Introduction to Fault Analysis

1.0 Timing, protective systems, and fault types

To begin, we introduce a common way of referring to time that you may not have heard used yet. Time is measured in "cycles" where 1 cycle=1/60 of a second=0.0167 second.

Another important term is "protection." A single protective system will have at least one circuit breaker, one relay, and instrument transformers to measure electrical quantities on the power system. Transmission lines generally have a protective system on each end of the line.

Following initiation of a "fault," a properly operating circuit breaker will open the circuit after 4-10 cycles.

A "fault" refers to a short circuit in a power system. Faults can be divided into two broad classes: temporary and permanent faults.

Temporary faults cause momentary disruption, but the fault is cleared without protection operation. The most common type of temporary faults are those from lightning.

Permanent faults cause sustained disruption if not cleared by protection. The most common permanent fault types are associated with one or more of the following

1. Wind
2. Ice loading
3. Thermal heating and sag
4. Various "rare" events such as:
    a. Trees growing into lines
    b. Automobile striking tower
    c. Earthquakes
    d. Flooding
    e. Airplanes

Recognizing that transmission systems are comprised of three phases, we may observe that a line may be faulted in a variety of ways. The four most common ways are

1. Three phase
2. Single line to ground
3. Double line to ground
4. Line to line

Most power systems find that #2 is the most common, comprising about 70% of all faults. Type 4 is next, at about 15%. Type 3 is next, at about 10%.

Type #1 is least common, usually only about 5 % are 3 phase faults. However, type #1 is a special kind of fault in that it is symmetric, i.e., all 3 phases see the same "load" even during the fault and therefore conditions remain balanced. The fact that conditions remain balanced is "good" for analysis purposes, as well shall see, but not at all "good" for the power system, as the 3 phase fault is usually the most severe.

➔ What do we mean by “severe”?

2.0 Problems caused by faults

Faults cause three kinds of problems.

a. <u>Currents</u>: Since faults are short circuits, they force the voltage at the fault location to zero, so that each generator sees a low impedance path to ground. So all internal generator voltage appears across the impedance between the generator and the fault. The situation is illustrated in Fig. 1.

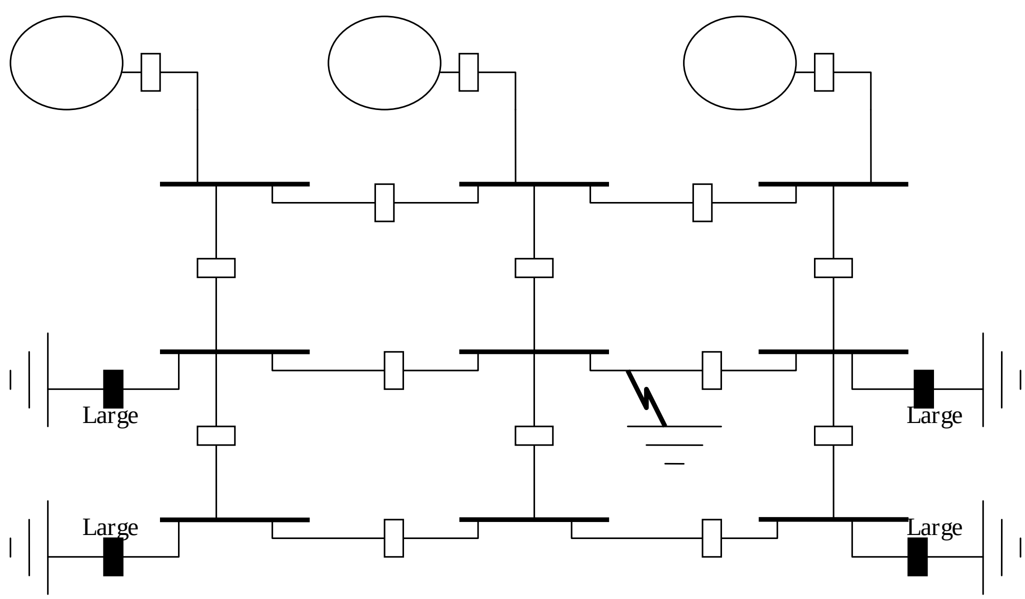

Fig. 1

In Fig. 1, the fault grounds the middle bus. There are five paths to ground in this figure. For the four loads, the currents are small, since the load impedance is large. For the short circuit path, all voltage appears across the generator and line impedances. Since these impedances are small, currents are large. A simpler illustration is shown in Fig. 2.

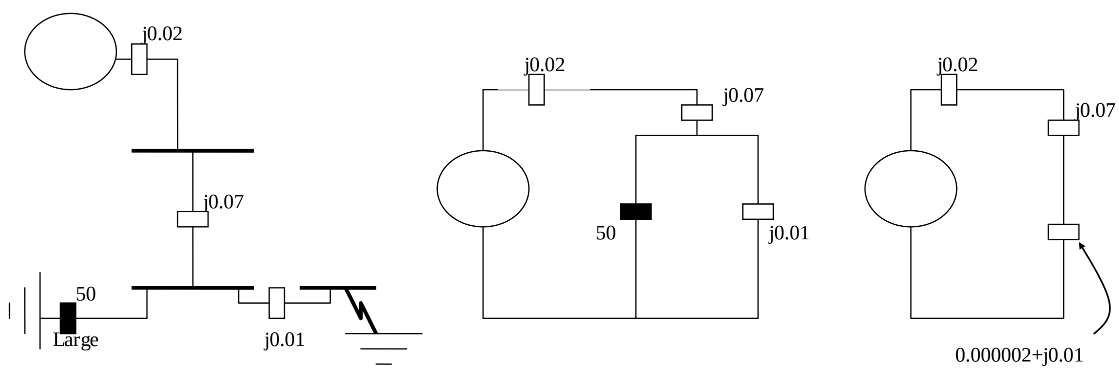

Fig. 2

In Fig. 2, we see that impedance seen by the generator is 0.000002+j0.1, so that if the generator voltage is 1.0, the current is 1.0/(0.000002+j0.1)=0.002-j10 pu, a large current. Protection equipment must be able to (a) carry the high current (for a short time); (b) interrupt that high current.

b. <u>Generator acceleration</u>: From the previous example, we see that the current is 0.002-j10, resulting in a power of VI^* (V=1), which is 0.002+j10. Without the fault the current is 1/(50+j0.09)=0.02-j0.00002, resulting in a power of 0.02+j0.00002. We see that the effect of the fault on the power is to dramatically increase the reactive and to decrease the real. This happens because the high current causes an increase in voltage drop across the reactive elements in the generator and lines and (because the total voltage drop does not change) a decrease in voltage drop across the load impedance, as shown in Fig. 3.

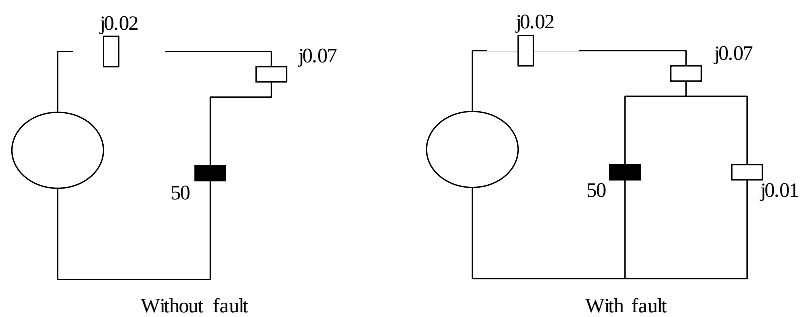

Fig. 3

The fact that the real power consumption of the network decreases is a problem because the real power mechanical input to the generator does not decrease (it comes from the turbine, from the steam, which is unaffected by the fault). So the generator sees same amount of real power in, but decreased real power out. What does it do? Accelerate! We must remove the faulted condition quickly, otherwise the generator will speed up too much and lose synchronism with the rest of the network. Loss of synchronism is a very bad thing, and we must prevent it.

c. Loss of a component: Proper action by the protection system to eliminate the faulted condition from the network results in loss of a component and therefore a weakening of the network. This can cause overloads, undervoltages, and voltage instability.

In this course, we will study in some depth the first two problems listed above, both of which relate to the "fault-on" time period. These two problems are generally called

1. Fault analysis
2. Transient instability

We may or may not get time to study problem (c) about loss of a component. It is goes under the term "security assessment."

Closely related to both problems (a) and (b) is a third issue that we will study

3. Protection: circuit breaker selection and relay settings

These topics will take us up to spring break, as observed on the course web page at home.eng.iastate.edu/~jdm/ee457/ee457schedule.htm

Question: why is protection closely related to fault analysis and transient instability?

- The objective of fault analysis is to establish the requirements for the circuit breaker, or to check that the existing circuit breaker is adequate. Critical information here includes the maximum current rating and the interrupting capability of the circuit breaker.
- A key issue for transient stability is the length of time for which the unit is

accelerating. The longer is this time, the more likely it is that the unit will lose synchronism. The length of time for which the unit is accelerating is determined by the time required for the circuit breaker to open following the fault.

We will also study several other topics, all of which relate to what is typically found in or related to an energy management system (EMS):

4. Automatic generation control
5. Economic dispatch and markets
6. State estimation

The amount of time we will spend on these topics can be seen at
home.eng.iastate.edu/~jdm/ee457/ee457schedule.htm

3.0 Transients in RL networks

A power transmission network is comprised of elements that have primarily resistance and inductance only (there is some capacitance but it tends to be small compared to the inductance). It is informative, therefore, to study the characteristics of an RL circuit. Our main goal in doing so is to see the relationship between the DC and steady-state components of the current after a fault.

Consider the circuit in Fig. 4.

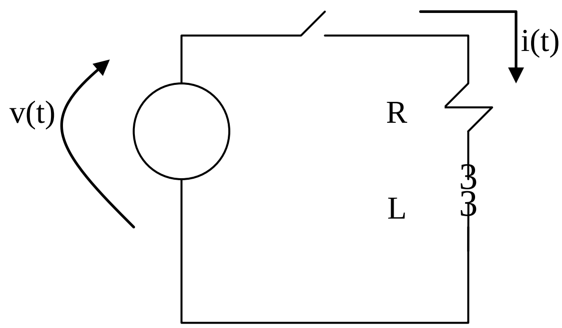

Fig. 4

The situation we will study is analogous to an open circuit generator that suddenly closes into a faulted power system, which is similar to the situation of a normally loaded generator suddenly experiencing the fault since the pre-fault current looks like a zero-current condition compared to the fault-on current (which is very large). The R+jL is the Thevenin impedance seen from the terminals of the generator looking into the faulted power system.

Assume that the voltage source is given by

$$v(t) = V_m \sin(\omega t + \alpha) \qquad (1)$$

The parameter α provides a way to control the timing of when the switch is closed (or when the fault occurs).

Using a trig identity, we can see that the above can be written as:

$$v(t) = V_m (\sin \omega t \cos \alpha + \cos \omega t \sin \alpha) \qquad (2)$$

Let's write the voltage equation for the circuit of Fig. 4:

$$Ri(t) + L\frac{di(t)}{dt} = V_m (\sin \omega t \cos \alpha + \cos \omega t \sin \alpha) \qquad (3)$$

Take LaPlace transform of (3) to obtain:

$$RI(s) + LsI(s) - Li(0) = V_m \left(\frac{s \sin \alpha}{s^2 + \omega^2} + \frac{\omega \cos \alpha}{s^2 + \omega^2} \right) \qquad (4)$$

When the switch is just closed at t=0, we have zero current, therefore, in this condition, (4) is:

$$RI(s) + LsI(s) = V_m \left(\frac{s \sin \alpha}{s^2 + \omega^2} + \frac{\omega \cos \alpha}{s^2 + \omega^2} \right) \qquad (5)$$

Solving for I(s) results in:

$$I(s) = \frac{V_m}{L}\left(\frac{1}{s+R/L}\right)\left(\frac{s\sin\alpha}{s^2+\omega^2} + \frac{\omega\cos\alpha}{s^2+\omega^2}\right) \qquad (6)$$

Distributing the two factors through yields:

$$I(s) = \frac{\frac{V_m}{L} s\sin\alpha}{(s+R/L)(s^2+\omega^2)} + \frac{\frac{V_m}{L}\omega\cos\alpha}{(s+R/L)(s^2+\omega^2)} \qquad (7)$$

We will skip some tedious steps at this point. These steps involve:

- Application of partial fraction expansion
- Some algebra
- Inverse LaPlace transform

These steps result in:

$$i(t) = \frac{V_m}{|Z|}\left\{\sin(\omega t + \alpha - \theta) - \sin(\alpha - \theta)e^{-Rt/L}\right\} \qquad (8)$$

where

$$\theta = \tan^{-1}\left(\frac{\omega L}{R}\right) \qquad (9)$$

is the power factor angle, i.e., the angle by which steady-state current lags voltage, and

$$|Z| = \sqrt{R^2 + (\omega L)^2} \qquad (10)$$

is the magnitude of the Thevenin impedance.

Notice the qualitative difference between the two terms inside the curly brackets of (8). The first term, call it $i_1(t)$, is a sinusoidal function of time, and provides that an oscillating current is present for all time.

$$i_1(t) = \frac{V_m}{|Z|}\sin(\omega t + \alpha - \theta) \tag{11}$$

The second term, call it $i_2(t)$, is an exponentially decreasing function of time, a "DC offset," given by:

$$i_2(t) = \frac{V_m}{|Z|}\sin(\alpha - \theta)e^{-Rt/L} \tag{12}$$

Notice that, at t=0, $i_2(0)$ is given by:

$$i_2(0) = \frac{V_m}{|Z|}\sin(\alpha - \theta) \equiv i_{20} \tag{13}$$

so that

$$i_2(t) = i_{20}e^{-Rt/L} \tag{14}$$

So the current, i(t), is composed of i_1 and i_2:

$$i(t) = i_1(t) - i_2(t) \tag{15}$$

One important observation here is that

- because the current in the inductor is zero just before the switch closes,
- then the current in the inductor must be zero just after the switch closes.

The reason for this is that current cannot change instantaneously in an inductor.
If the current could change instantaneously, then di/dt could be infinite, making Ldi/dt (voltage across the inductor) also infinite.

Therefore, it must be the case that $i(0)=0$ under all possible conditions. Since $i(0)=i_1(0)-i_2(0)$, then $i_1(0)=i_2(0)$, that is, at $t=0$, the sinusoidal component must be exactly the same as the DC component.

This observation allows us to consider the DC offset by considering $i_2(0)$ directly or by considering $i_1(0)$, since $i_1(0)=i_2(0)$. It does not really matter which one we choose. Let's choose $i_1(0)$ in what follows.

Question: For what value of α do we obtain minimum DC offset?

Consider (11), repeated here for convenience

$$i_1(t) = \frac{V_m}{|Z|} \sin(\omega t + \alpha - \theta) \qquad (11)$$

Equation (11) indicates that $i_1(0)$ depends on α-θ. Given t=0, if $\alpha-\theta=0$, or $\alpha=\theta$, then $i_1(0)=0$ implies $i_2(0)=i_{20}=0$, and so there will be no DC offset.

So the condition for min DC offset is $\alpha=\theta$.

But what is θ? It is the power factor angle, given by $\theta=\theta_v-\theta_i$.

Consider that a faulted circuit is highly inductive. For a purely inductive circuit, current lags voltage by 90°, and $\theta=\pi/2$. For a circuit where inductance is much larger than resistance, the power factor angle θ is very close to $\pi/2$.

This means that $i_1(0)=0$ if switch is closed so that $\alpha \approx \pi/2$. Then, the voltage is

$$v(t) = V_m \sin(\omega t + \alpha) = V_m \sin(\omega(0) + \pi/2) = V_m$$

which is a positive maximum. So we obtain minimum DC offset if fault occurs when the voltage is a positive maximum.

Similar reasoning (except using $\alpha-\theta=\pi$), leads to the same conclusion if voltage is a negative maximum.

Question: For what value of α do we obtain maximum DC offset?

➔ Again, drawing on the fact that $i_1(0)=i_2(0)$, we see that the DC offset is maximum when $i_1(0)$ is maximum.

Considering (11) again, repeated here for convenience

$$i_1(t) = \frac{V_m}{|Z|} \sin(\omega t + \alpha - \theta) \qquad (11)$$

we observe, with t=0, $i_1(0)$ is a maximum when α-θ=π/2, i.e., when α=π/2+θ.
Since we already established that θ≈π/2, then the condition for maximum DC offset is α≈π/2+π/2≈π.

Therefore we obtain maximum DC offset when the switched is closed so that α≈π.

Then, the voltage is

$$v(t) = V_m \sin(\omega t + \alpha) = V_m \sin(\omega(0) + \pi) = 0$$

which is a zero.
Therefore we obtain maximum DC offset if the fault occurs when the voltage is zero.

Let's take a look at some numerical data to illustrate.

- R=1 ohm
- L=0.05 henry
- V_m=10 volts
- ω=2*π*60 radians/sec
- α=π/2

The significance of the last bullet is that the switch is being closed when the voltage waveform is almost at a maximum, i.e.,

$$v(t) = V_m \sin(\omega t + \alpha) \Rightarrow v(0) = V_m \sin(\pi / 2) = V_m$$

A plot of the voltage waveform for this condition is given in Fig. 5.

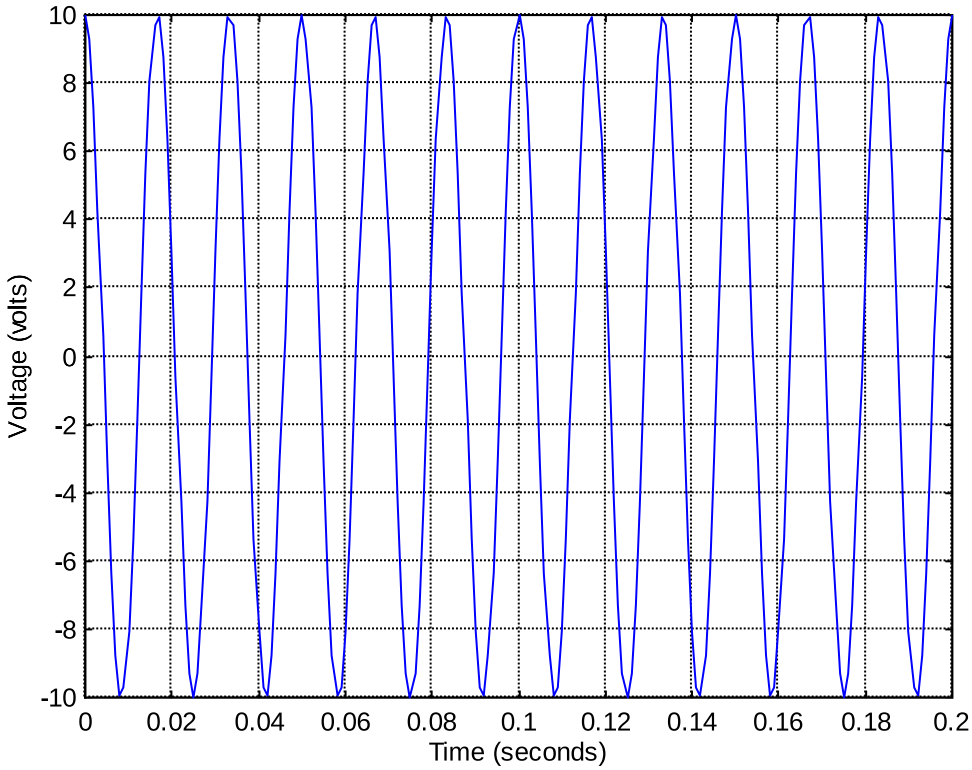

Fig. 5

We can calculate:

$$\theta = \tan^{-1}\left(\omega * L / R\right) = 1.5178 \text{ radians } (86.96°)$$

$$|Z| = \sqrt{R^2 + (\omega L)^2} = 18.8761 \text{ ohms}$$

$$\alpha - \theta = 0.0530 \text{ radians } (3.04°)$$

We use the following Matlab code to compute the currents i_1, i_2, and i.

```
R=1;
L=0.05;
Vm=10;
omega=2*pi*60;
alpha=pi/2;
theta=atan(omega*L/R);
zmag=sqrt(R^2+(omega*L)^2);
t=0:.001:0.2;
i1=(Vm/zmag)*sin(omega*t+alpha-theta);
i2=-(Vm/zmag)*sin(alpha-theta)*exp(-R*t/L);
i=i1+i2;
plot(t,i1,'r:',t,i2,'g--',t,i,'b-');
legend('i1','-i2','i=i1-i2');
ylabel('current (amperes)');
xlabel('time (sec)');
grid
```

Figure 6 shows the result.

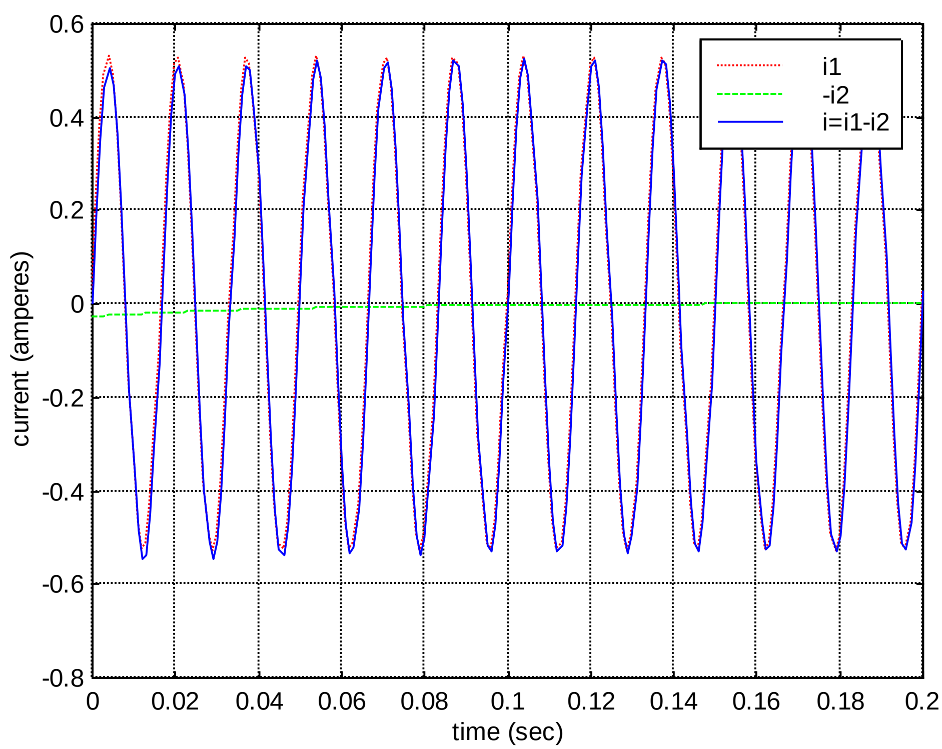

Fig. 6

Some observations:

- The dotted red curve, i_1, is the steady-state term, and oscillates for all time.
- The yellow dashed curve, i_2, is the DC offset term ($-i_2$). It is small to begin with and goes to almost 0 after about 0.1 sec.
- The blue solid curve, i, is the composite current. It becomes the same as i_1 after the DC offset has died (after about 0.1 sec).
- The DC offset is so small that it has almost no affect on the composite current.

Now change $\alpha=\pi$. In this case, the voltage waveform is almost at a zero, i.e.,

$$v(t) = V_m \sin(\omega t + \alpha) \Rightarrow v(0) = V_m \sin(\pi) = 0$$

A plot of the voltage waveform for this condition is given in Fig. 7.

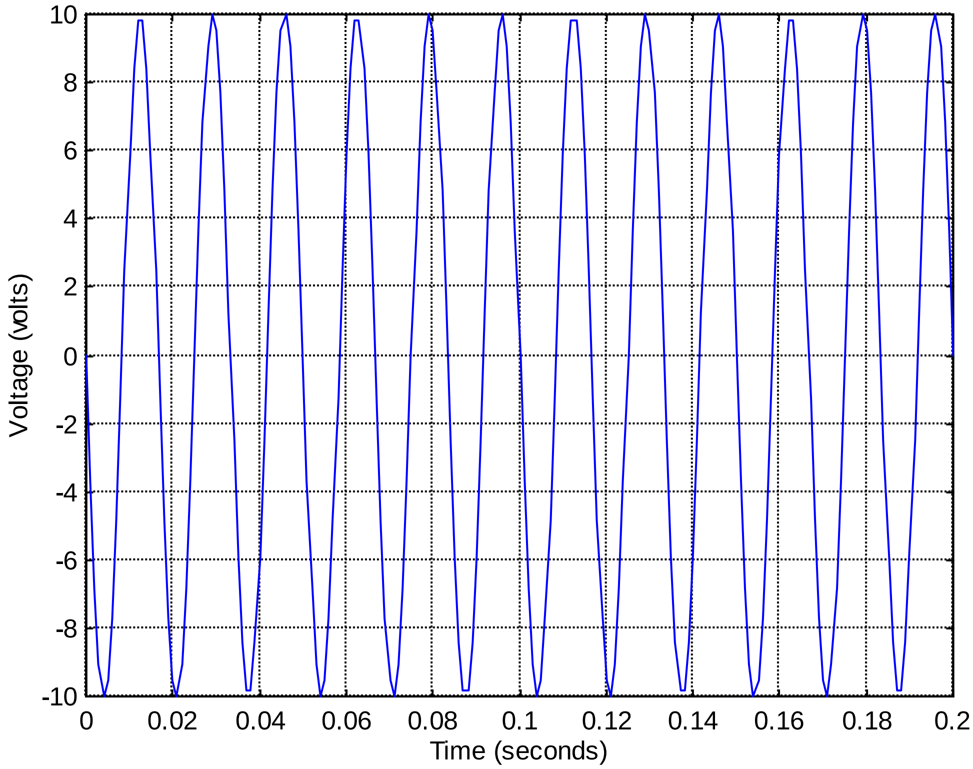

Fig. 7

In this case, we have that

$\alpha - \theta = 1.6238$ radians (93.04°)

Figure 8 shows the results.

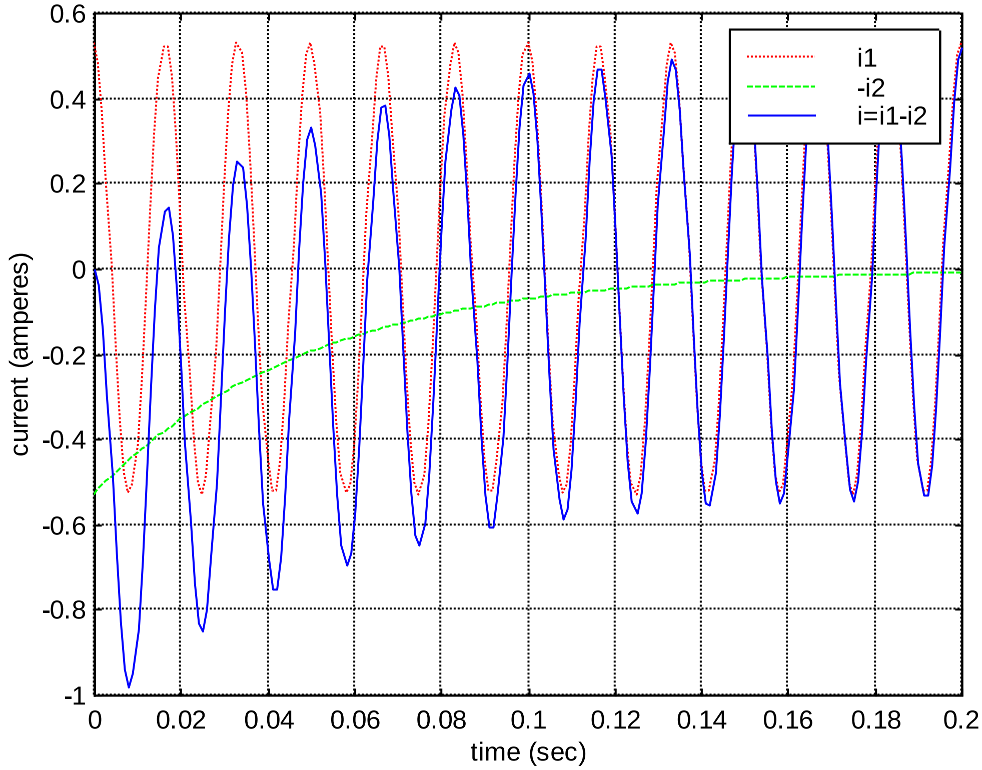

Fig. 8

Some observations:

- The dotted red curve, i_1, is the steady-state term, and oscillates for all time.
- The yellow dashed curve, i_2, is the DC offset term and goes to almost 0 after about 0.2 sec.
- The blue solid curve, i, is the composite current. It becomes the same as i_1 after the DC offset has died (after about 0.2 seconds).

- The DC offset term here is quite large. In fact, it causes the current to reach a value at about 0.01 second that is almost twice the amplitude of i_1.

Homework Part A (due Tuesday, Jan 20):

1. Using the output from the matlab code provided above, for $\alpha=\pi$, compute the ratio $K(\alpha)=|i|_{max}(\alpha)/|i_1|_{max}$, where the "max" indicates the maximum absolute value of the waveform.
2. Repeat for the following values of α: α=3, 2.5, 2, 1.5, 1, 0.5, 0.
3. Repeat parts (1) and (2) but use R=0.1.
4. Repeat parts (1) and (2) but use R=10.

The point of the above exercise is that, depending on where on the voltage waveform the breaker opens, the DC offset term i_2 can cause the current to be significantly higher than the steady-state term i_1. It is should be clear from the exercise, that an upper bound for $|i|_{max}(\alpha)/|i_1|_{max}$ is 2.0.

And you should consider to prove this by expressing $i(t)/i_1(t)$, using a trig identity sin(x-y)=sinxcosy-cosxsiny on numerator and denominator, and then evaluating for $\alpha-\theta=\pi/2$. You should get $-[\cos wt+e^{-Rt/L}]/\cos wt$.

Gross [1, p. 360] and Glover & Sarma [2, pg. 278] analyze this situation in terms of RMS current values, as follows.

Recall that the rms value of a periodic function is the square root of the sum obtained by adding the square of the rms value of each harmonic to the square of the DC value [3, pp. 729]. Stretching this concept a bit, if we assume that the DC value at some selected time t, $i_2(t)$, is constant, we may then compute an rms value of the composite current as

$$I(t) = \sqrt{I_1^2 + \left[i_2(t)\right]^2} \qquad (16)$$

where I is the rms value of the composite current, and I_1 is the rms value of the steady-state current. Because

$$i_1(t) = \frac{V_m}{|Z|}\sin\left(\omega t + \alpha - \theta\right)$$

we know $I_1=V_m/|Z|\sqrt{2} \Rightarrow V_m/|Z|=\sqrt{2}I_1$. Thus

$$i_2(t) = \frac{V_m}{|Z|}\sin\left(\alpha - \theta\right)e^{-Rt/L} = \sqrt{2}I_1 \sin(\alpha - \theta)e^{-Rt/L}$$

With α-θ=π/2, the last equation becomes

$$i_2(t) = \sqrt{2}I_1 \sin(\alpha - \theta)e^{-Rt/L} = \sqrt{2}I_1 e^{-Rt/L}$$

Substitution of the last equation into (16) results in

$$I(t) = \sqrt{I_1^2 + \left[\sqrt{2}I_1 e^{-Rt/L}\right]^2} = \sqrt{I_1^2 + I_1^2\left[2e^{-2Rt/L}\right]}$$

$$= I_1\sqrt{1 + 2e^{-2Rt/L}}$$

When t is very small, then the rms value of the composite current is given by

$$I(t) = I_1\sqrt{1+2} = I_1\sqrt{3}$$

So the upper bound of |I|(α)/|I_1| is $\sqrt{3} \approx 1.73$, consistent with the indicated references [1,2]

Thus, we see that the maximum value of rms current is 1.73 times the rms steady-state current I_1.

We will find it very convenient to only compute the steady-state fault currents. Then we can specify that the circuit breaker have an interruptible rating (rms) at least 1.73 times the steady-state rms value of the fault current.

4.0 Consideration of all three phases

We saw that the DC component depends on where on the voltage waveform the switch is closed. For a three-phase synchronous generator, however, the three phase voltages are out of phase by 120°. Assuming a three-phase fault shorts all three phases at exactly the same time, then each phase sees a different α. The matlab code for studying this situation is below, and Fig. 9 illustrates.

```
R=1;
L=0.05;
Vm=10;
omega=2*pi*60;
alpha=pi/2;
theta=atan(omega*L/R);
zmag=sqrt(R^2+(omega*L)^2);
t=0:.001:0.2;
% a-phase
i1a=(Vm/zmag)*sin(omega*t+alpha-theta);
i2a=-(Vm/zmag)*sin(alpha-theta)*exp(-R*t/L);
ia=i1a+i2a;
% b-phase
i1b=(Vm/zmag)*sin(omega*t+alpha-2.0944-theta);
i2b=-(Vm/zmag)*sin(alpha-2.0944-theta)*exp(-R*t/L);
ib=i1b+i2b;
% c-phase
i1c=(Vm/zmag)*sin(omega*t+alpha+2.0944-theta);
i2c=-(Vm/zmag)*sin(alpha+2.0944-theta)*exp(-R*t/L);
ic=i1c+i2c;
plot(t,ia,'r:',t,ib,'g--',t,ic,'b-');
legend('a-phase','b-phase','c-phase');
ylabel('current (amperes)');
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```
xlabel('time (sec)');
grid
```

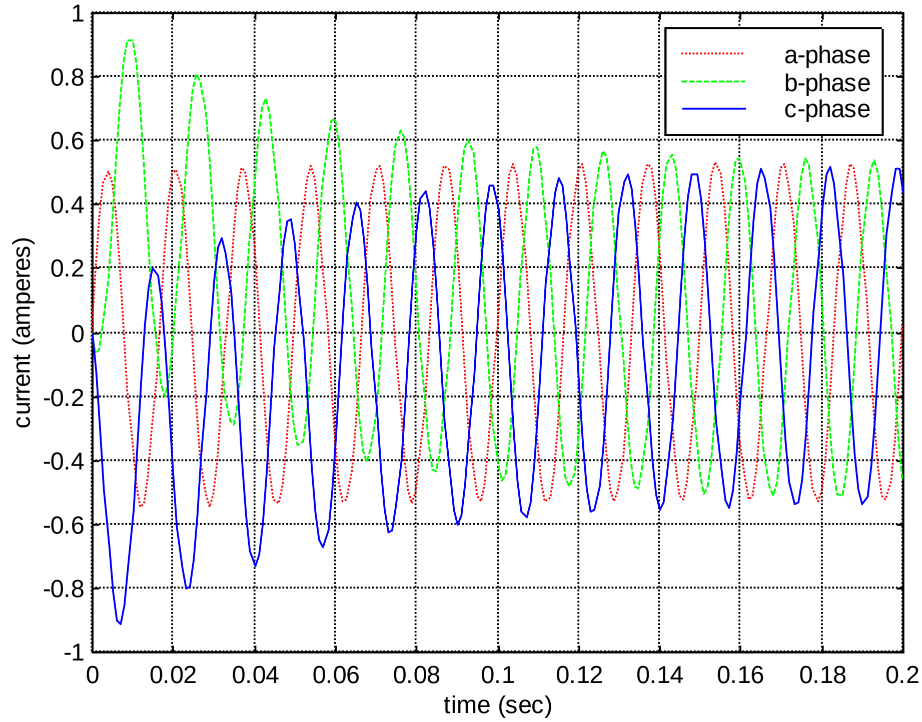


Fig. 9

And so we observe that independent of when the fault occurs, at least one phase is going to see a greatly increased current.

5.0 Decaying steady-state term

In previous sections, we learned that if a synchronous generator experiences faulted conditions, it will respond as an RL circuit. However, our treatment assumed that the steady-state terms for each phase current have constant amplitude. This is not actually the case. These terms actually have an amplitude that decays from a maximum value at the instant of fault initiation to a steady-state value following some time. This effect is due to the fact that the magnetic flux is initially forced to flow through high reluctance paths that do not link the field winding or the damper windings of the machine [2]. Detailed analysis of these effects is tedious and not worth the time it will take to do it. It is done in EE 554 [4].

Fig. 10 illustrates the actual response of what we previously called $i_1(t)$ for the a-phase i.e., the DC offset term is not considered here. Since we have removed the DC-offset term, we will call this current the *symmetrical* rms current.

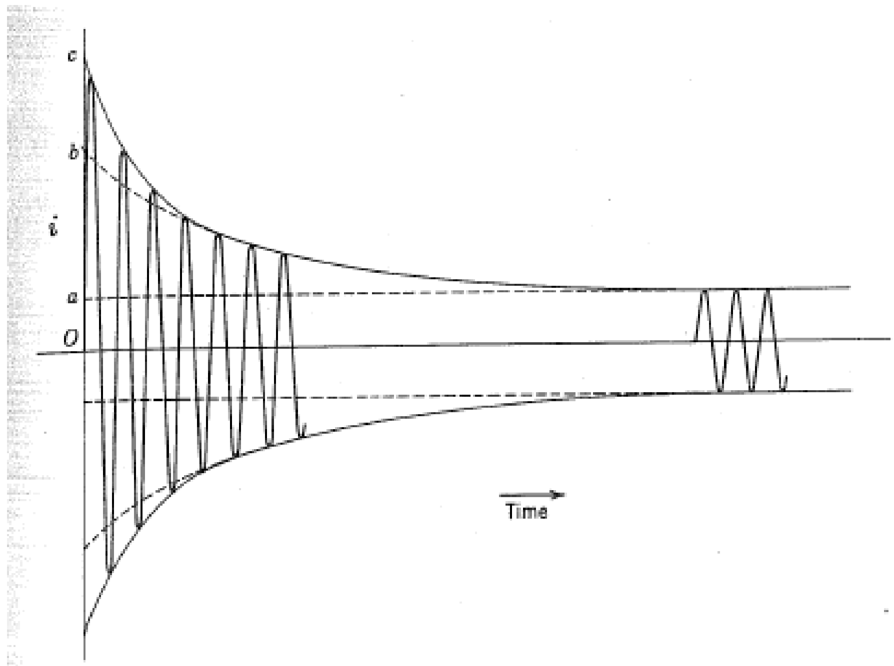


Fig. 10

The envelope $I_{max}(t)$ indicated in Fig. 10 is the maximum symmetrical current. Division of I_{max} by $\sqrt{2}$ gives the rms symmetrical current that we previously called I_1.

Suppose that we wish to use our R-L circuit to compute $I_1(t)$ as a phasor, as in Fig. 11.

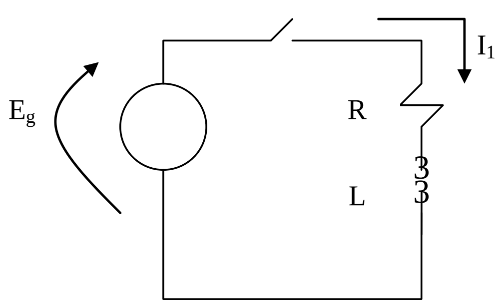


Fig. 11

We will get

$$I_1 = \frac{E_g}{R + j\omega L} = \frac{E_g}{R + jX} \qquad (17)$$

If we are interested only in current magnitude, then R+jX≈jX and so

$$I_1 = \frac{E_g}{X} \qquad (18)$$

But of course, (18) only gives us a single value of the current magnitude, and clearly the current magnitude decreases with time during the first few cycles.

We could always compute the exact transient as shown in Fig. 9=10. However, in order to enable simpler analysis, we will define three different generator reactances to use in approximating the rms symmetrical current. These are:

- X''_d: subtransient reactance, used to approximate current from $t=0^+$ to t=2 cycles

- X'_d: transient reactance, used to approximate current from t=2 cycles to t≈30 cycles.
- X_d: steady-state reactance: used to approximate current during the steady-state, which is generally after about 30 cycles.

Therefore we obtain three different currents corresponding to these three different reactances. They are:

- I'': subtransient current
- I': transient current
- I: steady-state current

The situation is illustrated in Fig. 12-a and b.

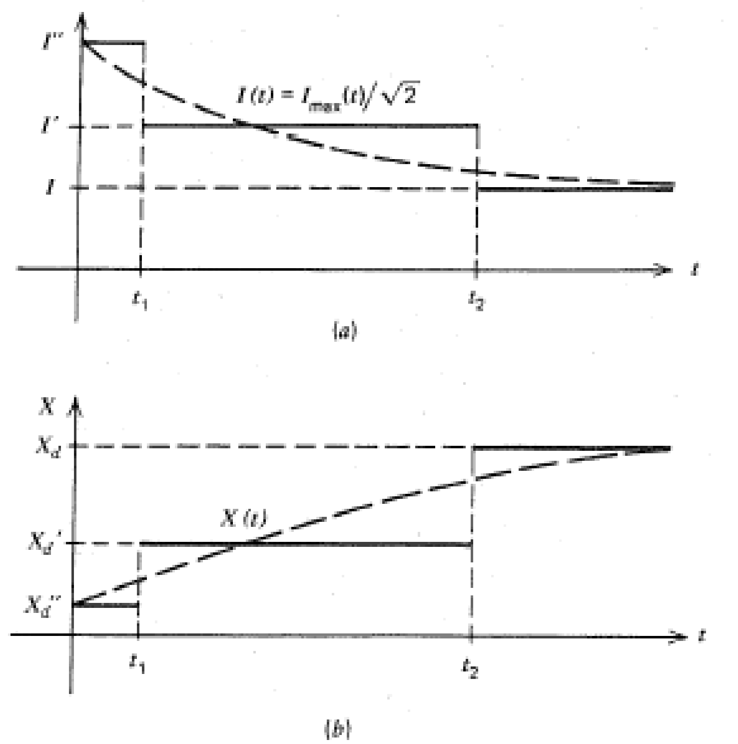


Fig. 12

In Fig. 12-a, we have discretized the current into three intervals corresponding to I'', I', and I.

Correspondingly, in Fig. 12-b, we have discretized the generator reactance into three intervals corresponding to X'', X', and X.

If the generator is *unloaded* when the fault occurs, with internal voltage of E_g, then the three different fault currents may be computed by:

$$I'' = \frac{E_g}{X_d''} \qquad (19)$$

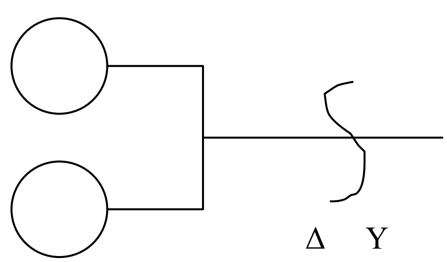
$$I' = \frac{E_g}{X_d''} \qquad (19)$$

$$I = \frac{E_g}{X_d} \qquad (19)$$

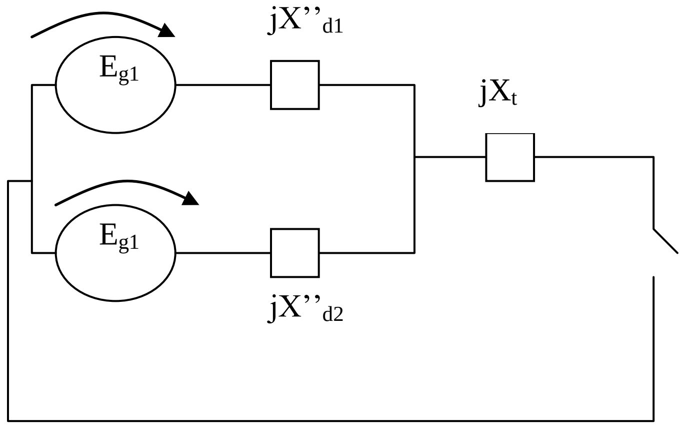
The above analysis applies to a smooth-rotor machine. It is more complex for a salient rotor machine, but smooth rotor analysis gives good approximations for a salient pole machine.

HW assignment Part B
(also due Tue, Jan 20):

Two generators are connected in parallel to the low-voltage side of a three-phase Δ-Y transformer, as shown in the figure below. Generator 1 is rated 50,000kVA, 13.8kV. Generator 2 is rated 25,000kVA, 13.8kV. Each generator has a subtransient reactance of 25% on its own base. The transformer is rated 75,000kVA, 13.8Δ/69Y kV, with a reactance of 10% on its own base. Before the fault occurs, the voltage on the high-voltage side of the transformer is 66kV. The transformer is unloaded and there is no circulating current between the generators. Find the subtransient current in each generator when a three-phase short circuit occurs on the high-voltage side of the transformer.



Hint: The circuit to analyze should appear as below.



[1] C. Gross, "Power System Analysis," second edition, Wiley, 1986.
[2] J. Glover and M. Sarma, "Power system analysis and design," PWS Publishers, Boston, 1987.
[3] J. Nilsson, "Electric Circuits," second edition, Addison Wesley, 1986.
[4] http://home.eng.iastate.edu/~jdm/ee554/schedule.htm