ .

Solving (32) for  $\Delta\omega(s)$ , we obtain

$$\begin{aligned} Ms\Delta\omega(s) + D\Delta\omega(s) &= \Delta P_m - \Delta P_L(s) \\ \Delta\omega(s)(Ms + D) &= \Delta P_m - \Delta P_L(s) \\ \Delta\omega(s) &= \underbrace{(\Delta P_m - \Delta P_L(s))}_{\text{Input}} \underbrace{\frac{1}{Ms + D}}_{\text{Transfer Function}} \end{aligned} \quad (33)$$

We model (33) as in Fig. 8.

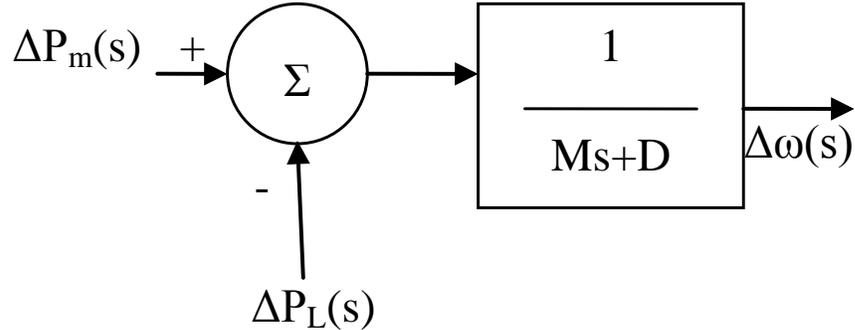


Fig. 8

## 8.0 Turbine (prime mover) model

The mechanical power is provided by the *prime-mover*, otherwise known as the *turbine*. For nuclear, coal, gas, and combined cycle units, the prime-mover is a steam turbine, and the mechanical power is controlled by a steam valve.

For hydroelectric machines, the prime mover is a hydro-turbine, and the mechanical power is controlled by the water gate.

We desire a turbine model which relates mechanical power control (steam valve or water gate) to mechanical power provided by the turbine.

Since the mechanical power provided by the turbine is the mechanical power provided to the generator, we can denote it as  $\Delta P_m$ . We will denote the mechanical power control as  $\Delta P_v$ .

Reference [5, p. 214-216] provides a brief but useful discussion about turbine models and indicates that a general turbine model is as shown in Fig. 9.

**→NEXT 6 PAGES PROVIDE BACKGROUND ON TURBINE MODELING. YOU SHOULD READ THIS ON YOUR OWN.**

**→IN LECTURE, WE SKIP TO PAGE 40.**

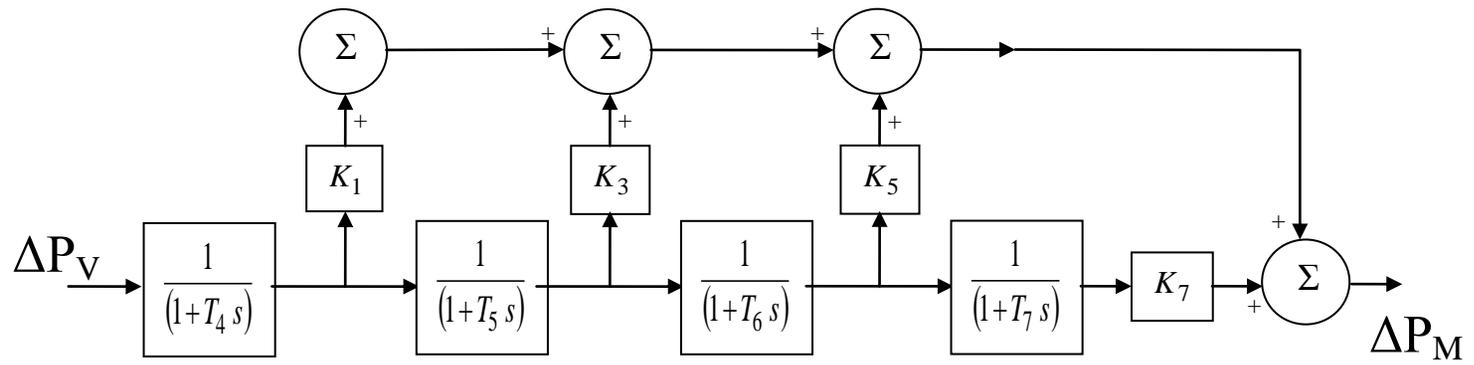


Fig. 9

This turbine model can be applied to a multi-stage steam turbine, a single-stage steam turbine, or a hydro-turbine.

The multi-stage steam turbine is a common type of turbine that uses reheating to provide additional power from the same steam, and as a result is most often called a reheat turbine. The principle behind a reheat turbine is that the energy of the steam is dependent upon two of its attributes: pressure and temperature.

This can be observed in a very simple way by recalling that work done by exerting a force  $F$  over a distance  $d$  is given by  $W=F \times d$ . Dividing  $F$  by Area  $A$  and multiplying  $d$  by  $A$ , we get  $W=(F/A) \times dA$ , and recognizing  $F/A$  as pressure,  $P$ , and  $dA$  as volume  $V$ , we see that  $W=P \times V$ .

Now the only other thing we need to know is that volume  $V$  increases with temperature, and we see quickly that the energy exerted by an amount of steam flowing over turbine blades increases with pressure and temperature.

A typical 2-stage reheat turbine is shown in Fig. 10 where we observe that the reheater provides increased temperature to utilize the reduced pressure steam a second time in the low pressure turbine.

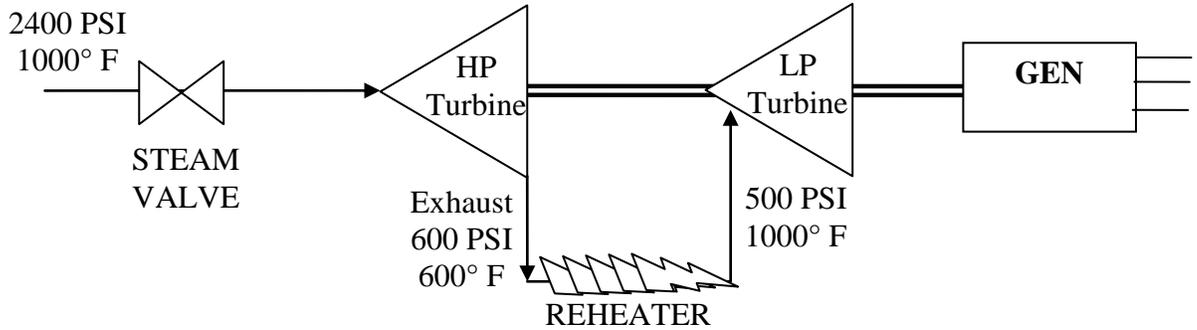


Fig 10

Referring once again to Fig. 9, the time constant  $T_4$  represents the first stage, often called the *steam chest*. If the turbine is non-reheat, then this is the only time constant needed, and the desired transfer function is given by

$$\Delta P_m(s) = \frac{1}{1 + T_4 s} \Delta P_V(s) \quad (34)$$

For multistage turbines, reference [5] states that

“The time constants  $T_5$ ,  $T_6$ , and  $T_7$  are associated with time delays of piping systems for reheaters and cross-over mechanisms. The coefficients  $K_1$ ,  $K_3$ ,  $K_5$ , and  $K_7$  represent fractions of total

mechanical power outputs associated with very high, high, intermediate, and low pressure components, respectively.”

Reference [5] also provides some typical data for steam turbine systems, reproduced in Table 2 below.

Table 2

	$T_4$	$T_5$	$T_6$	$T_7$	$K_1$	$K_3$	$K_5$	$K_7$
Non-reheat	0.3	0	0	0	1	0	0	0
Single-reheat	0.2	7.0	0	0.4	0.3	0.4	0.3	0
Double-reheat	0.2	7.0	7.0	0.4	0.22	0.22	0.3	0.26

Finally, reference [5] addresses hydro turbines:

“In the case of hydro turbines, the situation depends on the geometry of the system, among other factors. The overall transfer function of a hydro turbine is given as

$$\Delta P_m(s) = \frac{1 - sT_w}{1 + 0.5sT_w} \Delta P_V(s)$$

where  $T_w$  is known as the water time constant. The significance of the above transfer function is that it contains a zero in the complex right-half plane. From a stability viewpoint, this may cause some problems since this is a non-minimum phase system. Using the model of Figure 6.5 (Fig. 9 in our notes) one identifies the following parameters:  $T_4=0$ ,  $T_5=T_w/2$ ,  $T_6=T_7=0$ ,  $K_1=-2$ ,  $K_3=3$ ,  $K_5=K_7=0$ . Typical values of  $T_w$  range from .5 to 5 sec.”

The above system is called “non-minimum phase” because it has a right-half-plane zero and therefore, in frequency-response (Bode) plots, we will see a greater phase contribution at frequencies corresponding to the RHP zero.

We will be able to illustrate the basic attributes of AGC by using the model for the simplest turbine system – the nonreheat turbine, with transfer function given by (34).

For convenience, we write this transfer function here, together with that of the generator with load frequency sensitivity given by (33):

$$\Delta\omega(s) = \underbrace{(\Delta P_m - \Delta P_L)}_{\text{Input}} \underbrace{\frac{1}{Ms + D}}_{\text{Transfer Function}} = \underbrace{(\Delta P_m - \Delta P_L(s))G(s)}_{\text{Input}} \quad (33)$$

$$\Delta P_m(s) = \frac{1}{1 + T_4 s} \Delta P_V(s) = T(s) \Delta P_V(s) \quad (34)$$

Substituting (34) into (33) results in

$$\Delta\omega(s) = \left( \frac{1}{1 + T_4 s} \Delta P_V(s) - \Delta P_L(s) \right) \frac{1}{Ms + D} \quad (35)$$

We see in (35) transfer functions providing frequency deviation as a function of:

- change in steam valve setting and
- change in connected load.

The block diagram for this appears as in Fig. 11. Fig. 12 illustrates the action of the primary speed controller, which we will describe next.

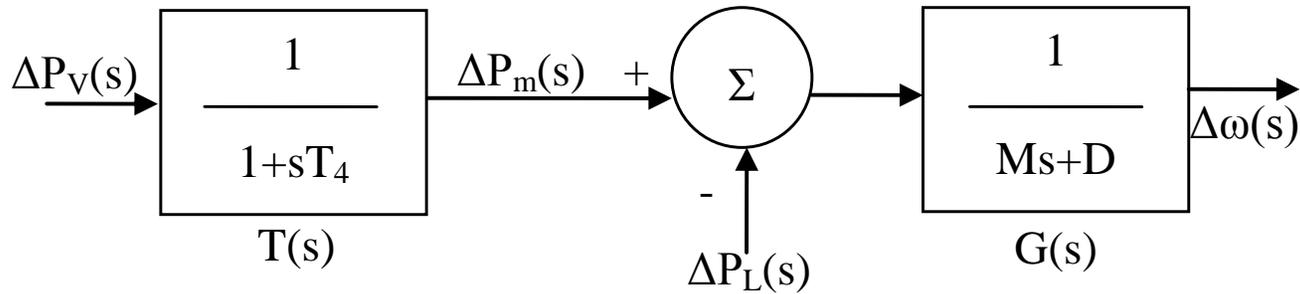


Fig. 11

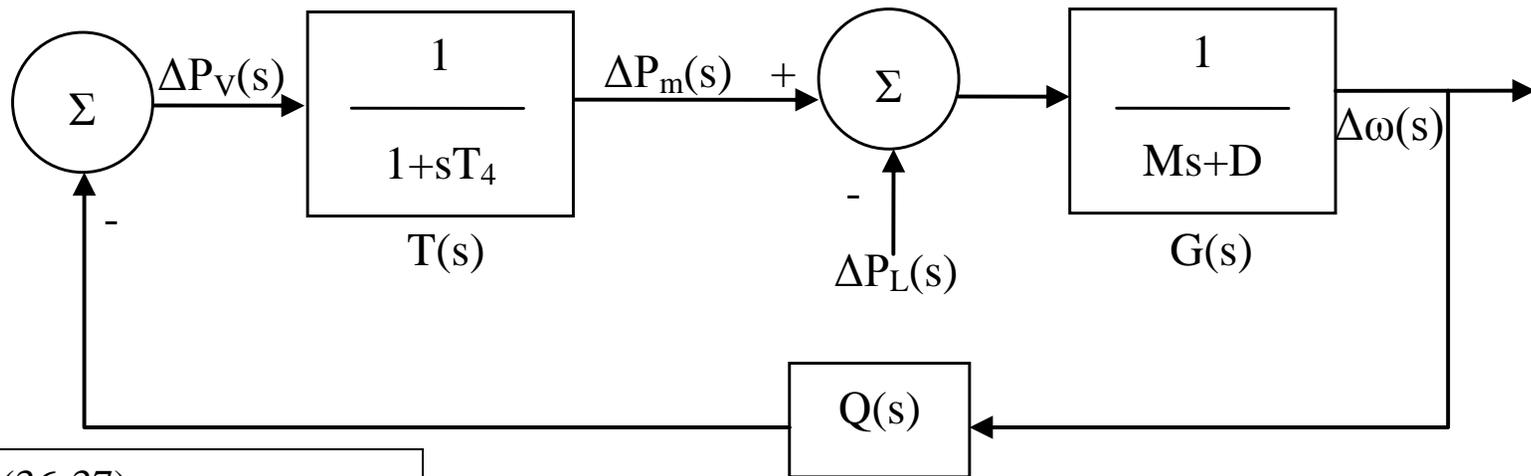


Fig. 12

Equations (36-37):

$$\Delta\omega(s) = (\Delta P_m(s) - \Delta P_L(s))G(s)$$

$$\Delta P_m(s) = T(s)\Delta P_V(s) = -T(s)Q(s)\Delta\omega(s)$$

## 9.0 Primary speed control

The primary speed controller is also referred to as the speed governor. It has three purposes:

1. Regulate the speed of the machine.
2. Aid in matching system MW generation with system MW load.
3. Provide a mechanism through which secondary speed control can act.

Regulation of speed means we will control  $\Delta\omega$ . To control  $\Delta\omega$ , we must regulate the input power to the machine, denoted in Fig. 11 by  $\Delta P_V$ . This requires having feedback from  $\Delta\omega$  to  $\Delta P_V$ . We denote this feedback as  $Q(s)$ , as shown in Fig. 12.

Using the simpler notation of  $T(s)$  and  $G(s)$ , we can see from Fig. 12 that

$$\Delta\omega(s) = (\Delta P_m(s) - \Delta P_L(s))G(s) \quad (36)$$

and

$$\Delta P_m(s) = T(s)\Delta P_V(s) = -T(s)Q(s)\Delta\omega(s) \quad (37)$$

Substitution of (37) into (36) results in

$$\Delta\omega(s) = (-T(s)Q(s)\Delta\omega(s) - \Delta P_L(s))G(s) \quad (38)$$

Solving for  $\Delta\omega$ ...

$$\Delta\omega(s) + T(s)Q(s)\Delta\omega(s)G(s) = -\Delta P_L(s)G(s)$$

$$\Delta\omega(s)(1 + T(s)Q(s)G(s)) = -\Delta P_L(s)G(s)$$

$$\Delta\omega(s) = \frac{-\Delta P_L(s)G(s)}{1 + T(s)Q(s)G(s)} \quad (39)$$

Appendix A of these notes provides an analysis of a mechanical-hydraulic speed governor that was used for many years (and still is in some older power plants). Newer plants today use computer-based digital controllers. But the concepts are the same; we utilize eq. (A22) from App A to illustrate the relations (in notation of App A, the circumflex above variables indicates the Laplace domain in these equations).

$$\Delta P_V = \Delta \hat{x}_E = \frac{-k_5 k_3}{s + k_5 k_4} (k_1 \Delta \hat{\omega} - k_A \Delta \hat{P}_C) \quad (A22)$$

Here,  $\Delta \hat{x}_E$  is the same as  $\Delta P_V$ . We drop the  $\Delta \hat{x}_E$  notation, and we ignore  $\Delta P_C$  (which is the set-point power output) for now, so that

$$\Delta P_V = \frac{-k_5 k_3}{s + k_5 k_4} k_1 \Delta \hat{\omega} \quad (40)$$

The first kind of governors ever designed provided simple integral feedback. This is obtained from (40) if  $k_4$  is set to 0 (in Appendix A, this corresponds to disconnecting rod CDE at point E in Fig. A4 and A5 – see eq. (A11a)). Then we obtain

$$\Delta P_V = \frac{-k_5 k_3}{s} k_1 \Delta \hat{\omega} \quad (41)$$

The constants in (41) are combined to obtain:

$$\Delta P_V = \frac{-K_G}{s} \Delta \hat{\omega} \quad (42)$$

Comparing (42) to the block diagram of Fig. 12, we see that

$$Q(s) = \frac{K_G}{s} \quad (43)$$

Substitution of (43) into (39) results in

$$\Delta \omega(s) = \frac{-\Delta P_L(s)G(s)}{1 + \frac{K_G}{s} T(s)G(s)} \quad (44)$$

The block diagram for (44) is shown in Fig. 14.

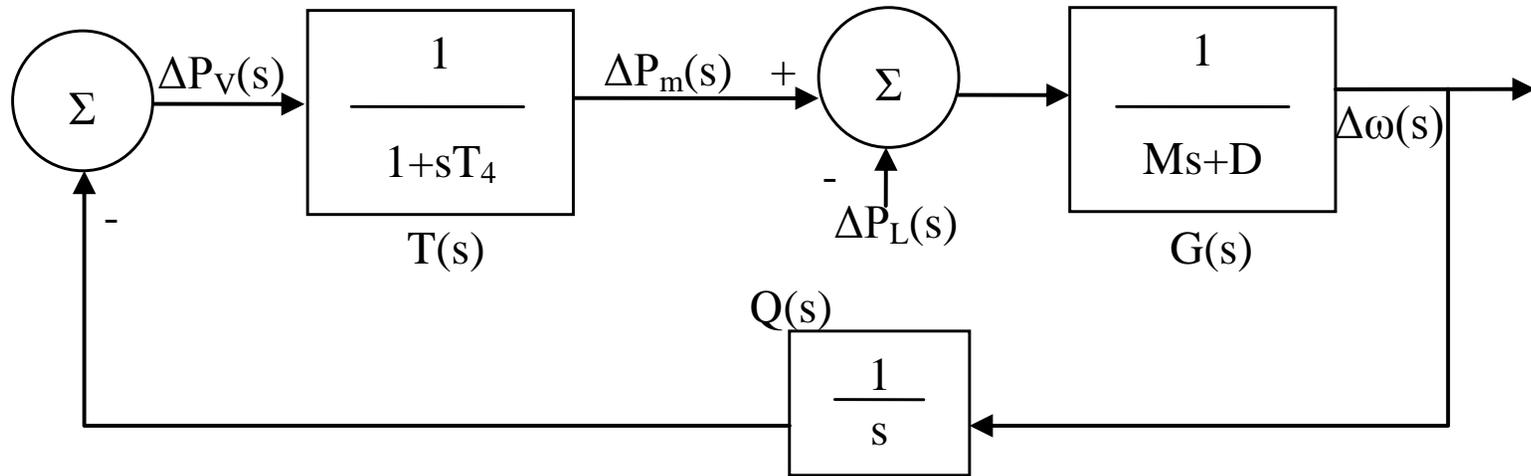


Fig. 14

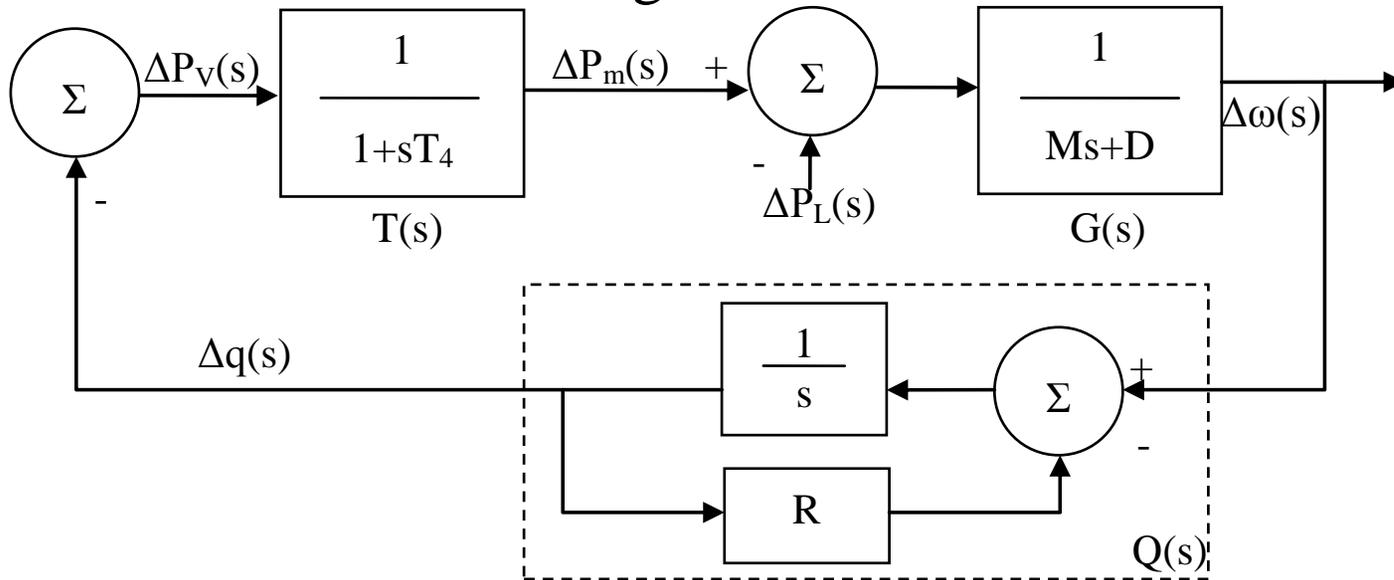


Fig. 15

It is of interest to understand the steady-state response of the system characterized by (44) to a load change.

Let's assume that the load change is an instantaneous change. Mathematically, we can model this using the step function  $u(t)$ . If the amount of load change is  $L$ , then the appropriate functional notation is

$$\Delta P_L(t) = Lu(t) \quad (45)$$

Taking the Laplace transform, we obtain

$$\Delta P_L(s) = \frac{L}{s} \quad (46)$$

Substituting (46) into (44), we obtain

$$\Delta\omega(s) = \frac{-\frac{L}{s}G(s)}{1 + \frac{K_G}{s}T(s)G(s)} \quad (47)$$

Multiplying through by  $s$  results in

$$\Delta\omega(s) = \frac{-LG(s)}{s + K_G T(s)G(s)} \quad (48)$$

Recall the final-value theorem (FVT) from Laplace transform theory, which says

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

We can use the FVT to obtain the steady-state response of (48) according to:

$$\lim_{t \rightarrow \infty} \Delta\omega(t) = \lim_{s \rightarrow 0} s\Delta\omega(s) = \lim_{s \rightarrow 0} s \frac{-LG(s)}{s + K_G T(s)G(s)} = 0 \quad (49)$$

Equation (49) indicates that the steady-state response to a step-change in load is 0. In classical control theory, this is said to be a “Type 1” system, implying that the response to a step change gives 0 steady-state error.

Therefore, this governor forces  $\Delta\omega(t)$  to 0 after a long enough time. Very nice!

Or is it...?

To fully appreciate the implications of this governor design, we need to recognize that the frequency deviation signal  $\Delta\omega(t)$  actually comes from comparing the measured turbine speed  $\omega$  with a desired reference speed  $\omega_{ref}$ , so that

$$\Delta\omega(t) = \omega_{ref} - \omega(t) \quad (50)$$

Since there are many generators in a power system, each having their own equation (50), we can write

$$\Delta\omega_1(t) = \omega_{ref1} - \omega(t)$$

$$\Delta\omega_2(t) = \omega_{ref2} - \omega(t)$$

⋮

$$\Delta\omega_n(t) = \omega_{refn} - \omega(t)$$

It is physically not possible to ensure

$$\omega_{ref1} = \omega_{ref2} = \cdots = \omega_{refn}$$

This governor design would work fine if there were only a single machine. But with multiple machines, there will always be some units in the system that see a non-zero actuation signal  $\Delta\omega$ . This causes machines to “fight” against one another.

That is, for the two-machine case, we will see that machine 1 will correct causing machine 2 to see actuation, machine 2 will correct causing machine 1 to see actuation, and so on.

To correct this problem, we add a proportional feedback loop around the integrator, as shown in Fig. 15. Note that the transfer function for the speed governor is now given by

$$Q(s) = \frac{\Delta\omega(s)}{\Delta q(s)} = \frac{K_G / s}{1 + \frac{K_G R}{s}} = \frac{K_G}{s + K_G R} \quad (51)$$

Factoring out the  $K_G R$  term from the denominator, we obtain

$$Q(s) = \frac{K_G}{K_G R \left( \frac{s}{K_G R} + 1 \right)} = \frac{1}{R} \frac{1}{\left( 1 + \frac{s}{K_G R} \right)} \quad (52)$$

Now define  $T_G = 1/K_G R$  as the governor time constant, we can write (52) as

$$Q(s) = \frac{1}{R} \frac{1}{(1 + sT_G)} \quad (53)$$

We can use (53) in redrawing Fig. 15 as shown in Fig. 16. We may derive the overall transfer function via Fig. 16. Alternatively, we may substitute (53) into (39), repeated here for convenience.

$$\Delta\omega(s) = \frac{-\Delta P_L(s)G(s)}{1 + T(s)Q(s)G(s)} \quad (39)$$

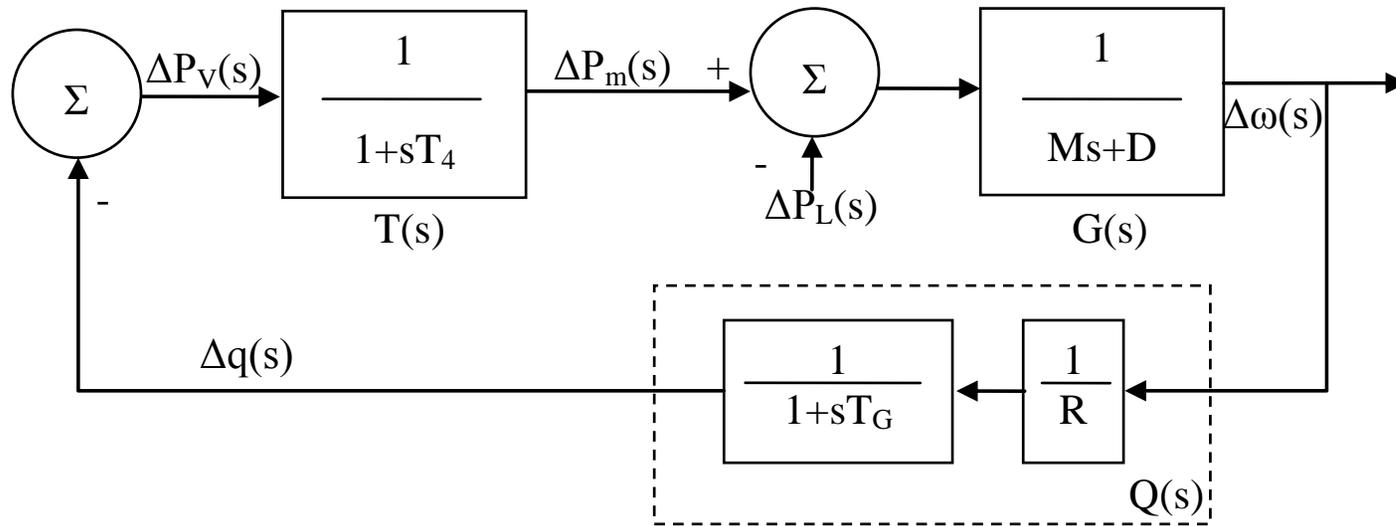


Fig. 16

$$\Delta\omega(s) = \frac{-\Delta P_L(s)G(s)}{1 + \frac{1}{R} \frac{1}{(1 + sT_G)} T(s)G(s)} \quad (54)$$

Again, we desire to look at steady-state frequency error to a step change in load. Following the same procedure as before, with

$$\Delta P_L(s) = \frac{L}{s} \quad (46)$$

the FVT provides that we can write

$$\begin{aligned} \lim_{t \rightarrow \infty} \Delta\omega(t) &= \lim_{s \rightarrow 0} s\Delta\omega(s) \\ &= \lim_{s \rightarrow 0} s \frac{-(L/s)G(s)}{1 + \frac{1}{R} \frac{1}{(1 + sT_G)} T(s)G(s)} \\ &= \lim_{s \rightarrow 0} \frac{-LG(s)}{1 + \frac{1}{R} \frac{1}{(1 + sT_G)} T(s)G(s)} \end{aligned} \quad (55)$$

To better see the significance of (55) we need to substitute into it the transfer functions for  $G(s)$  and  $T(s)$ , which can be seen from Fig. 16 to be

$$G(s) = \frac{1}{Ms + D} \quad T(s) = \frac{1}{1 + sT_4} \quad (56)$$

Substitution into (55) results in

$$\lim_{t \rightarrow \infty} \Delta\omega(t) = \lim_{s \rightarrow 0} \frac{-L \frac{1}{Ms + D}}{1 + \frac{1}{R} \frac{1}{(1 + sT_G)} \frac{1}{(1 + sT_4)} \frac{1}{(Ms + D)}} \quad (57)$$

Clearing the fraction in the denominator,

$$\lim_{t \rightarrow \infty} \Delta\omega(t) = \lim_{s \rightarrow 0} \frac{-L \frac{R(1 + sT_G)(1 + sT_4)(Ms + D)}{Ms + D}}{R(1 + sT_G)(1 + sT_4)(Ms + D) + 1} \quad (58)$$

Canceling the Ms+D term in the numerator,

$$\lim_{t \rightarrow \infty} \Delta\omega(t) = \lim_{s \rightarrow 0} \frac{-LR(1 + sT_G)(1 + sT_4)}{R(1 + sT_G)(1 + sT_4)(Ms + D) + 1} \quad (59)$$

Now we can see that as  $s \rightarrow 0$ , the expression becomes

$$\lim_{t \rightarrow \infty} \Delta\omega(t) = \frac{-LR}{RD + 1} = \frac{-L}{D + 1/R} \quad (60)$$

We denote the steady-state response to a step load change of L as  $\Delta\omega_\infty$ ,

$$\Delta\omega_\infty = \frac{-LR}{RD + 1} = \frac{-L}{D + 1/R} \quad (61)$$

We will address how to eliminate this steady-state error. Before doing that, however, let's look at what happens to the mechanical power delivered to the generator in the steady-state.

To do that, we follow the same procedure as we did when inspecting steady-state frequency deviation, except now we are investigating  $\Delta P_{m,\infty}$ .

This requires that we first express  $\Delta P_m(s)$ . This can be done easily based on Fig. 16, where we observe that

$$\Delta P_m(s) = -T(s)Q(s)\Delta\omega(s) \quad (61)$$

Substituting (39) into (61), we obtain

$$\Delta P_m(s) = -T(s)Q(s) \frac{-\Delta P_L(s)G(s)}{1 + T(s)Q(s)G(s)} \quad (62)$$

Substituting (53) and (56) into (62) results in

$$\Delta P_m(s) = -\frac{1}{(1+sT_4)} \frac{1}{R} \frac{1}{(1+sT_G)} \frac{-\Delta P_L(s) \frac{1}{Ms+D}}{1 + \frac{1}{(1+sT_4)} \frac{1}{R} \frac{1}{(1+sT_G)} \frac{1}{(Ms+D)}} \quad (63)$$

Clearing the fraction in the denominator

$$\Delta P_m(s) = -\frac{1}{(1+sT_4)} \frac{1}{R} \frac{1}{(1+sT_G)} \frac{-\Delta P_L(s) \frac{R(1+sT_4)(1+sT_G)(Ms+D)}{Ms+D}}{R(1+sT_4)(1+sT_G)(Ms+D)+1} \quad (64)$$

Several terms will cancel:

$$\Delta P_m(s) = -\frac{-\Delta P_L(s)}{R(1+sT_4)(1+sT_G)(Ms+D)+1} \quad (65)$$

Note the two negative signs make a positive.

Again using

$$\Delta P_L(s) = \frac{L}{s} \quad (46)$$

and applying the FVT, we obtain:

$$\begin{aligned} \lim_{t \rightarrow \infty} \Delta P_m(t) &= \lim_{s \rightarrow 0} s \Delta P_m(s) \\ &= \lim_{s \rightarrow 0} s \frac{L/s}{R(1+sT_4)(1+sT_G)(Ms+D)+1} \\ &= \lim_{s \rightarrow 0} \frac{L}{R(1+sT_4)(1+sT_G)(Ms+D)+1} \quad (66) \\ &= \frac{L}{RD+1} \end{aligned}$$

Therefore,

$$\Delta P_{m,\infty} = \frac{L}{RD+1} \quad (67)$$

But recall (61):

$$\Delta \omega_\infty = \frac{-LR}{RD+1} \quad (61)$$

Dividing both sides of (61) by  $-R$ , we obtain

$$\frac{-\Delta \omega_\infty}{R} = \frac{L}{RD+1} \quad (68)$$

Equating (67) and (68) results in

$$\Delta P_{m,\infty} = \frac{-\Delta \omega_\infty}{R} \quad (69)$$

Finally, we use the fact that, at steady-state,  $\Delta P_{m,\infty} = \Delta P_{e,\infty}$ , and therefore (69) can be expressed as

$$\Delta P_{e,\infty} = \frac{-\Delta\omega_\infty}{R} \quad (70)$$

Or, equivalently,

$$\Delta\omega_\infty = -R\Delta P_{e,\infty} \quad (71)$$

Figure 17 plots  $\omega_\infty$  as a function of  $P_{e,\infty}$ .

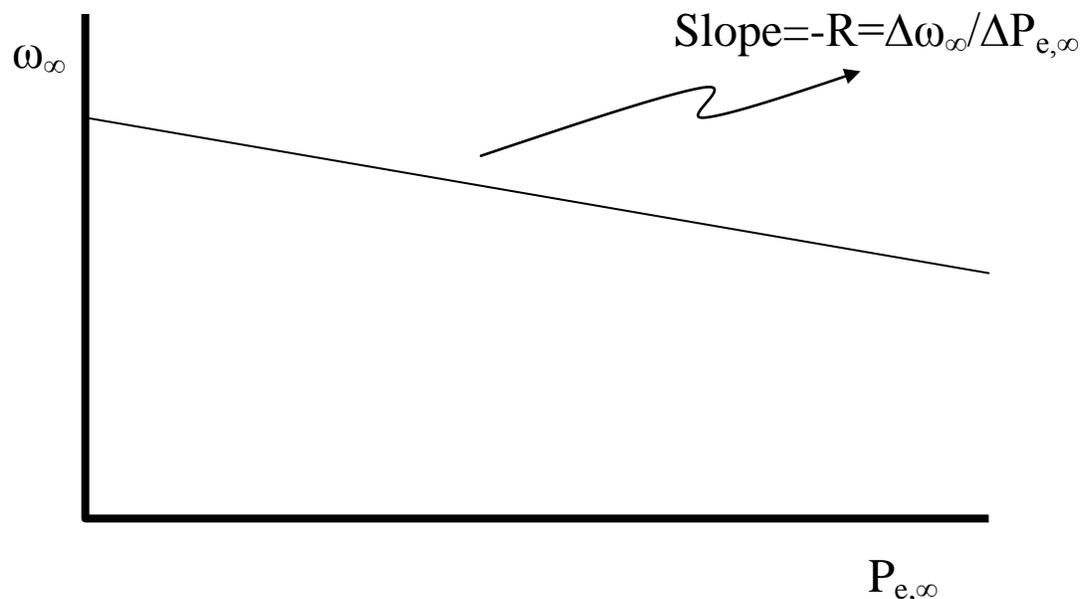


Fig. 17

Figure 17 displays the so-called *droop characteristic* of the speed governor, since the plot “droops” moving from left to right.

It is important to understand that the droop characteristic (Fig. 17) displays steady-state frequency and power. You can think of  $\Delta P_L(t)=Lu(t)$  as the initiating change, then we wait a minute, at which time all transients have died, and the speed governor will have operated in such a way so that the frequency will decrease (for positive L) by  $\Delta\omega_\infty$ , and the generation will have increased (for positive L) by  $\Delta P_{e,\infty}$ .

The parameter R can be understood via

$$R = -\frac{\Delta\omega_\infty}{\Delta P_{e,\infty}} \quad (72)$$

which shows that R is the value of per-unit frequency deviation required to produce a 1 per unit change in electric output power.

R is called the regulation coefficient, or droop constant. It is typically set to 5% (0.05 pu) in the US (frequency and power are given in per-unit and the power per-unit base is the generator rated MVA). At 5% droop, the frequency deviation corresponding to a 100% change in machine output is 3 Hz.

Question: Would 4% droop be tighter or looser frequency control than 5%?<sup>1</sup>

---

<sup>1</sup> Westinghouse machines used to be set to 4% and GE machines to 5%. It was then recommended to set all machines to 5%, and some confused engineers were supportive on the basis that the higher value of R was better for frequency regulation.

Answer: It would mean that a machine is required to move more for a given frequency. At 4% droop, the frequency necessary to cause a 100% change in machine output is 2.4 Hz. So answer is “tighter.”

Recalling (70),

$$\Delta P_{e,\infty} = \frac{-\Delta\omega_\infty}{R} \quad (70)$$

we see that if all units in the interconnection have the same  $R$ , in per-unit (on each units base MVA), and recalling that in the steady-state, the entire interconnection sees the same  $\Delta\omega_\infty$ , (70) tells us that all units will make the same per-unit power change (when power is given on the generator rated MVA). This means that generators “pick-up” in proportion to their rating, i.e., larger sized machines “pick-up” more than smaller sized machines. Lovely.

Observe the benefit. Before, with integral control only where  $Q(s)=K_G/s$ , we found steady-state error to be zero, but the machines would continuously “fight” against each other if their  $\omega_{\text{ref}}$  were not exactly equal.

Now, with proportional-integral control,  $Q(s)=(1/R)(1/(1+sT_G))$ , we have non-zero steady-state error, but, machines will not “fight” one another; instead, they will load share in proportion to their MVA rating, and in proportion to the final steady-state frequency error. This situation is illustrated in Fig. 18 below.

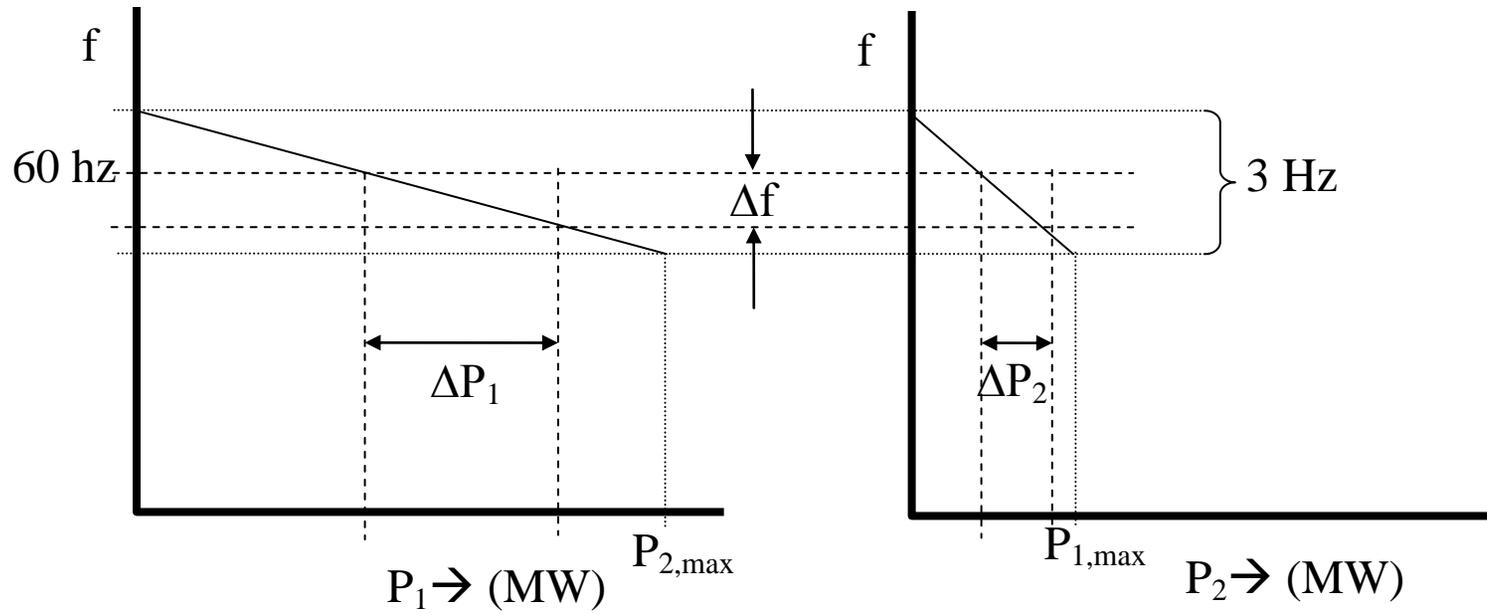


Fig. 18

Notice in Fig. 18, that when  $\Delta f=3\text{Hz}$ , for both units,  $j=1$  and  $j=2$ ,

$$R = -\left[ \frac{\Delta f / 60}{\Delta P_j / P_{j,\max}} \right] = \left[ \frac{3 / 60}{P_{j,\max} / P_{j,\max}} \right] = 0.05$$

Question: What can we do about the fact that we have a steady-state frequency deviation?

Answer: Modify the real power generation set point of the units.

The control point in which to accomplish this is called the “speed changing motor.” For a system with a single generator, this control changes turbine speed (and therefore the name) and consequently frequency. But for interconnected systems with a large number of generators (such that frequency is almost constant), this control mainly changes the power output of the machine and is situated so that the governor can do the work of actually opening and closing the valve (or gate), as shown in Fig. 19 below, which we redraw as shown in Fig. 20 and then Fig. 21. W&W calls the control the machine’s “load reference set point.” I denote it by  $\Delta P_{\text{ref}}$ .

The modification to the speed changer motor will cause a shift as shown in Fig. 22. For constant power, speed (or frequency) changes. For constant frequency, power changes.

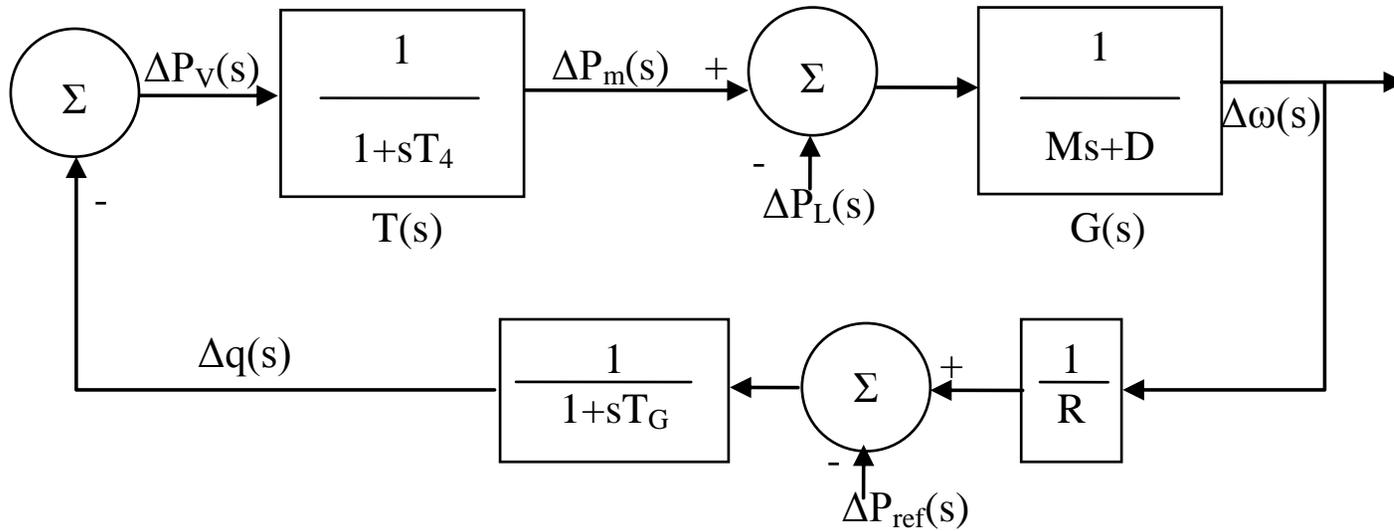


Fig. 19

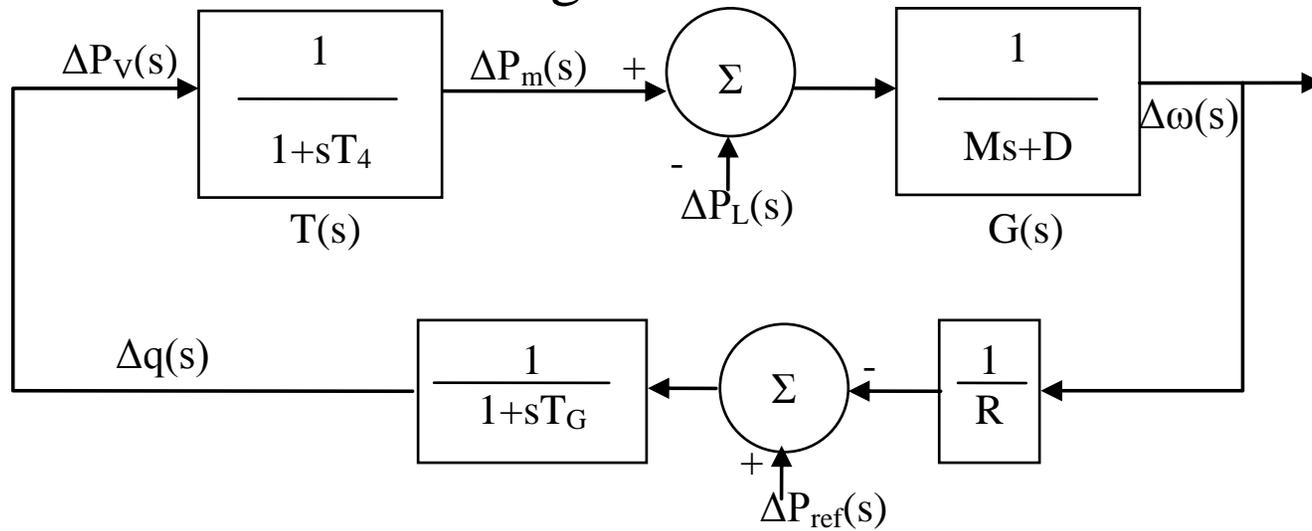


Fig. 20

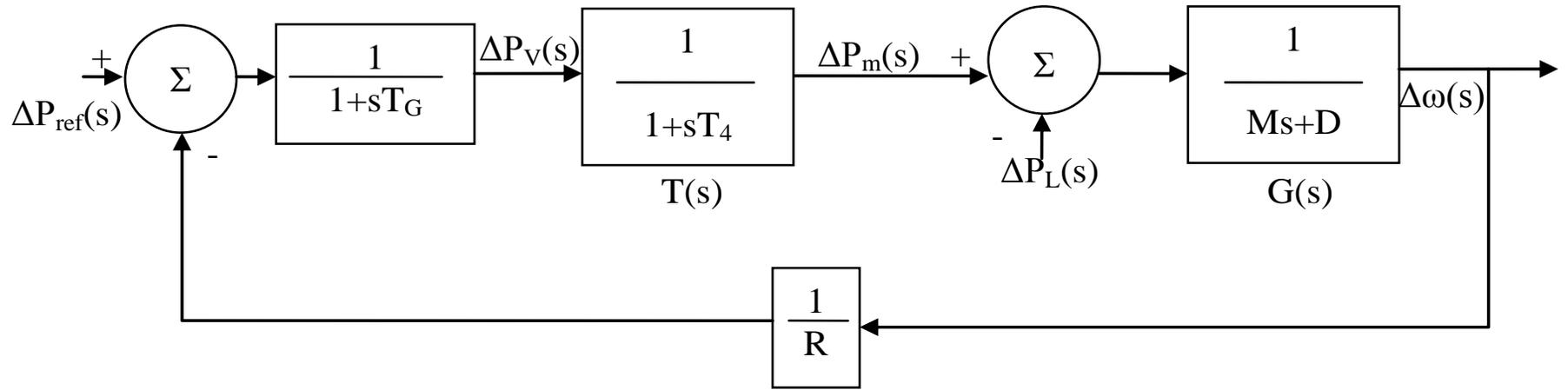


Fig. 21

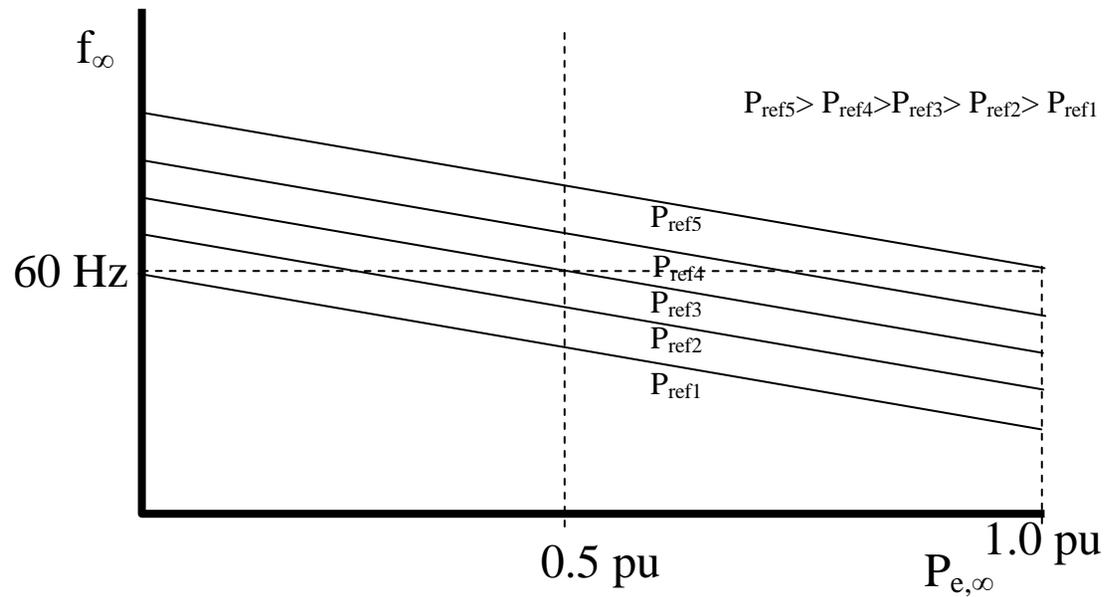


Fig. 22

## 10.0 Two-area system

Let's now consider applying governor control to a system comprised of two balancing areas (BA) with each BA having only one generator (and so we have temporarily relieved ourselves from having to worry about how to allocate the demand among the various generators). Fig 23 below illustrates.

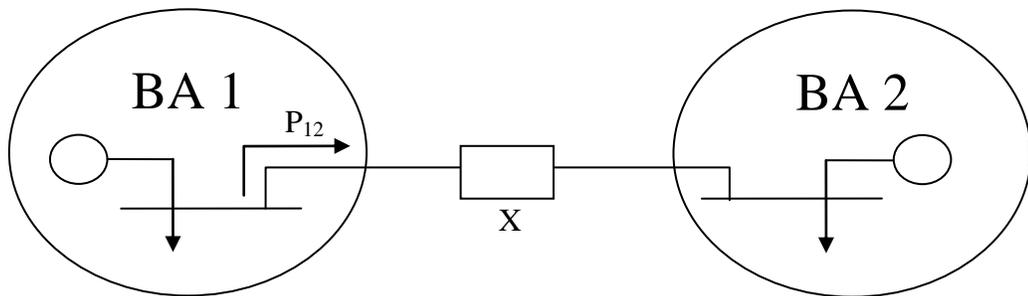


Fig. 23

The power flowing from BA1 to BA2 may be expressed as

$$P_{12} = \frac{V_1 V_2}{X} \sin(\theta_1 - \theta_2) \quad (71)$$

Assume that  $V_1 = V_2 = 1.0$  and that  $(\theta_1 - \theta_2)$  is small (and in radians). Then (71) becomes

$$P_{12} = \frac{1}{X} (\theta_1 - \theta_2) \quad (72)$$

Now assume that we have a small perturbation (perhaps a small  $\Delta P_L$ ). Then

$$P_{12} + \Delta P_{12} = \frac{1}{X} [(\theta_1 + \Delta\theta_1) - (\theta_2 + \Delta\theta_2)] \quad (73)$$

which can be rewritten as

$$P_{12} + \Delta P_{12} = \frac{1}{X} [(\theta_1 - \theta_2) + (\Delta\theta_1 - \Delta\theta_2)] \quad (74)$$

where we see that

$$\Delta P_{12}(t) = \frac{1}{X} (\Delta\theta_1(t) - \Delta\theta_2(t)) \quad (75)$$

In (75), we have made the dependency on time explicit. Taking the Laplace transform of (75), we obtain:

$$\Delta P_{12}(s) = \frac{1}{X} (\Delta\theta_1(s) - \Delta\theta_2(s)) \quad (76)$$

But

$$\Delta\theta_i(t) = \int \Delta\omega_i(t) dt \quad (77a)$$

In Laplace, (77a) becomes

$$\Delta\theta_i(s) = \frac{\Delta\omega_i(s)}{s} \quad (77b)$$

Substituting (77b) into (76) results in

$$\Delta P_{12}(s) = \frac{1}{sX} (\Delta\omega_1(s) - \Delta\omega_2(s)) \quad (78)$$

Equation (78) assumes that  $\Delta\omega_i(s)$  is in rad/sec. However, we desire it to be in per-unit, which can be achieved by (79):

$$\Delta P_{12}(s) = \frac{\omega_{e0}}{sX} (\Delta\omega_1(s) - \Delta\omega_2(s)) \quad (79)$$

Notation for  $\Delta\omega_i$  in (78) and (79) is the same, but in (78),  $\Delta\omega_i$  is in rad/sec whereas in (79),  $\Delta\omega_i$  is in pu, with this difference in the two equations being compensated by the multiplication of base frequency  $\omega_{e0}$  in (79).

Now we define the tie-line stiffness coefficient

$$T = \frac{\omega_{e0}}{X} \quad (80)$$

This tie-line stiffness coefficient is proportional to what is often referred to as the synchronizing power coefficient. For a single-generator connected through a tie-line having reactance  $X$  to an infinite bus, the power transfer is given by

$$P_e = \frac{V_1 V_2}{X} \sin \delta$$

where  $V_1$  and  $V_2$  are the voltage magnitudes of the two buses and  $\delta$  is the angular difference

between them. In this case, the synchronizing power coefficient is defined as

$$\frac{\partial P_e}{\partial \delta} = \frac{V_1 V_2}{X} \cos \delta$$

The synchronizing power coefficient, or the stiffness coefficient as defined in (80), increases as the line reactance decreases. The larger the  $T$ , the greater the change in power for a given change in bus phase angle.

Equation (79) becomes, then,

$$\Delta P_{12}(s) = \frac{T}{s} (\Delta \omega_1(s) - \Delta \omega_2(s)) \quad (81)$$

The text (pp. 341-344) uses  $P_{tie}$  instead of  $P_{21}$ . To clarify, we have

$$-\Delta P_{21} = \Delta P_{12} = \Delta P_{tie} \quad (82)$$

Observe that  $+\Delta P_{12}$  causes increase generation from BA1 – it appears to BA1 as a load increase and should therefore have the same sign as  $\Delta P_L$ . We draw the block diagram for the 2-area system using two of Fig. 21 with (81) and (82), as shown in Fig. 24, which is the same (except for some nomenclature) as Fig. 9.16 in W&W.

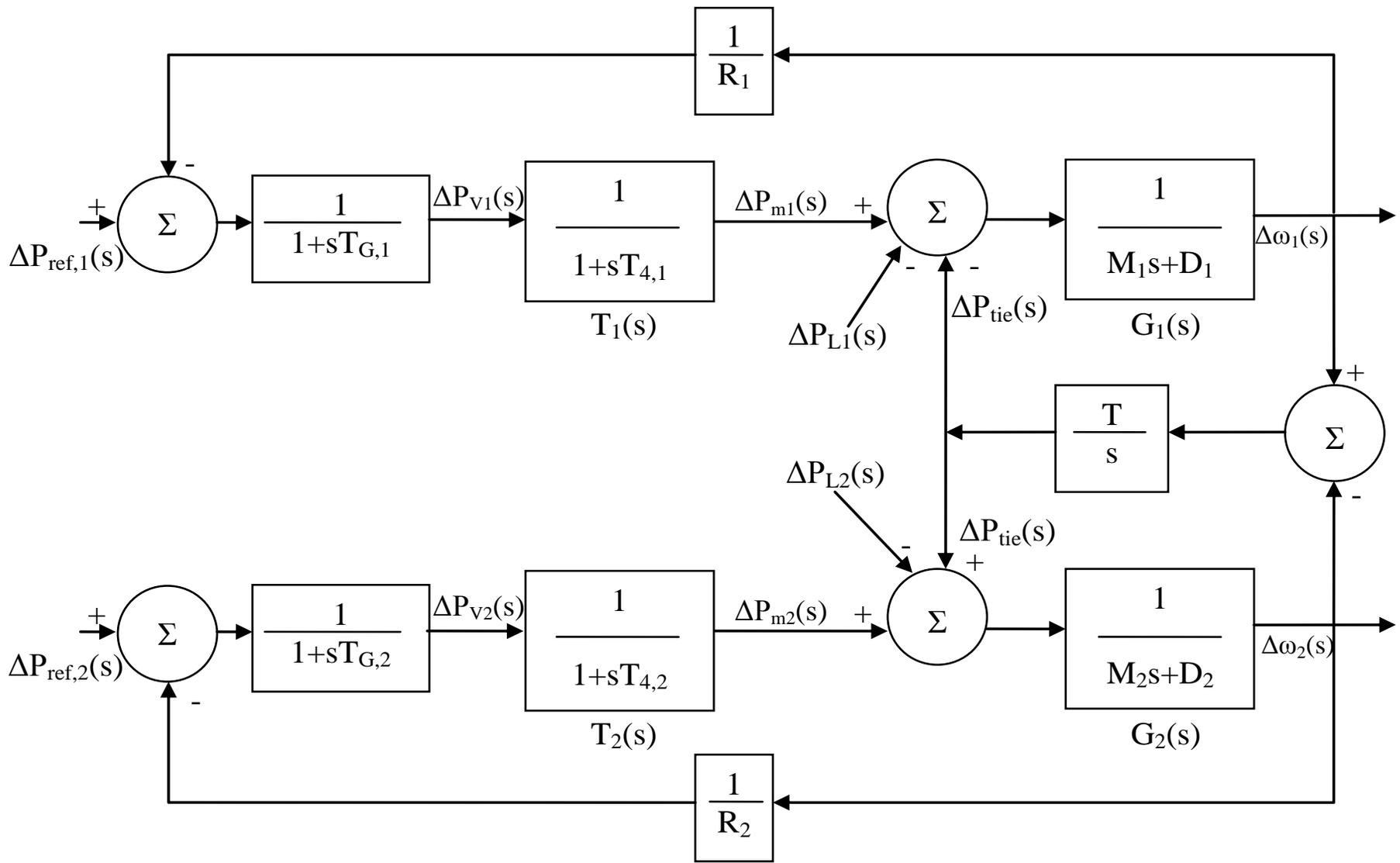


Fig. 24

We would like to find steady-state values of frequency  $\Delta\omega_\infty$  and mechanical power  $\Delta P_{m1\infty}$  and  $\Delta P_{m2\infty}$ . To do this, we begin by recalling the expression for  $\Delta\omega(s)$  for the single balancing area model, given by (54):

$$\Delta\omega(s) = \frac{-\Delta P_L(s)G(s)}{1 + \frac{1}{R} \frac{1}{(1+sT_G)} T(s)G(s)} \quad (54)$$

where,

$$G(s) = \frac{1}{Ms + D} \quad T(s) = \frac{1}{1 + sT_4} \quad (56)$$

Substitution yields

$$\Delta\omega(s) = \frac{-\Delta P_L(s) \frac{1}{Ms + D}}{1 + \frac{1}{R} \frac{1}{(1+sT_G)} \frac{1}{1+sT_4} \frac{1}{Ms + D}} \quad (83)$$

Simplifying,

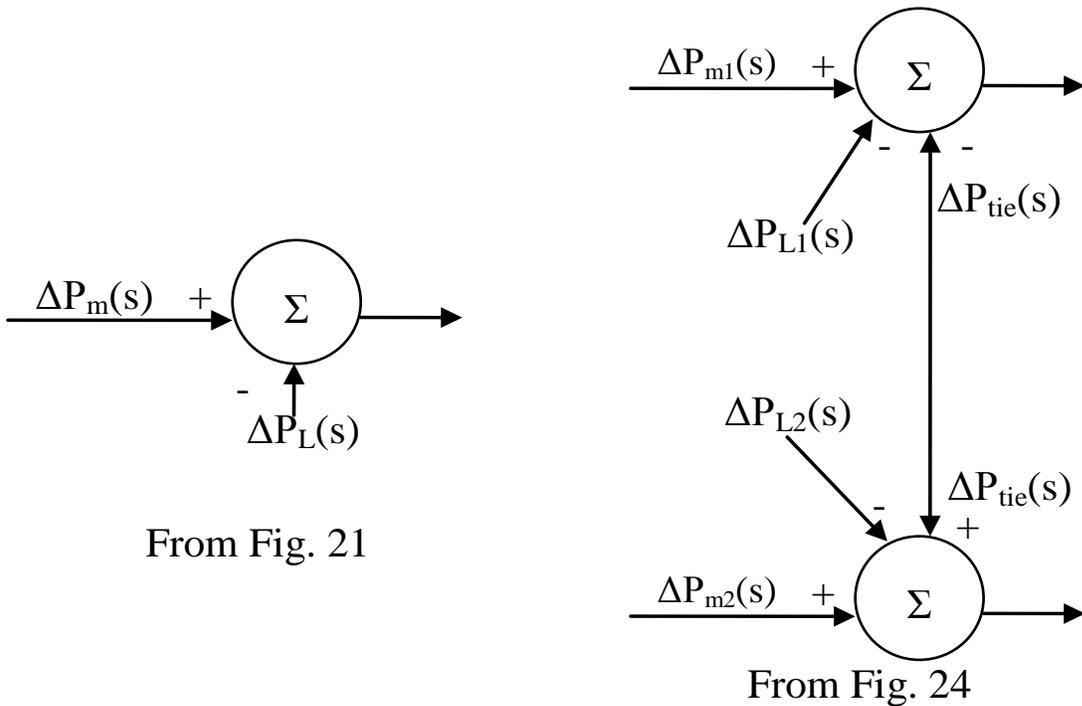
$$\Delta\omega(s) = \frac{-\Delta P_L(s)R(1+sT_G)(1+sT_4)}{R(1+sT_G)(1+sT_4)(Ms + D) + 1} \quad (84)$$

Remember, (84) is for a single balancing area.

Question: If you are BA1, how does being connected to BA2 differ from operating as a single BA?

In developing data for the 2-area system, all quantities must be normalized to the same power base. This pertains to M, D, R, and all power quantities.

Answer: The answer can be seen by comparing the *external inputs* (to the “demand summer”) of Fig. 24, which models 2 interconnected balancing areas, and that of Fig. 21, which models just one.



Whereas Fig. 21 sees only its own load change,  $\Delta P_L$ , as an external input, Fig. 24 sees,

1. in the case of BA1, the BA1 load change plus the tie-line flow change,  $\Delta P_{L1} + \Delta P_{tie}$ , i.e.,

$$\Delta P_L \rightarrow \Delta P_{L1} + \Delta P_{tie} = \Delta P_{L1}(s) + \frac{T}{s}(\Delta \omega_1(s) - \Delta \omega_2(s)) \quad (85a)$$

2. and in the case of BA2, the BA2 load change less the tie-line flow change,  $\Delta P_{L2} - \Delta P_{tie}$ , i.e.,

$$\Delta P_L \rightarrow \Delta P_{L2} - \Delta P_{tie} = \Delta P_{L1}(s) - \frac{T}{s}(\Delta \omega_1(s) - \Delta \omega_2(s)) \quad (85b)$$

So what we can do is to just make the substitution of (85a) into the  $\Delta \omega$  expression of (84), i.e., replace  $\Delta P_L$  in (84) with  $\Delta P_L + \Delta P_{tie}$  from the RHS of (85a). Alternatively, we can make the substitution of (85b) into the  $\Delta \omega$  expression of (84), i.e., replace  $\Delta P_L$  in (84) with  $\Delta P_L - \Delta P_{tie}$  from the RHS of (85b). In the first case, we obtain  $\Delta \omega_1$ , and in the second, we obtain  $\Delta \omega_2$ . It does not matter which one we use because we are interested in the steady-state frequency, and we know within an interconnection, the steady-state frequency is the same everywhere, so the expressions will be identical.

Although it gets a little algebraically messy, making one of the above substitutions, with  $\Delta P_{L1}(s) = L_1/s$ , and use of the FVT, we obtain:

Again, for the 2-area system, all quantities must be normalized to the same power base. So in (86),  $R_1$ ,  $R_2$ ,  $D_1$ , and  $D_2$  must be on the same power base. Let's denote them with a subscript of "Bsys" in what follows.

$$\Delta\omega_{1\infty} = \Delta\omega_{2\infty} = \frac{-L_1}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2} = \Delta\omega_\infty \quad (86)$$

You can also algebraically find  $\Delta P_{m1,\infty}$  and  $\Delta P_{m2,\infty}$ , or you can reason as follows:

1. We found before that  $\Delta P_{m,\infty} = \frac{-\Delta\omega_\infty}{R}$  for a one balancing area system.
2. As we already observed, the only difference between the single area and the two area model, from the point of view of either area, is what the "demand summer" sees.
3. But observing the model of Fig. 24, we see that  $\Delta P_m$  depends on the input to the demand summer only through  $\Delta\omega$ ; as long as we know  $\Delta\omega$ , we do not need any additional information about the input to the demand summer to obtain  $\Delta P_m$ . Therefore, we have

$$\Delta P_{m1,\infty} = \frac{-\Delta\omega_\infty}{R_{1,Bsys}} \quad \Delta P_{m2,\infty} = \frac{-\Delta\omega_\infty}{R_{2,Bsys}} \quad (87)$$

Question: How does the generation get distributed?

There are two "demand summers." The one discussed here is the one in the right-center of the Fig. 24 diagram, where  $\Delta P_L$  is input.

Define  $R_{1,B1}=R_{2,B2}$  ( $=0.05$ ) as droop constants on the machine base.

But (87) are derived from a *system* of governors, necessitating that  $\Delta P_{m1}$ ,  $\Delta P_{m2}$ ,  $R_1$ , and  $R_2$  be given on the same MVA base – the system MVA base, which we refer to as  $S_{Bsys}$  (as opposed to the machine MVA base,  $S_B$ ). Recalling  $\Delta\omega_\infty = -R\Delta P_{m,\infty}$ , we can write that the per-unit frequency deviation is:

$$\Delta\omega_\infty = -R_{1,Bsys} \frac{\Delta P_{m1\infty}^{(MW)}}{S_{Bsys}} = -R_{1,B1} \frac{\Delta P_{m1\infty}^{(MW)}}{S_{B1}} \quad (88)$$

where  $\Delta P_{m1\infty}^{(MW)}$  is in MW. Solving (88) for  $R_{1,Bsys}$ , we obtain

$$R_{1,Bsys} = R_{1,B1} \frac{S_{Bsys}}{S_{B1}} \quad (89a)$$

Similarly, we can derive

$$R_{2,Bsys} = R_{2,B2} \frac{S_{Bsys}}{S_{B2}} \quad (89b)$$

Substitute (89a) and (89b) into (87) results in

$$\Delta P_{m1,\infty} = \frac{-\Delta\omega_\infty}{R_{1,B1} \frac{S_{Bsys}}{S_{B1}}} = \frac{-\Delta\omega_\infty}{R_{1,B1} S_{Bsys}} S_{B1} \quad (90a)$$

$$\Delta P_{m2,\infty} = \frac{-\Delta\omega_\infty}{R_{2,B2} \frac{S_{Bsys}}{S_{B2}}} = \frac{-\Delta\omega_\infty}{R_{2,B2} S_{Bsys}} S_{B2} \quad (90b)$$

Equations (90a) and (90b) express the steady-state power deviations in per-unit on the system base,  $S_{Bsys}$ . To express them in MW, we have

$$\Delta P_{m1,\infty}^{MW} = \frac{-\Delta\omega_\infty}{R_{1,B1}} S_{B1} \quad (91a)$$

$$\Delta P_{m2,\infty}^{MW} = \frac{-\Delta\omega_\infty}{R_{2,B2}} S_{B2} \quad (91b)$$

If we require that  $R_{1,B1}=R_{2,B2}$  be equal (and in North America they are 0.05), then (91a) and (91b) indicate that the governor action responds in such a way so that each balancing area compensates for load change in proportion to its size. This was a conclusion we guessed at in the analysis of a single area – see equation (70) – but here we have confirmed it using rigorous analysis for a two area case.

As a last comment regarding the two-area model, let's take a look at the electrical power out of the machines. In what follows, all quantities are on a common power base ( $S_{Bsys}$ ).

In the single area case, you will recall that the steady-state deviation in electrical power out of the machine is equal to the change in load  $\Delta P_L$  plus the change in load resulting from the steady-state frequency deviation per (30), i.e.,

$$\Delta P_e = \Delta P_L + D\Delta\omega \quad (30)$$

The situation for area 1 will be exactly the same, except now we also have to pay attention to changes in tie-line flow, i.e.,

$$\Delta P_{e1\infty} = \Delta P_{L1} + D_1\Delta\omega_\infty + \Delta P_{tie} \quad (92)$$

where  $\Delta P_{tie} = \Delta P_{12} = -\Delta P_{21}$ . If the only load change is in BA1, i.e.,  $\Delta P_{L2} = 0$ , then

$$\Delta P_{e2\infty} = D_1\Delta\omega_\infty - \Delta P_{tie} \quad (93)$$

But recall from (87)

$$\Delta P_{e1,\infty} = \frac{-\Delta\omega_\infty}{R_{1,Bsys}} \quad \Delta P_{e2,\infty} = \frac{-\Delta\omega_\infty}{R_{2,Bsys}} \quad (94)$$

Substitution of the  $\Delta P_{e2\infty}$  of (94) into (93) gives

$$\Delta P_{e2,\infty} = \frac{-\Delta\omega_\infty}{R_2} = D_2\Delta\omega_\infty - \Delta P_{tie} \quad (95)$$

Solving for  $\Delta P_{tie}$  results in

$$\Delta P_{tie} = \Delta\omega_\infty \left( D_2 + \frac{1}{R_2} \right) \quad (96)$$

Substitution of (86) into (96) results in

$$\Delta P_{tie} = \frac{-L_1 \left( D_2 + \frac{1}{R_2} \right)}{\frac{1}{R_1} + \frac{1}{R_2} + D_1 + D_2} \quad (97)$$

## 11.0 Secondary control

Question: What sort of additional control do we need for our two-area system?

For a step load change in area 1, we desire:

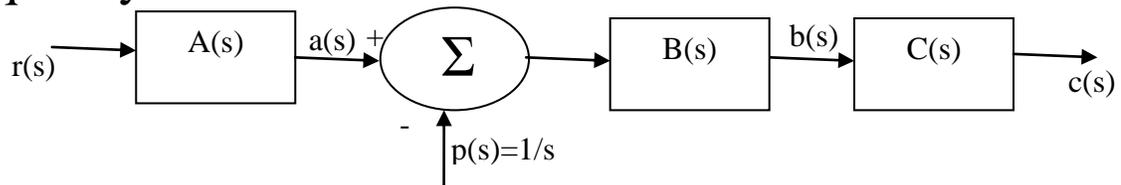
- |   |   |   |
|---|---|---|
| <ol style="list-style-type: none"> <li>1. <math>\Delta P_{m1\infty} = \Delta P_{L1}</math></li> <li>2. <math>\Delta P_{m2\infty} = 0</math></li> <li>3. <math>\Delta\omega_\infty = 0</math></li> <li>4. <math>\Delta P_{tie\infty} = 0</math></li> </ol> | } | <p>Each BA compensates for its own load change.</p> |
|---|---|---|

How to develop such a control?

There are three basic ways:

1. Derive steady-state relations for  $\Delta\omega_\infty$  and  $\Delta P_{tie\infty}$  and ask: What do we need to do to make their sum = 0? This is what W&W do, see page 349.
2. Derive expressions for  $\Delta\omega(s)$ ,  $\Delta P_{tie}(s)$ ,  $\Delta P_{m1}(s)$ , and  $\Delta P_{m2}(s)$  given an unknown control input  $u(s)$ , and then apply conditions 1-4 above. This works, but it is algebraically messy.
3. Reason via knowledge of process control. This is what we will do. The reasoning is provided in what follows.

Integral control action: Consider the following system that is conceptually similar to our own two-area load-frequency model.



We desire to ensure that  $b(t)$  and  $c(t)$  are driven to 0 in the steady-state ( $t=\infty$ ) following a step change in  $p(t)$ , i.e.,  $p(s)=1/s$ . In the above system, we do not achieve this as shown below (assume  $r(s)=0$ ).

$$b(s) = -B(s)p(s) = \frac{-B(s)}{s}$$

$$c(s) = C(s)b(s) = \frac{-C(s)B(s)}{s}$$

$$b_\infty = \lim_{t \rightarrow \infty} b(t) = \lim_{s \rightarrow 0} sb(s) = \lim_{s \rightarrow 0} [-B(s)]$$

$$c_\infty = \lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sc(s) = \lim_{s \rightarrow 0} [C(s)B(s)]$$

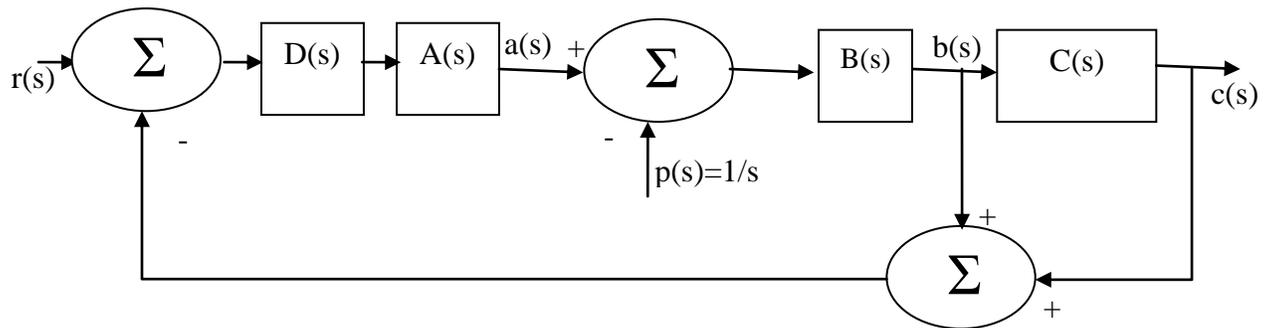
We observe that neither term goes to zero.

The only way we will achieve  $b_\infty=0$  is if we have a “zero” in  $B(s)$ , and the only way we will achieve  $c_\infty=0$  is if we have a “zero” in  $B(s)$  or  $C(s)$  or both.

Assumption 1: We do not have a zero in  $B(s)$  or  $C(s)$ .

Assumption 2: We do not have a zero in  $A(s)$ .

Let’s add a controller in front of  $A(s)$  and feedback the sum of  $c(s)$  and  $b(s)$  through it as follows:



Again, assume that  $r(s)=0$  and  $p(s)=1/s$ . We may derive the expression for  $b(s)$  as follows:

$$a(s) = -A(s)D(s)[b(s) + c(s)] = -A(s)D(s)[b(s) + b(s)C(s)]$$

$$b(s) = B(s)[a(s) - p(s)] = B(s)\left[a(s) - \frac{1}{s}\right] = B(s)a(s) - \frac{B(s)}{s}$$

$$\Rightarrow b(s) = -B(s)A(s)D(s)[b(s) + b(s)C(s)] - \frac{B(s)}{s}$$

$$= -b(s)\{B(s)A(s)D(s)[1 + C(s)]\} - \frac{B(s)}{s}$$

$$\Rightarrow b(s)\{1 + B(s)A(s)D(s)[1 + C(s)]\} = -\frac{B(s)}{s}$$

$$b(s) = \frac{-B(s)/s}{1 + B(s)A(s)D(s)[1 + C(s)]}$$

And because  $c(s)=C(s)b(s)$ ,

$$c(s) = \frac{-B(s)C(s) / s}{1 + B(s)A(s)D(s)[1 + C(s)]}$$

We can now look at the steady-state error of these two terms:

$$b_{\infty} = \lim_{t \rightarrow \infty} b(t) = \lim_{s \rightarrow 0} sb(s) = \lim_{s \rightarrow 0} \left[ \frac{-B(s)}{1 + B(s)A(s)D(s)[1 + C(s)]} \right]$$

$$c_{\infty} = \lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sc(s) = \lim_{s \rightarrow 0} \left[ \frac{-B(s)C(s)}{1 + B(s)A(s)D(s)[1 + C(s)]} \right]$$

Now let  $D=1/s$ , an integrator.

In that case,

$$b_{\infty} = \lim_{t \rightarrow \infty} b(t) = \lim_{s \rightarrow 0} sb(s) = \lim_{s \rightarrow 0} \left[ \frac{-B(s)}{1 + B(s)A(s) \frac{1}{s} [1 + C(s)]} \right]$$

$$= \lim_{s \rightarrow 0} \left[ \frac{-sB(s)}{s + B(s)A(s)[1 + C(s)]} \right] = 0$$

Likewise,

$$c_{\infty} = \lim_{t \rightarrow \infty} c(t) = \lim_{s \rightarrow 0} sc(s) = \lim_{s \rightarrow 0} \left[ \frac{-B(s)C(s)}{1 + B(s)A(s) \frac{1}{s} [1 + C(s)]} \right]$$

$$= \lim_{s \rightarrow 0} \left[ \frac{-sB(s)C(s)}{s + B(s)A(s)[1 + C(s)]} \right] = 0$$

What is the point? We may zero the steady-state error of a signal, when system input is a step response, by passing the signal through an integral-controller via a negative feedback loop.

→ That is, we desire to make  $\Delta\omega_\infty$  and  $\Delta P_{tie,\infty}$  zero just like we made  $b(t)$ ,  $c(t)$  zero in the above exercise. So, let's sum them & pass them through an integral controller via a negative feedback loop. Fig. 9.21, W&W, illustrates.

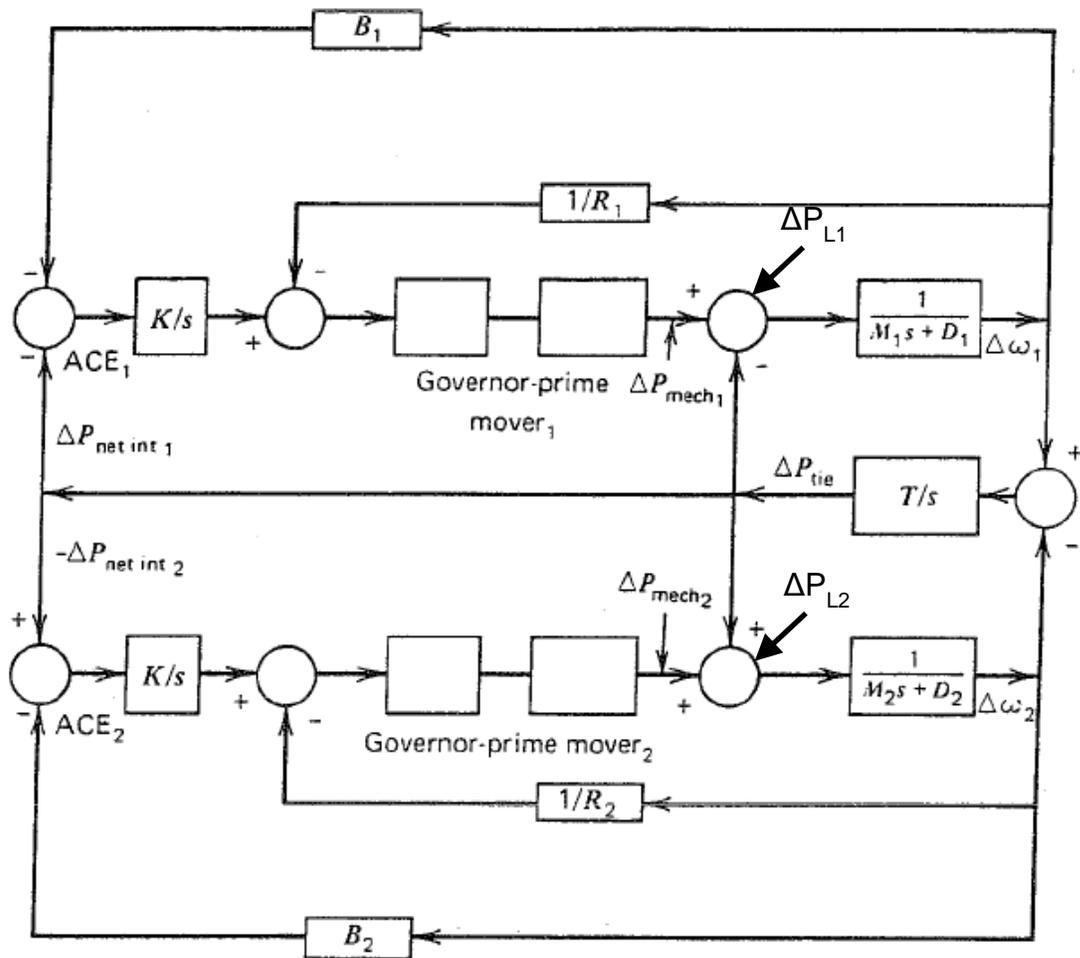


FIG. 9.21 Tie-line bias supplementary control for two areas.

Notice (1)  $D(s)=K/s$ ; (2) frequency bias factors  $B_1$  &  $B_2$ ; and (3) the signals  $ACE_1=-B_1\Delta\omega-\Delta P_{tie}$ ,  $ACE_2=-B_2\Delta\omega+\Delta P_{tie}$  where

$$B_1 = D_1 + \frac{1}{R_1}$$

$$B_2 = D_2 + \frac{1}{R_2}$$

The  $ACE_1$  and  $ACE_2$  signals become positive (to increase generation) when  $\Delta\omega$  and  $\Delta P_{tie}$  are negative.

## 12.0 State equations

State equations may be written for the system illustrated in Fig. 9.21 of W&W. These equations enable one to simulate the system using Simulink.

$$\dot{\Delta P_{V1}}(t) = -\frac{1}{T_{G1}} \Delta P_{V1}(t) - \frac{1}{T_{G1}R_1} \Delta\omega_1(t) + \frac{1}{T_{G1}} \Delta P_{ref1}(t)$$

$$\dot{\Delta P_{m1}}(t) = \frac{1}{T_{T1}} \Delta P_{V1}(t) - \frac{1}{T_{T1}} \Delta P_{m1}(t)$$

$$\dot{\Delta\omega_1}(t) = \frac{1}{M_1} \Delta P_{m1}(t) - \frac{D_1}{M_1} \Delta\omega_1(t) - \frac{1}{M_1} \Delta P_{tie}(t) - \frac{1}{M_1} \Delta P_{L1}(t)$$

$$\dot{\Delta P_{ref1}}(t) = -KB_1\Delta\omega_1(t) - K\Delta P_{tie}$$

$$\Delta P_{V_2}^{\cdot}(t) = -\frac{1}{T_{G_2}} \Delta P_{V_2}(t) - \frac{1}{T_{G_2} R_2} \Delta \omega_2(t) + \frac{1}{T_{G_2}} \Delta P_{ref_2}(t)$$

$$\Delta P_{m_2}^{\cdot}(t) = \frac{1}{T_{T_2}} \Delta P_{V_2}(t) - \frac{1}{T_{T_2}} \Delta P_{m_2}(t)$$

$$\Delta \omega_2^{\cdot}(t) = \frac{1}{M_2} \Delta P_{m_2}(t) - \frac{D_2}{M_2} \Delta \omega_2(t) + \frac{1}{M_2} \Delta P_{ie}(t) - \frac{1}{M_2} \Delta P_{L_2}(t)$$

$$\Delta P_{ref_2}^{\cdot}(t) = -KB_2 \Delta \omega_2(t) + K \Delta P_{ie}$$

$$\Delta P_{ie}^{\cdot}(t) = T \Delta \omega_1(t) - T \Delta \omega_2(t)$$

### **13.0 Base point calculation**

A last comment is that the ACE, being a measure of how much the total system generation needs to change, is allocated to the various units that comprise the balancing area via **participation factors**. This is illustrated in Fig. 9.25 of W&W. The participation factors are obtained by linearizing the economic (market) dispatch about the last **base point** solution (see Wood & Wollenberg, section 3.8). Base point calculation is performed by the real-time market every 5 mins, as indicated in the slide at the top of the next page.

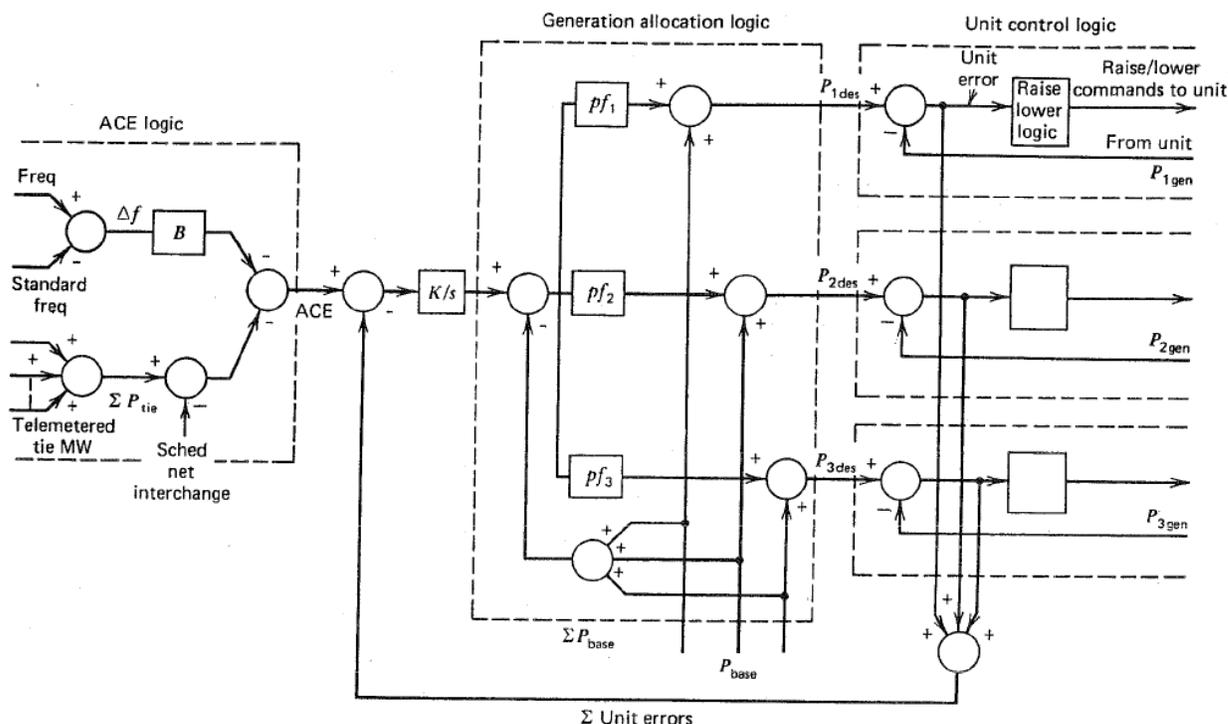


FIG. 9.25 Overview of AGC logic.

## Base point calculation via real-time market

Focus on interval 2, {t+5, t+10}.

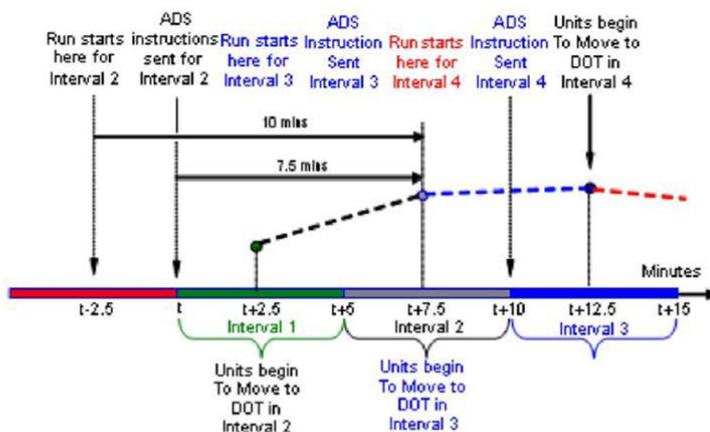
For interval 2, a short-term net load forecast is made 7.5 min before interval 2 begins, at t-2.5, and generation set points are computed accordingly.

At t+2.5, which is 2.5 minutes before interval 2 begins, the units start to move.

The units are ramped at a rate which provides that they reach the desired base point at t+7.5 min, which is 2.5 min after the interval begins.

Source: Y. Makarov, C. Loutan, J. Ma, and P. de Mello, "Operational impacts of wind generation on California power systems," IEEE Trans on Power Systems, Vol. 24, No. 2, May 2009.

ADS: automatic dispatch system  
DOT: dispatch operating target



**Key point:** The base point is computed from a net load forecast. There is error in this forecast, which typically increases as wind penetration increases. This error contributes to power imbalance and therefore frequency deviation.

## 14.0 Control performance standards

Control Performance Standards CPS1 and CPS2 are two performance metrics associated with load frequency control. These measures depend on area control error (ACE), given for control area  $i$  as

$$ACE_i = \Delta P_i + B\Delta f \quad (12)$$

$$\Delta P_i = AP_i - SP_i \quad (13)$$

where  $AP_i$  and  $SP_i$  are actual and scheduled exports, respectively.  $ACE_i$  is computed on a continuous basis.

With this definition, we can define CPS1 and CPS2 as

- CPS1: It measures ACE variability, a measure of short-term error between load and generation [6]. It is an average of a function combining ACE and interconnection frequency error from schedule [7]. It measures control performance by comparing how well a control area's ACE performs in conjunction with the frequency error of the interconnection. It is given by

$$CPS1 = (2 - CF) \times 100\% \quad (14a)$$

$$CF = \frac{ControlParameter_{12-Month}}{(\varepsilon_1)^2} \quad (14b)$$

$$ControlParameter = \frac{ACE_{minute}}{-10B} \times \Delta F_{minute} \quad (14c)$$

where

- CF is the compliance factor, the ratio of the 12 month average control parameter divided by the square of the frequency target  $\varepsilon_1$ .
- $\varepsilon_1$  is the maximum acceptable steady-state frequency deviation – it is 0.018 Hz=18 mHz in the eastern interconnection. It is illustrated in Fig. 9 [8].

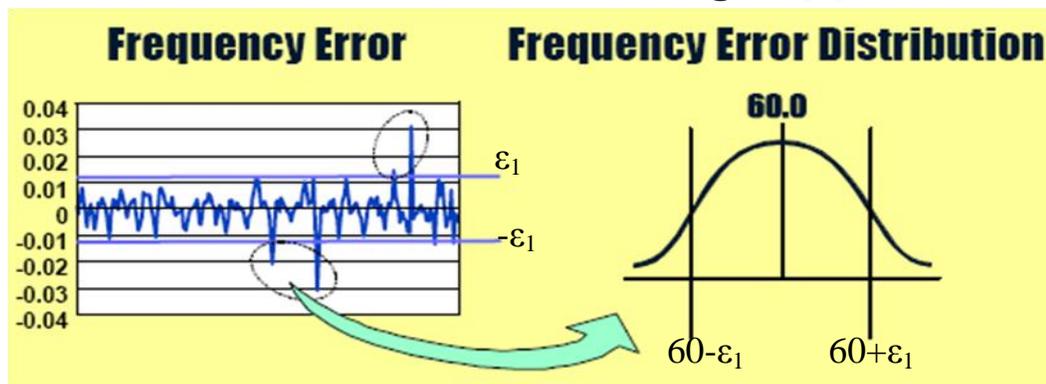


Fig. 9 [8]

- The control parameter, a “MW-Hz,” indicates the extent to which the control area is contributing to or hindering correction of the interconnection frequency error, as illustrated in Fig. 10 [8].

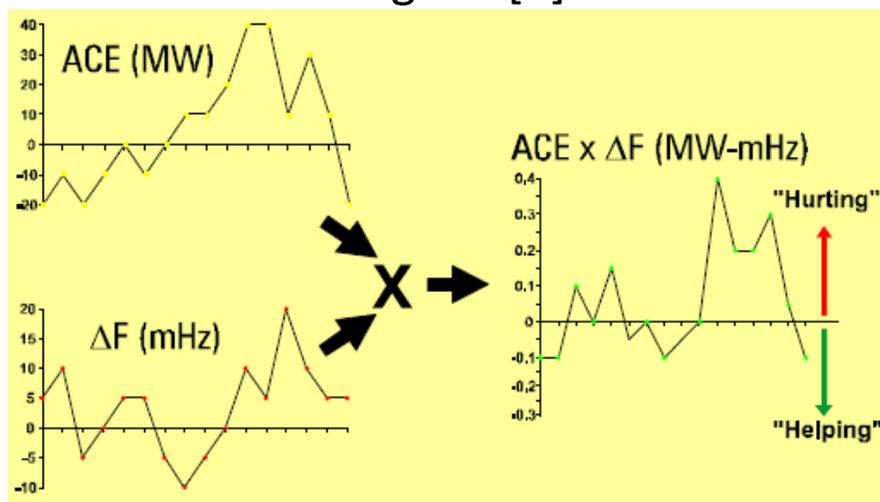


Fig. 10 [8]

If ACE is positive, the control area will be increasing its generation, and if ACE is negative, the control area will be decreasing its generation. If  $\Delta F$  is positive, then the overall interconnection needs to decrease its generation, and if  $\Delta F$  is negative, then the overall interconnection needs to increase its generation. Therefore if the sign of the product  $ACE \times \Delta F$  is positive, then the control area is hindering the needed frequency correction, and if the sign of the product  $ACE \times \Delta F$  is negative, then the control area is contributing to the needed frequency correction.

- A CPS1 score of 200% is perfect (actual measured frequency equals scheduled frequency over any 1-minute period)
- The minimum passing long-term (12-month rolling average) score for CPS1 is 100%
- CPS2: The ten-minute average ACE.

In summary, from [9], “CPS1 measures the relationship between the control area’s ACE and its interconnection frequency on a one-minute average basis. CPS1 values are recorded every minute, but the metric is evaluated and reported annually. NERC sets minimum CPS1 requirements that each control area must exceed each year. CPS2 is a monthly performance standard that sets control-area-specific limits on the maximum average ACE for every 10-minute period.” The underlying issue here is that control area operators are penalized if they do

not maintain CPS. The ability to maintain these standards is decreased as inertia decreases.

## Appendix A

These notes are adapted from treatment in [10].

Speed governing equipment for steam and hydro turbines are conceptually similar. Most speed governing systems are one of two types; mechanical-hydraulic or Electro-hydraulic. Electro-hydraulic governing equipment use electrical sensing instead of mechanical, and various other functions are implemented using electronic circuitry. Some Electro-hydraulic governing systems also incorporate digital (computer software) control to achieve necessary transient and steady state control requirements. The mechanical-hydraulic design illustrated in Fig. A4 is used with older generators. We review this older design here because it provides good intuitive understanding of the primary speed loop operation.

Basic operation of this feedback control for turbines operating under-speed (corresponding

to the case of losing generation or adding load) is indicated by movement of each component as shown by the vertical arrows.

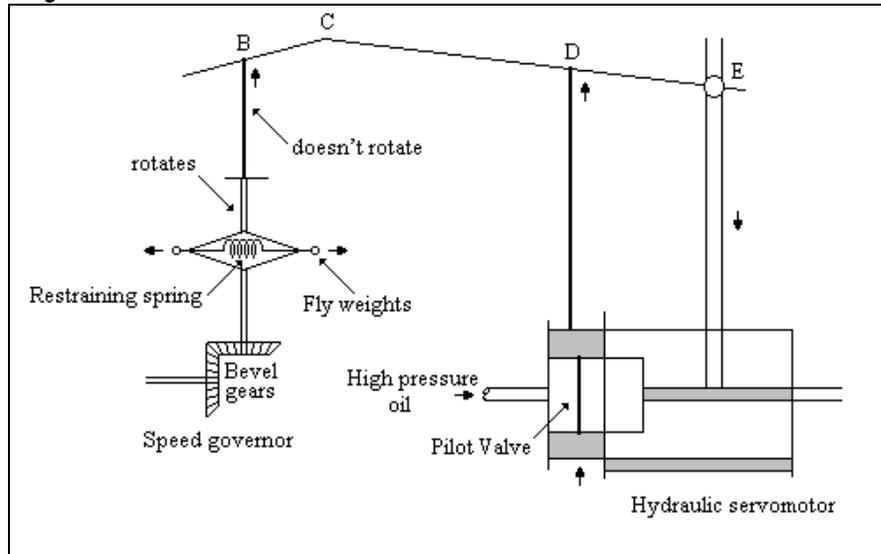


Fig. A4

- As  $\omega_m$  decreases, the bevel gears decrease their rotational speed, and the rotating flyweights pull together due to decreased centrifugal force. This causes point B and therefore point C to raise.
- Assuming, initially, that point E is fixed, point D also raises causing high pressure oil to flow into the cylinder through the upper port and release of the oil through the lower port.
- The oil causes the main piston to lower, which opens the steam valve (or water gate in the

case of a hydro machine), increasing the energy supply to the machine in order to increase the speed.

- To avoid over-correction, Rod CDE is connected at point E so that when the main piston lowers, and thus point E lowers, Rod CDE also lowers. This causes a reversal of the original action of opening the steam valve. The amount of correction obtained in this action can be adjusted. This action provides for an intentional non-zero steady-state frequency error.

There is really only one input to the diagram of Fig. A4, and that is the speed of the governor, which determines how the point B moves from its original position and therefore also determines the change in the steam-valve opening.

However, we also need to be able to set the input of the steam-valve opening directly, so that we can change the MW output of the

generator in order to achieve economic operation. This is achieved by enabling direct control of the position of point C via a servomotor, as illustrated in Fig. A5. For example, as point A moves down, assuming constant frequency, point B remains fixed and therefore point C moves up. This causes point D to raise, opening the valve to increase the steam flow.

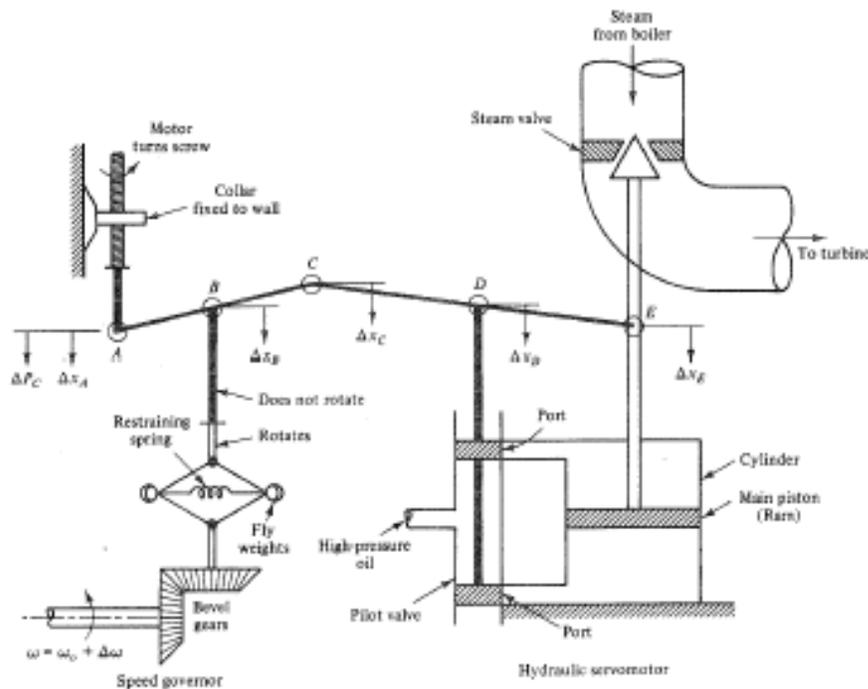


Figure 11.1 Servo-assisted speed governor.

Fig. A5

**A model for small changes**

We desire an analytic model that enables us to study the operation of the Fig. A5 controller

when it undergoes small changes away from a current state. We utilize the variables shown in Fig. A5, which include  $\Delta P_C$ ,  $\Delta x_A$ ,  $\Delta x_B$ ,  $\Delta x_C$ ,  $\Delta x_D$ ,  $\Delta x_E$ . We provide statements indicating the conceptual basis and then the analytical relation. In each case, we express an “output” or dependent variable as a function of “inputs” or independent variables of a certain portion of the controller.

1. Basis: Points A, B, C are on the same rod. Point C is the output. When A is fixed, C moves in same direction as B. When B is fixed, C moves in opposite direction as A.

$$\text{Relation: } \Delta x_C = k_B \Delta x_B - k_A \Delta x_A \quad (\text{A7})$$

2. Basis: Change in point B depends on the change in frequency  $\Delta \omega$ .

$$\text{Relation: } k_B \Delta x_B = k_1 \Delta \omega \quad (\text{A8})$$

3. Basis: Change in point A depends on the change in set point  $\Delta P_C$ .

$$\text{Relation: } k_A \Delta x_A = k_2 \Delta P_C \quad (\text{A9})$$

Substitution of (A8) & (A9) into (A7) result in  $\Delta x_C = k_1 \Delta \omega - k_2 \Delta P_C$  (A10a)

4. Basis: Points C, D, and E are on the same rod. Point D is the output. When E is fixed, D moves in the same direction as C. When C is fixed, D moves in the same direction as E.

Relation:  $\Delta x_D = k_3 \Delta x_C + k_4 \Delta x_E$  (A11a)

5. Basis: Time rate of change of oil through the ports determines the time rate of change of E.

Relation:  $\frac{d\Delta x_E}{dt} = \frac{d}{dt}(\text{oil through ports})$  (A12a)

6. Basis: A change in D determines the time rate of change of oil through the ports.

Relation:  $\left| \frac{d\Delta x_E}{dt} \right| = k_5 \Delta x_D$  (A12b)

7. Basis: The pilot valve is positioned so that when position D is moved by a positive  $\Delta x_D$ , the rate of change of oil through the ports decreases.

Relation:  $\frac{d\Delta x_E}{dt} = -k_5 \Delta x_D$  (13a)

Now we will take the LaPlace transform of eqs. (A10a), (A11a), and (A13a) to obtain:

$$\Delta \hat{x}_C = k_1 \Delta \hat{\omega} - k_2 \Delta \hat{P}_C \quad (\text{A10b})$$

$$\Delta\hat{x}_D = k_3\Delta\hat{x}_C + k_4\Delta\hat{x}_E \quad (\text{A11b})$$

$$s\Delta\hat{x}_E - \Delta x_E(0) = -k_5\Delta\hat{x}_D \quad (\text{A13b})$$

where the circumflex above the variables is used to indicate the LaPlace transform of the variables.

In eq. (A13b), we have used the LaPlace transform of a derivative which depends on the initial conditions. We will assume that the initial condition, i.e., the initial change, is 0, so that  $\Delta x_E(t=0)=0$ . Therefore, eq. (A13b) becomes:

$$s\Delta\hat{x}_E = -k_5\Delta\hat{x}_D \quad (\text{A13c})$$

and solving for  $\Delta\hat{x}_E$  results in

$$\Delta\hat{x}_E = \frac{-k_5}{s}\Delta\hat{x}_D \quad (\text{A13d})$$

Let's draw block diagrams for each of the equations (A10b), (A11b), and (A13d).

Starting with (A10b), which is  $\Delta\hat{x}_C = k_1\Delta\hat{\omega} - k_2\Delta\hat{P}_C$ , we can draw Fig. 6.

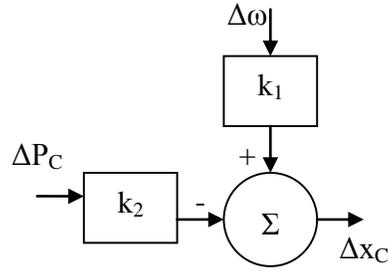


Fig. A6

Moving to (A11b), which is  $\Delta \hat{x}_D = k_3 \Delta \hat{x}_C + k_4 \Delta \hat{x}_E$ , we can draw Fig. A7.

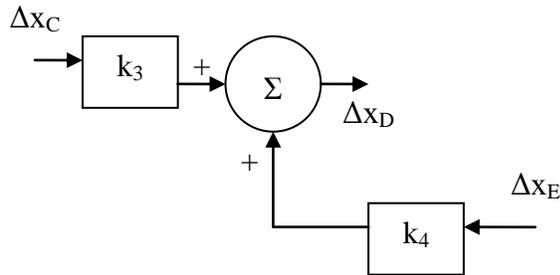


Fig. A7

Finally, considering (A13d), which is

$$\Delta \hat{x}_E = \frac{-k_5}{s} \Delta \hat{x}_D, \text{ we can draw Fig. A8.}$$

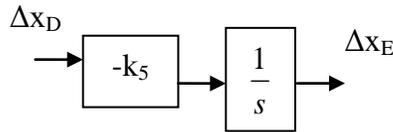


Fig. A8

Combining Figs. A6, A7, and A8, we have:

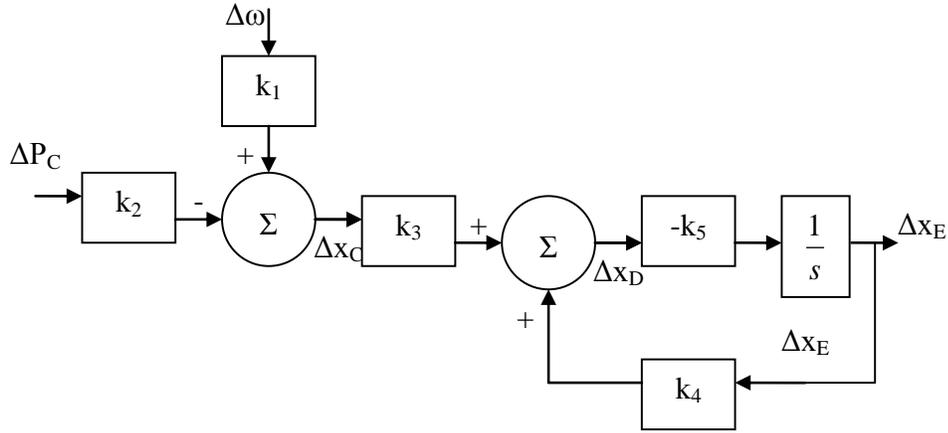


Fig. A9

We can derive the relation between the output which is  $\Delta x_E$  and the inputs which are  $\Delta P_C$  and  $\Delta \omega$  using our previously derived equations. Alternatively, we may observe from the block diagram that

$$\Delta \hat{x}_E = \frac{-k_5}{s} \Delta \hat{x}_D \quad (\text{A14})$$

$$\Delta \hat{x}_D = k_3 \Delta \hat{x}_C + k_4 \Delta \hat{x}_E \quad (\text{A15})$$

Substitution of (A15) into (A14) yields:

$$\Delta \hat{x}_E = \frac{-k_5}{s} (k_3 \Delta \hat{x}_C + k_4 \Delta \hat{x}_E) \quad (\text{A16})$$

Expanding (A16) results in:

$$\Delta \hat{x}_E = \frac{-k_5}{s} k_3 \Delta \hat{x}_C - \frac{k_5}{s} k_4 \Delta \hat{x}_E \quad (\text{A17})$$

Moving terms in  $\Delta x_E$  to the left-hand-side gives:

$$\Delta \hat{x}_E + \frac{k_5}{s} k_4 \Delta \hat{x}_E = \frac{-k_5}{s} k_3 \Delta \hat{x}_C \quad (\text{A18})$$

Factoring out the  $\Delta x_E$  yields:

$$\Delta \hat{x}_E \left( 1 + \frac{k_5}{s} k_4 \right) = \frac{-k_5}{s} k_3 \Delta \hat{x}_C \quad (\text{A19})$$

Dividing both sides by the term in the bracket on the left-hand-side provides:

$$\Delta \hat{x}_E = \frac{\frac{-k_5}{s} k_3 \Delta \hat{x}_C}{1 + \frac{k_5}{s} k_4} \quad (\text{A20})$$

Multiplying top and bottom by s gives:

$$\Delta \hat{x}_E = \frac{-k_5 k_3 \Delta \hat{x}_C}{s + k_5 k_4} \quad (\text{A21})$$

Now recognizing from Fig. A9 or (A10b), that  $\Delta \hat{x}_C = k_1 \Delta \hat{\omega} - k_A \Delta \hat{P}_C$ , we may make the appropriate substitution into (A21) to get:

$$\Delta \hat{x}_E = \frac{-k_5 k_3}{s + k_5 k_4} (k_1 \Delta \hat{\omega} - k_A \Delta \hat{P}_C) \quad (\text{A22})$$

Distributing the negative sign through:

$$\Delta \hat{x}_E = \frac{k_5 k_3}{s + k_5 k_4} (-k_1 \Delta \hat{\omega} + k_2 \Delta \hat{P}_C) \quad (\text{A23})$$

Now factor out  $k_2$  to obtain:

$$\Delta \hat{x}_E = \frac{k_2 k_5 k_3}{s + k_5 k_4} \left( \frac{-k_1}{k_2} \Delta \hat{\omega} + \Delta \hat{P}_C \right) \quad (\text{A24})$$

Simply switching the order of the terms in the parentheses:

$$\Delta \hat{x}_E = \frac{k_2 k_5 k_3}{s + k_5 k_4} \left( \Delta \hat{P}_C - \frac{k_1}{k_2} \Delta \hat{\omega} \right) \quad (\text{A25})$$

Divide top and bottom by  $k_5 k_4$  to get:

$$\Delta \hat{x}_E = \frac{k_2 k_3 / k_4}{s / k_5 k_4 + 1} \left( \Delta \hat{P}_C - \frac{k_1}{k_2} \Delta \hat{\omega} \right) \quad (\text{A26})$$

Now we make three definitions:

$$K_G = k_2 k_3 / k_4$$

$$T_G = \frac{1}{k_5 k_4}$$

$$R = \frac{k_2}{k_1}$$

(A27)

where  $K_G$  is the controller gain,  $T_G$  is the controller time constant, and  $R$  is the regulation constant. Using these parameters in (A26) gives:

$$\Delta \hat{x}_E = \frac{K_G}{1 + T_G s} \left( \Delta \hat{P}_C - \frac{1}{R} \Delta \hat{\omega} \right) \quad (\text{A28})$$

$T_G$  is typically around 0.1 second. Since  $T_G$  is the time constant of this system, it means that the response to a unit step change achieves about 63% of its final value in about 0.1 second.

# Appendix B

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- [1] [http://www.nerc.com/files/Glossary\\_of\\_Terms\\_2010April20.pdf](http://www.nerc.com/files/Glossary_of_Terms_2010April20.pdf)
- [2] N. Cohn, "Control of Generation and power flow on interconnected systems," second edition, Wiley,
- [3] G. Chown, "The economic analysis of relaxing frequency control," PhD Dissertation, University of Witwatersrand, Johannesburg, 2007.
- [4] "Interconnected Power System Dynamics Tutorial," Electric Power Research Institute EPRI TR-107726, March 1997.
- [5] A. Debs, "Modern Power Systems Control and Operation," Kluwer, 1988.
- [6] M. Shahidehpour, Hatim Yamin, and Zuyi Li, "Market Operations in Electric Power Systems," Wiley, 200?.
- [7] N. Jaleeli and L. VanSlyck, "NERC's New Control Performance Standards," IEEE Transactions on Power Systems, Vol. 14, No. 3, August 1999.
- [8] M. Terbruggen, "Control Performance Standards," a NERC Operators Training Document."
- [9] B. Kirby, M. Milligan, Y. Makarov, D. Hawkins, K. Jackson, H. Shiu "California Renewables Portfolio Standard Renewable Generation Integration Cost Analysis, Phase I: One Year Analysis Of Existing Resources, Results And Recommendations, Final Report," Dec. 10, 2003, available at [http://www.consultkirby.com/files/RPS\\_Int\\_Cost\\_PhaseI\\_Final.pdf](http://www.consultkirby.com/files/RPS_Int_Cost_PhaseI_Final.pdf).
- [10] A. Bergen and V. Vittal, "power systems analysis," second edition, Prentice Hall, 2000.