

State Estimation 2

1.0 Introduction

In these notes, we explore two very practical and related issues in regards to state estimation:

- use of pseudo-measurements
- network observability

2.0 Exact Pseudo-measurements

It is important to keep in mind that the objective of state estimation is to obtain a computer model that accurately represents the current conditions in the power system. So if we can think of ways to improve the model using something other than actual measurements, we should feel free to do that.

Pseudo-measurements are not measurements but are used in the state-estimation algorithm as if they were. If we can know with certainty that a particular pseudo-measurement is accurate, we should use it as it will increase the accuracy of our state estimate.

The most common “exact” pseudo-measurement is the bus injection at a substation that has no generation and serves no load. Figure 1 below illustrates.

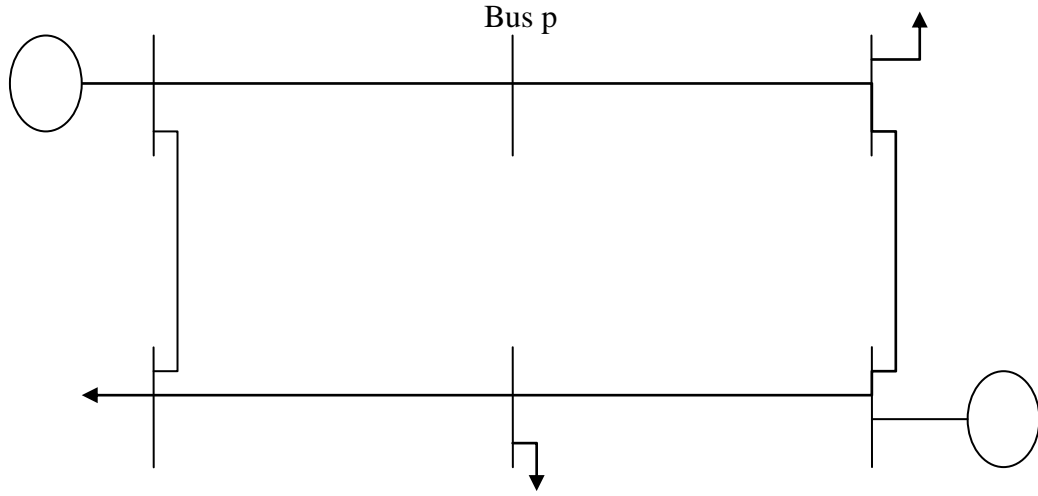


Fig. 1

In Fig. 1, bus p has no generation or load. We therefore know the real and reactive power injection of this bus with precision; it is 0. And so we can add two more measurements to the measurements that we actually have:

$$z_i = h_i(\underline{x}) + \eta_i \quad (1)$$

$$z_{i+1} = h_{i+1}(\underline{x}) + \eta_{i+1} \quad (2)$$

where:

$$\text{“Measurements”} \quad z_i = P_{p,inj} = 0 \quad (3)$$

$$z_{i+1} = Q_{p,inj} = 0 \quad (4)$$

$$h_i(\underline{x}) = P_{p,inj} = \sum_{k=1}^n |V_p| |V_k| (G_{pk} \cos(\theta_p - \theta_k) + B_{pk} \sin(\theta_p - \theta_k)) = 0 \quad (5)$$

$$h_{i+1}(\underline{x}) = Q_{p,inj} = \sum_{k=1}^n |V_p| |V_k| (G_{pk} \sin(\theta_p - \theta_k) - B_{pk} \cos(\theta_p - \theta_k)) = 0 \quad (6)$$

We recognize the summations of eqs. (5) and (6) as the power flow equations for real and reactive power injection, respectively.

The terms η_i and η_{i+1} are zero-mean Gaussian distributed errors for the pseudo-measurements. We can account for the fact that these pseudo-measurements are exact by letting σ_i and σ_{i+1} be very small. The weighted least-square estimation algorithm is then carried out as usual.

3. Observability

Recall our very first example at the beginning of the first set of state estimation notes. It is below.

In the circuit given of Fig. 2, current injections I_1 , I_2 , and voltage E are unknown. Let $R_1=R_2=R_3=1.0 \ \Omega$. The measurements are as follows:

- meter A_1 : $i_{1,2}=1.0$ Ampere
- meter A_2 : $i_{3,1}=-3.2$ Ampere
- meter A_3 : $i_{2,3}=0.8$ Ampere
- meter V : $e=1.1$ volt

The problem is to determine the state of the circuit, which in this case is nodal voltages v_1 , v_2 , and the voltage e across the voltage source.

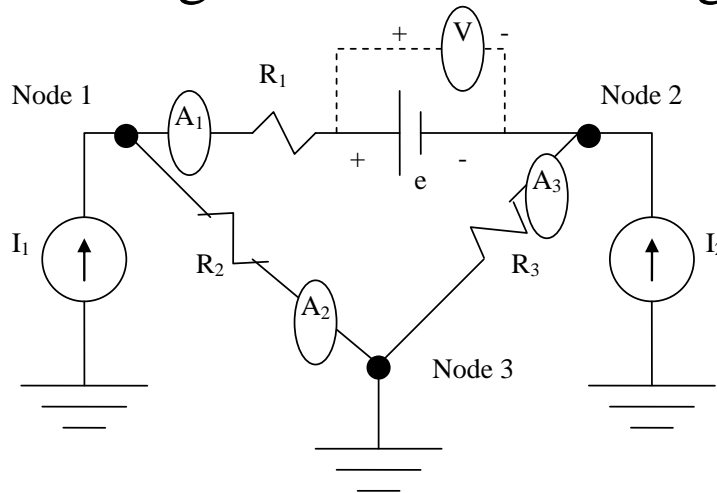


Fig. 2

To determine the state of the circuit (v_1 , v_2 , and e), we wrote each one of the measurements in terms of the states. We then expressed these four equations in matrix form:

$$\begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 1.0 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \quad (7)$$

Let's denote terms in eq. (7) as \underline{A} , \underline{x} , and \underline{b} , so:

$$\underline{A}\underline{x} = \underline{b} \quad (8)$$

We solved eq. (8) using:

$$\underline{x} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{b} = \underline{G}^{-1} \underline{A}^T \underline{b} = \underline{A}' \underline{b} \quad (9)$$

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 & -1 \\ -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix} \quad (10)$$

The inverse of the gain matrix is then found from Matlab as

$$\underline{G}^{-1} = \frac{1}{4} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} \quad (11)$$

The pseudo-inverse is then

$$\underline{A}' = \underline{G}^{-1} \underline{A}^T = \frac{1}{4} \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \quad (12)$$

Then we obtained the least squares estimate of the 3 states from the 4 measurements as

$$\underline{x} = \underline{A}' \underline{b} = \frac{1}{4} \begin{bmatrix} 1 & -3 & 1 & 1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1.0 \\ -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} = \begin{bmatrix} 3.125 \\ 0.875 \\ 1.175 \end{bmatrix} \quad (13)$$

Question: What would happen to this problem if we lost a measurement? Let's say that we lost A_1 , the measurement on the current flowing from bus 1 to bus 2. Let's see what happens.

To solve it, we just remove the first equation (corresponding to, in eq. (7), the first row of the A-matrix and the top element in b-vector).

$$\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \quad (14)$$

Actually, here the matrix is 3×3 and therefore we can solve exactly as:

$$\begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \quad (15)$$

But let's go ahead and use eq. (9) to see what happens.

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

The inverse of the gain matrix is then

$$\underline{G}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (17)$$

The pseudo-inverse is then

$$\underline{A}^I = \underline{G}^{-1} \underline{A}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (18)$$

The least squares estimate of the 3 states from the 3 measurements is then

$$\underline{x} = \underline{A}^I \underline{b} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -3.2 \\ 0.8 \\ 1.1 \end{bmatrix} = \begin{bmatrix} 3.2 \\ 0.8 \\ 1.1 \end{bmatrix} \quad (19)$$

Compared to the solution of eq. (13), our estimate can be assumed to be less accurate (since it is based on fewer measurements), but at least we still did obtain a solution.

Question: What if we lost two measurements? Let's say that we lost A_1 , the measurement on the current flowing from bus 1 to bus 2, and A_2 , the measurement on the current flowing from bus 1 to bus 3. Let's see what happens.

To solve it, we just remove the first two equations (corresponding to, in eq. (7), the first row of the A-matrix and the top element in b-vector).

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ e \end{bmatrix} = \begin{bmatrix} 0.8 \\ 1.1 \end{bmatrix} \quad (20)$$

The matrix is once again non-square, so we must use our least-squares procedure.

First, the gain matrix is given as

$$\underline{G} = \underline{A}^T \underline{A} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (16)$$

The inverse of the gain matrix is, however, singular – its determinant is zero (or equivalently, it has a zero eigenvalue). As a result, it cannot be inverted. In this case, our process must stop since we need \underline{G}^{-1} to evaluate \underline{x} , as indicated in eq. (9), repeated below for convenience.

$$\underline{x} = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{b} = \underline{G}^{-1} \underline{A}^T \underline{b} = \underline{A}^+ \underline{b} \quad (9)$$

What is the problem here?

The basic problem is that we do not have enough measurements. In this case, the system is said to be unobservable. This means that despite the availability of some measurements, it is not possible to provide an estimate of the states with those available measurements.

4. Approximate pseudo-measurements

A key step in state estimation is to test for observability. If the network is not observable, i.e., if we do not have enough independent measurements, then we will not be able to obtain a network model.

When the state estimator detects that the network is unobservable, it can make use of approximate measurements. Examples of such approximate pseudo-measurements include:

- Information obtained from plant operators over phone or e-mail.
- Information obtained from previous measurements.
- Information obtained from a power flow calculation.

In using approximate pseudo-measurements, it is generally good practice to pair it with a relatively large variance in the weighting matrix, given that it is in fact “approximate.”