

Homework #1 Solution

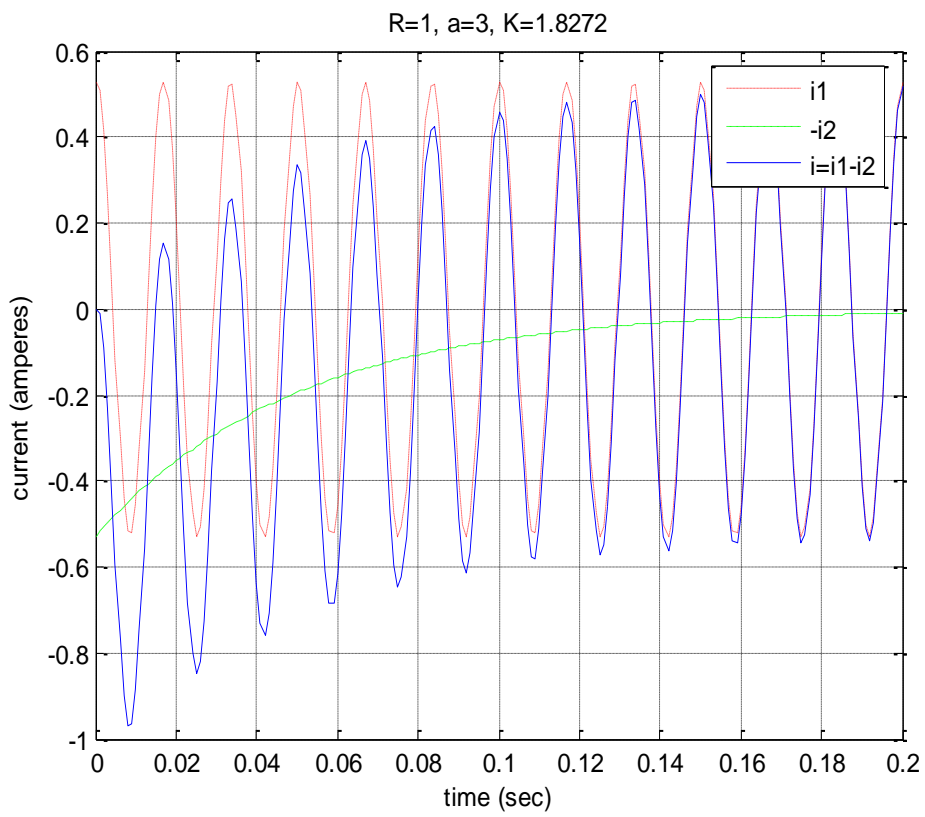
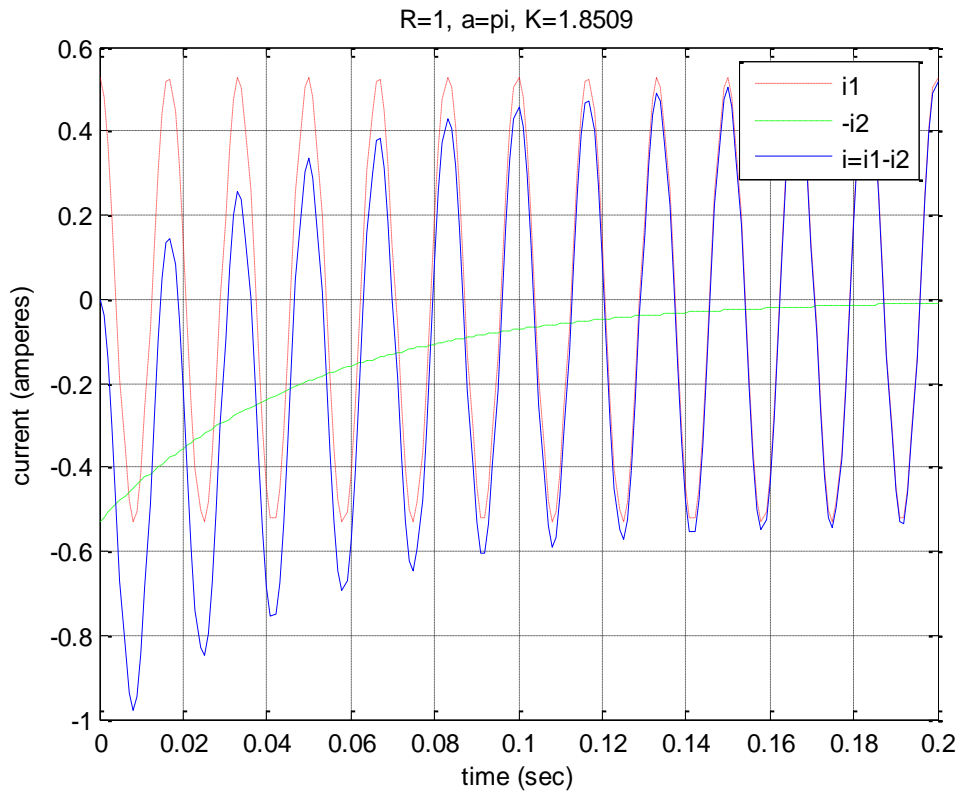
Problem #1:

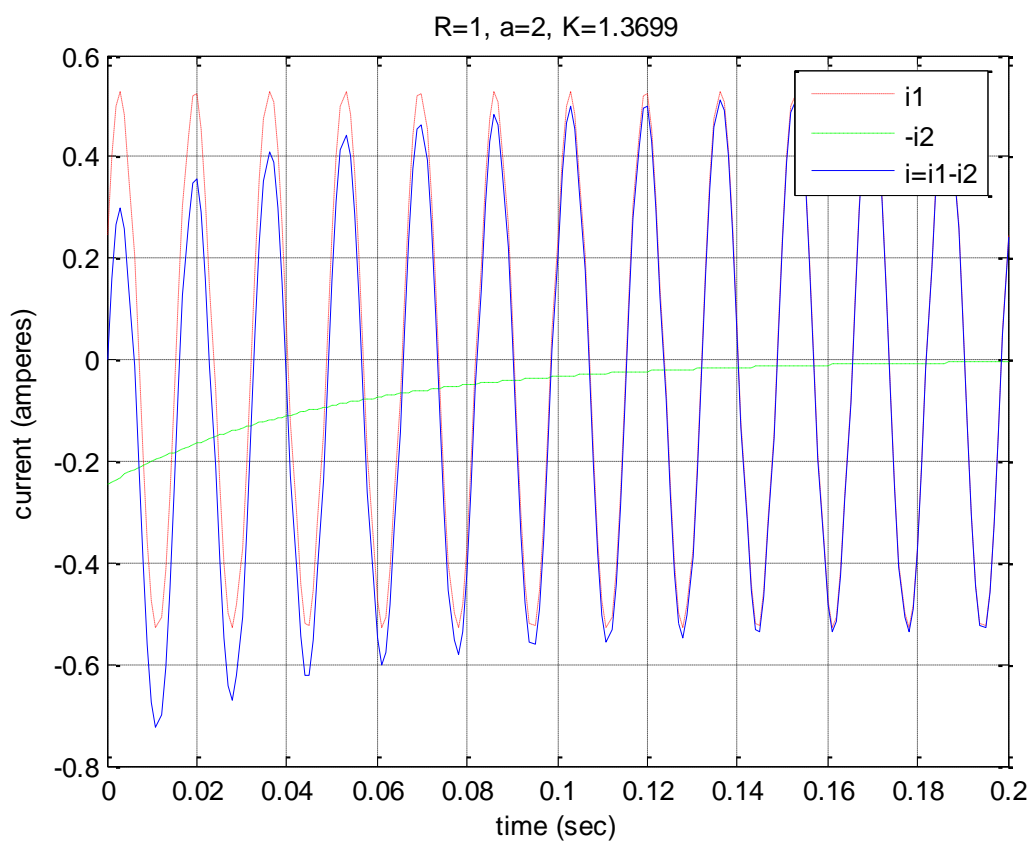
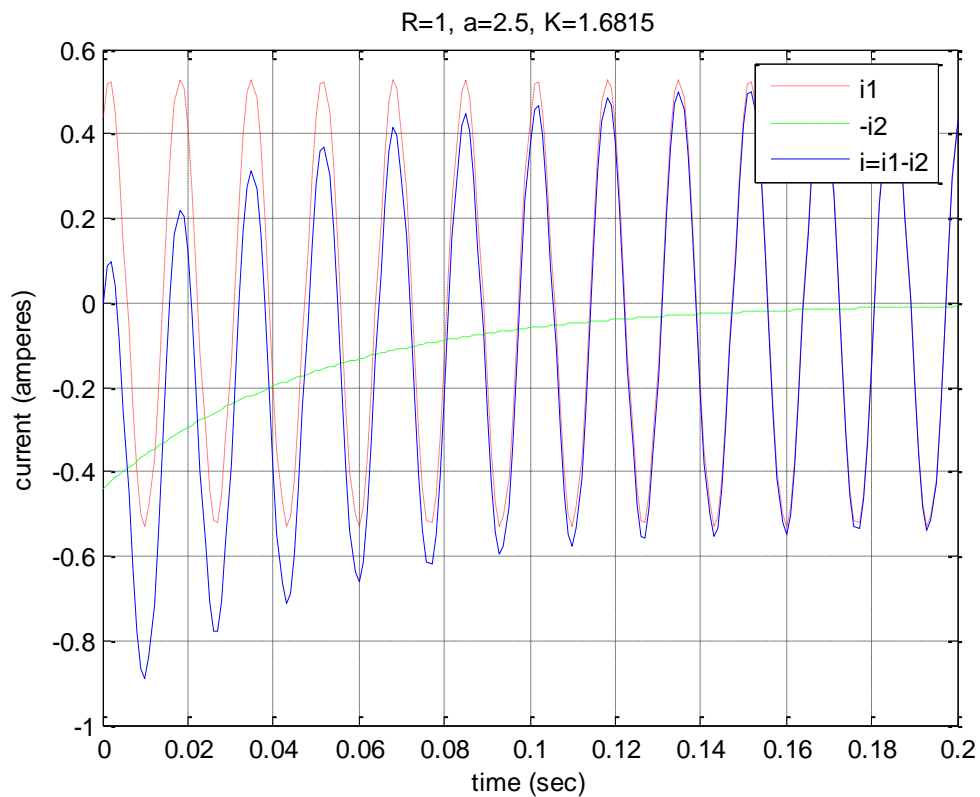
1. Using the output from the Matlab code provided above, for $\alpha=\pi$, compute the ratio $K(\alpha)=|i|_{\max}(\alpha)/|i_1|_{\max}$, where the “max” indicates the maximum absolute value of the waveform.
2. Repeat for the following values of α :
 $\alpha=3, 2.5, 2, 1.5, 1, 0.5, 0$.
3. Repeat parts (1) and (2) but use $R=0.1$.
4. Repeat parts (1) and (2) but use $R=10$.

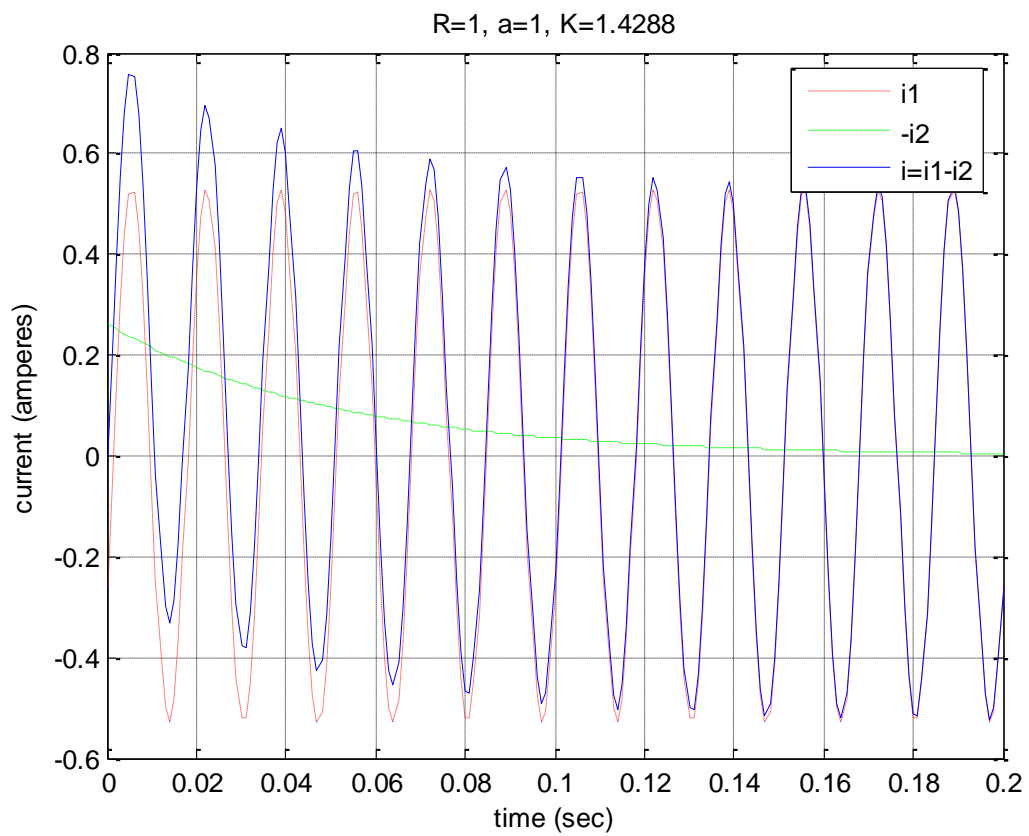
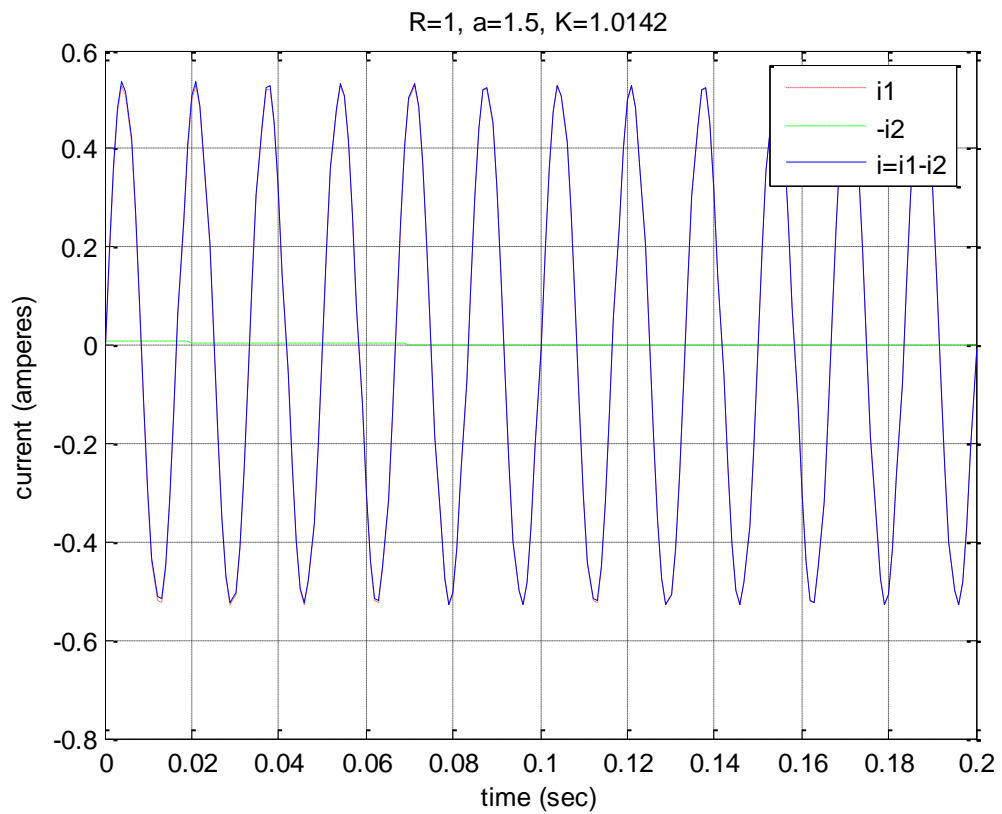
Solution:

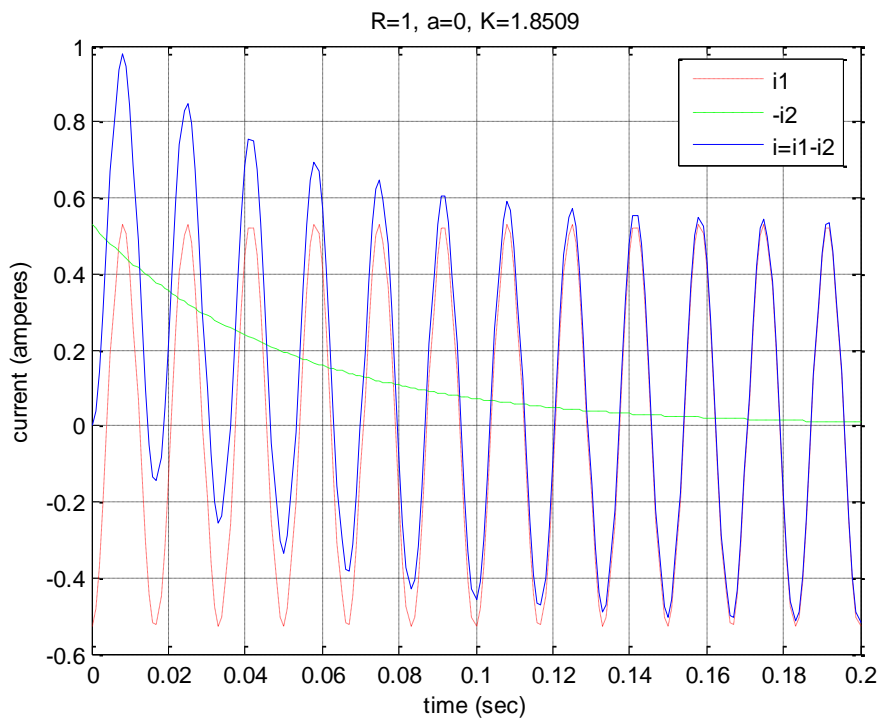
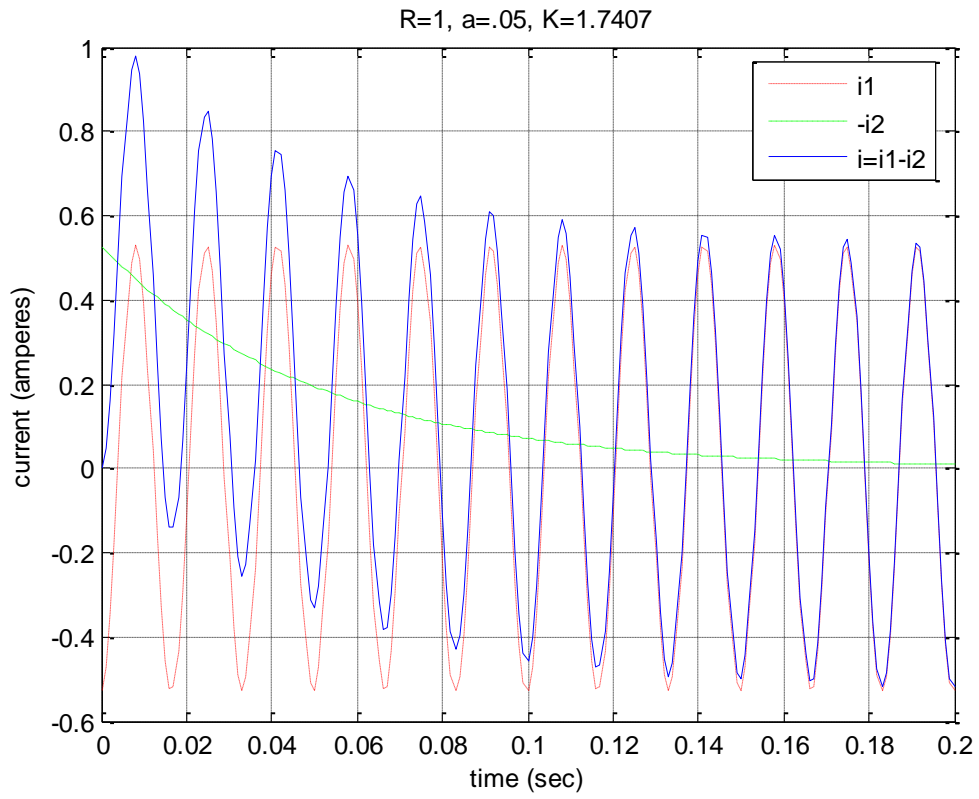
This problem simply involves changing the R and α values in the code below and computing K based on the numerical output.

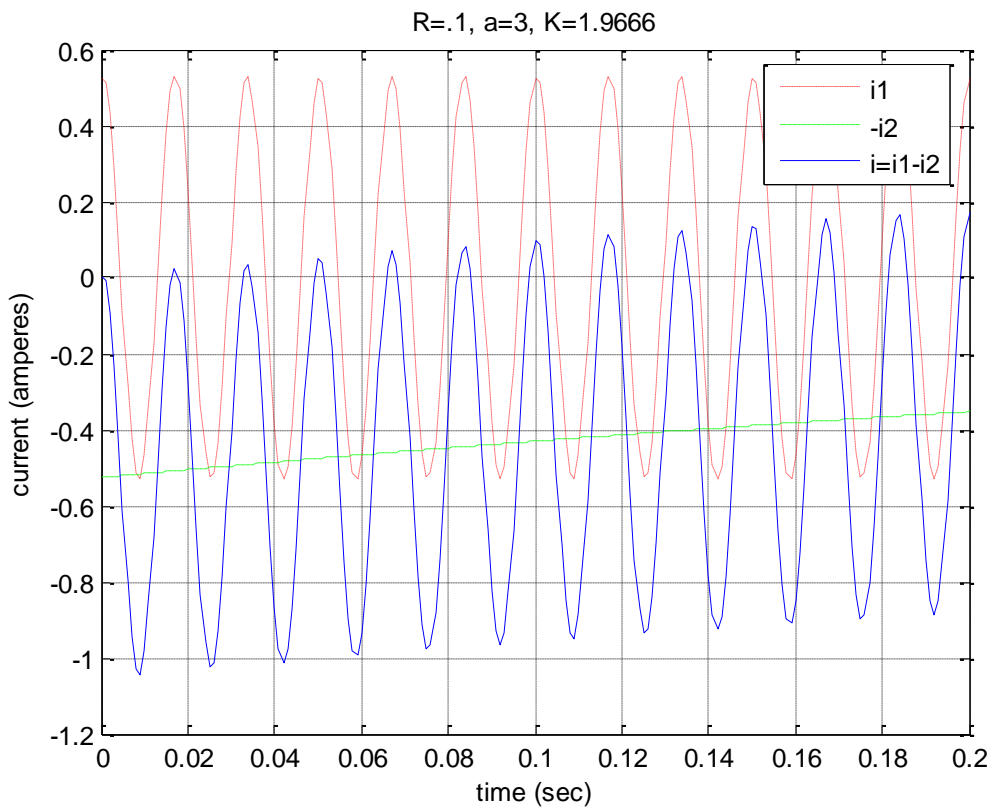
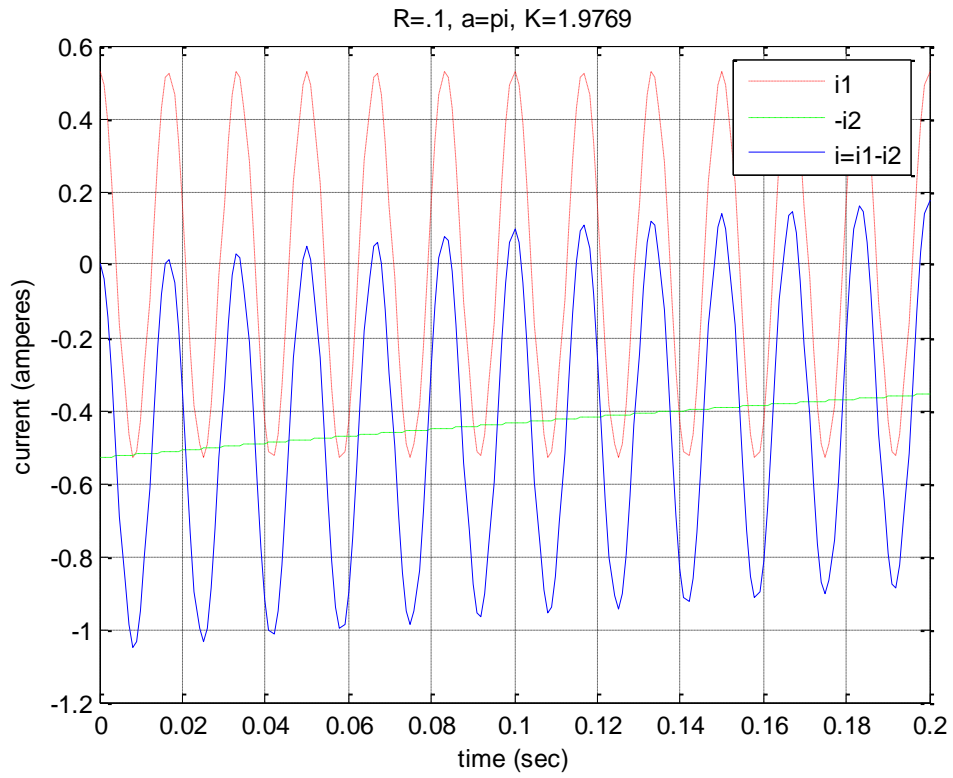
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R=1; %This value will change
L=0.05;
Vm=10;
omega=2*pi*60;
alpha=pi; %This value will change
theta=atan(omega*L/R);
zmag=sqrt(R^2+(omega*L)^2);
t=0:.001:0.2;
i1=(Vm/zmag)*sin(omega*t+alpha-theta);
i2=- (Vm/zmag)*sin(alpha-theta)*exp(-R*t/L);
i=i1+i2;
k=max(abs(i))/max(i1)
plot(t,i1,'r:',t,i2,'g--',t,i,'b-');
legend('i1','-i2','i=i1-i2');
ylabel('current (amperes)');
xlabel('time (sec)');
grid
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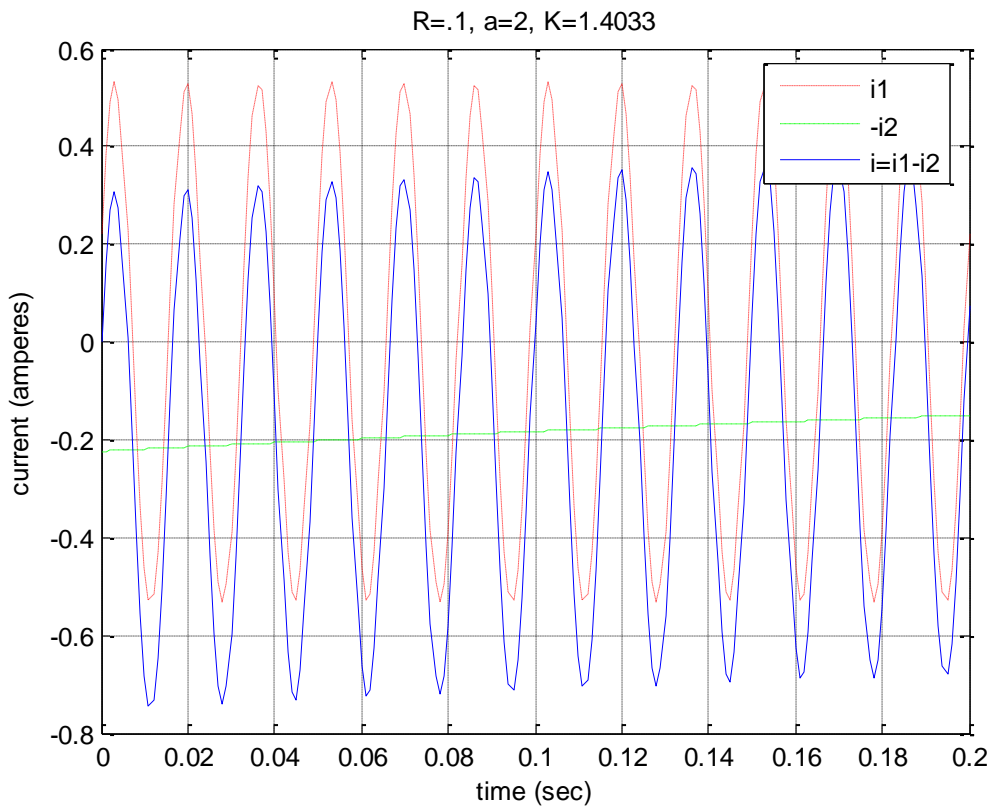
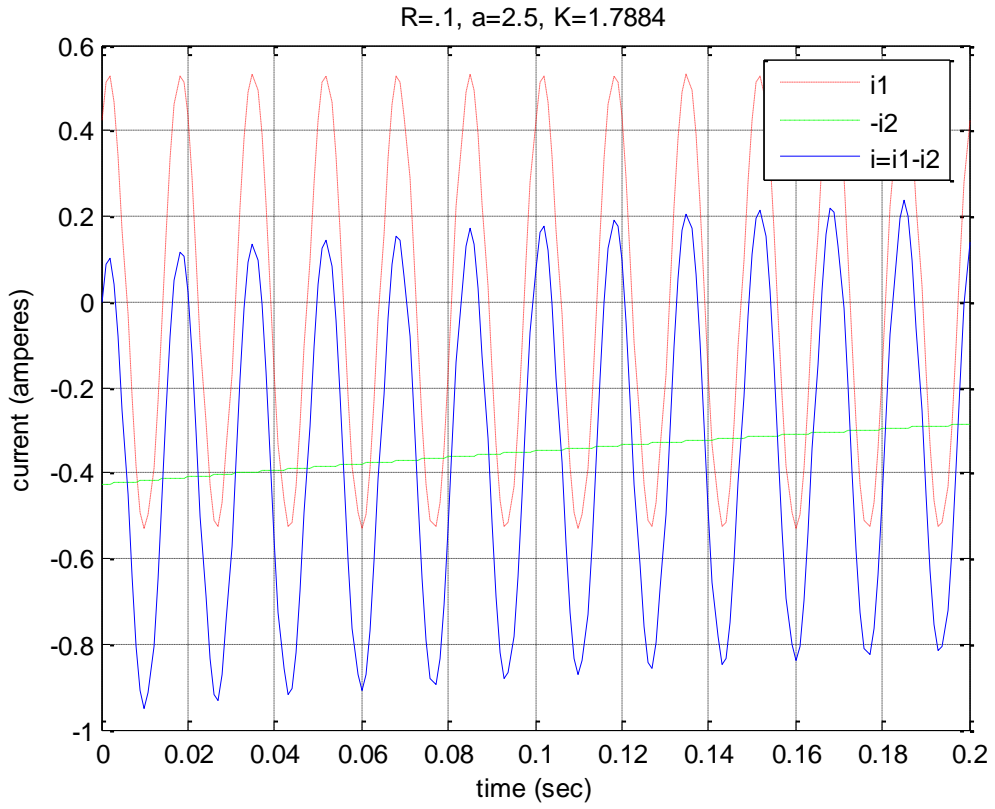


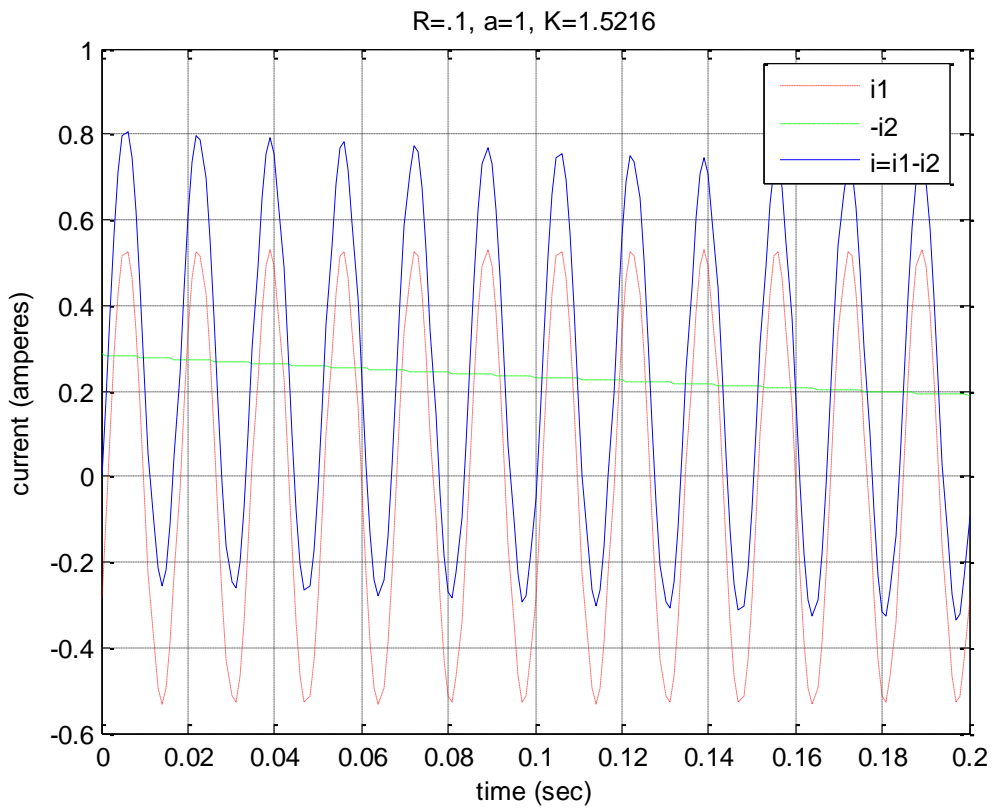
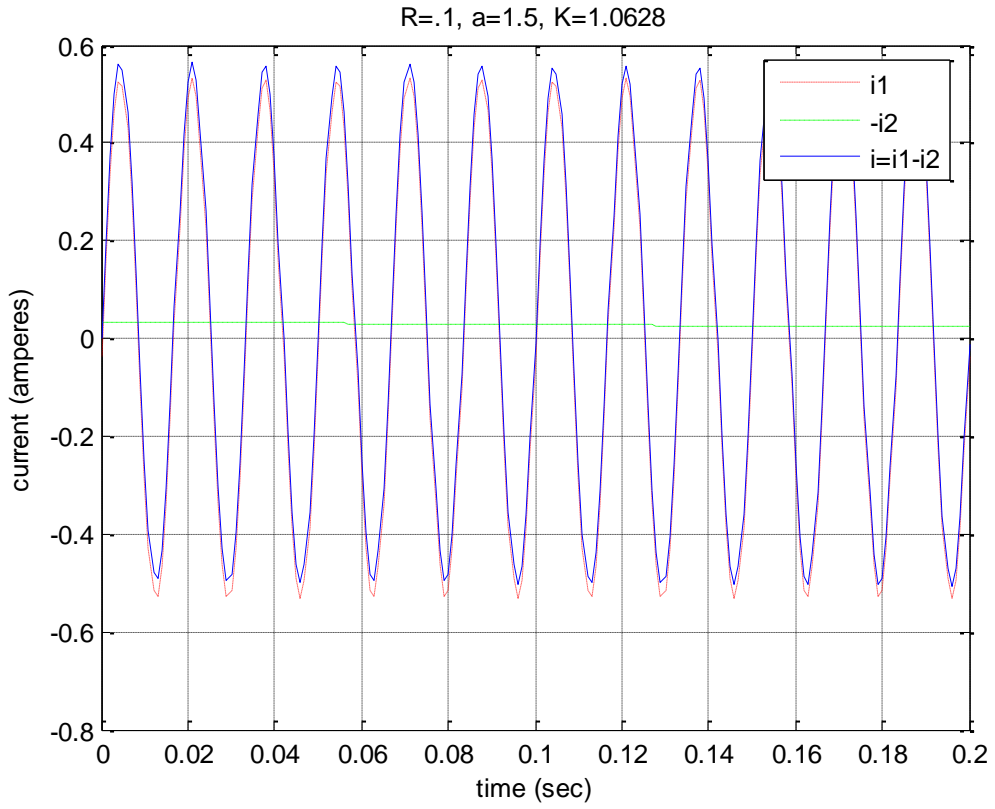


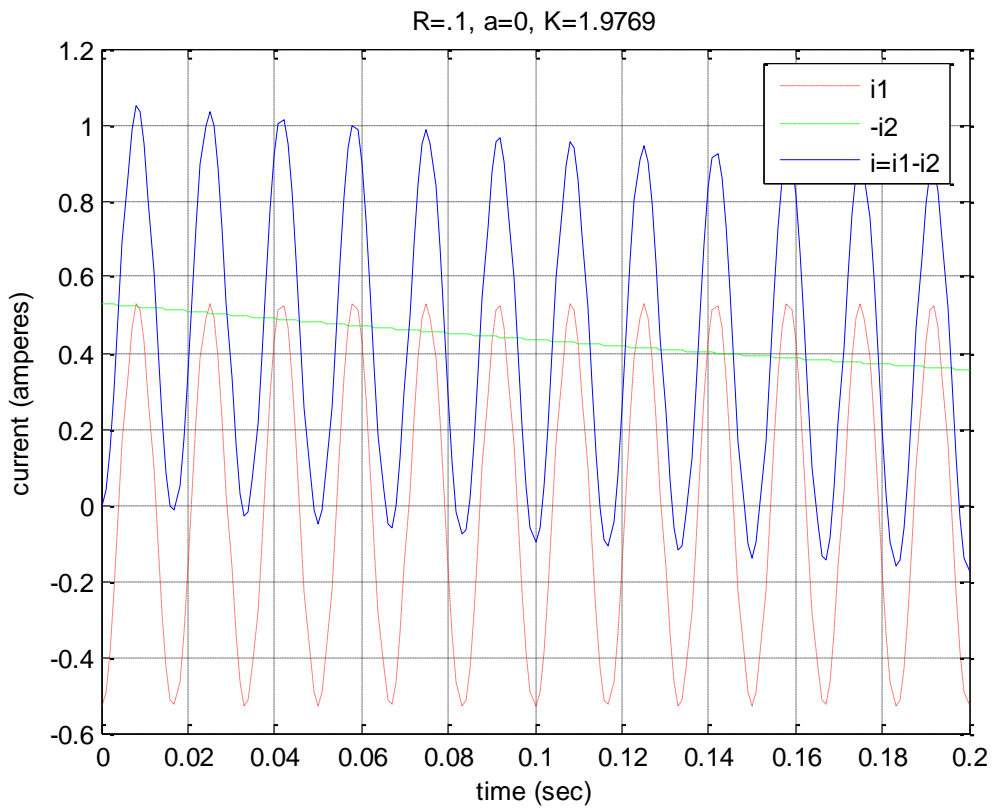
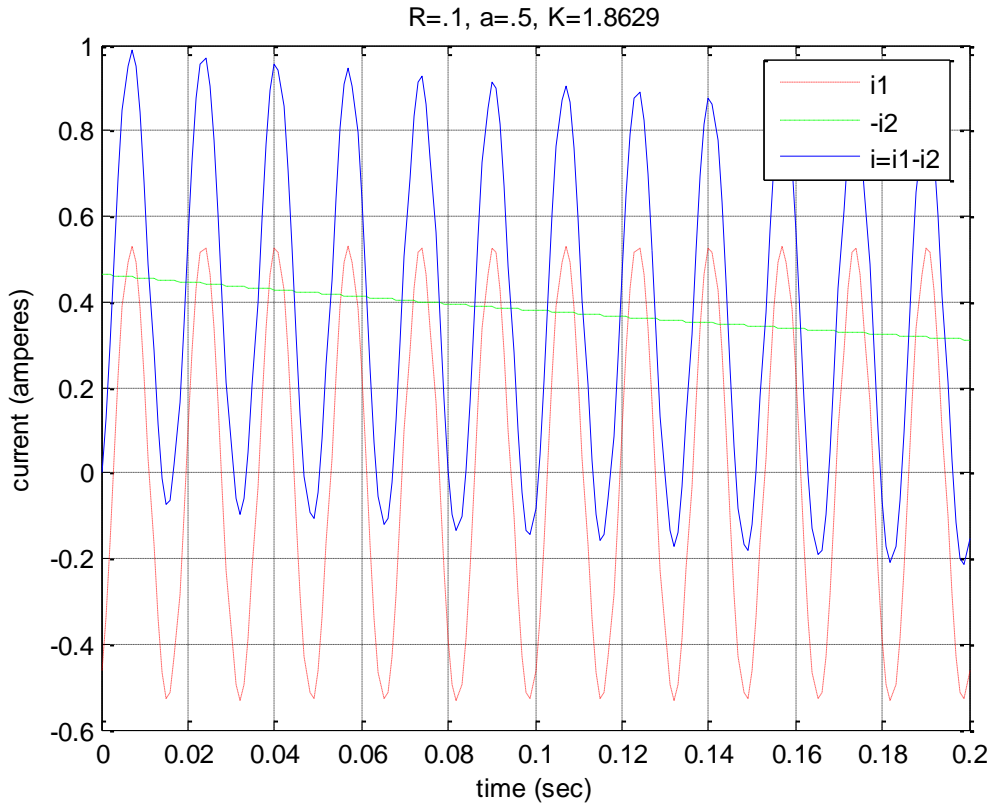


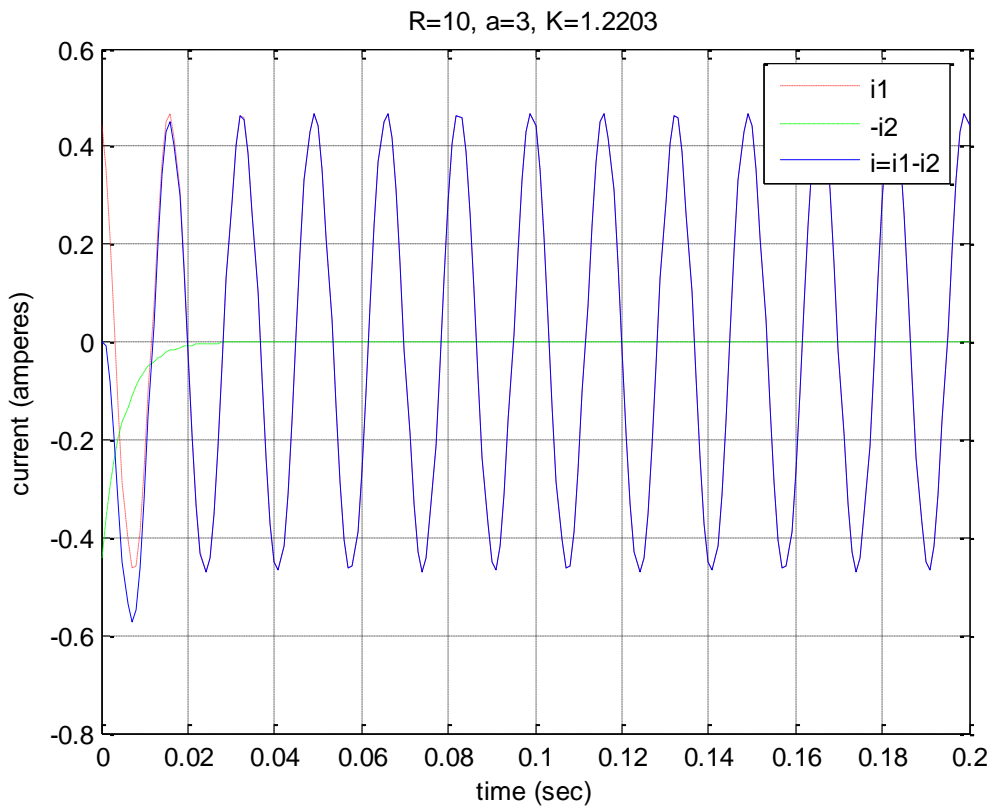
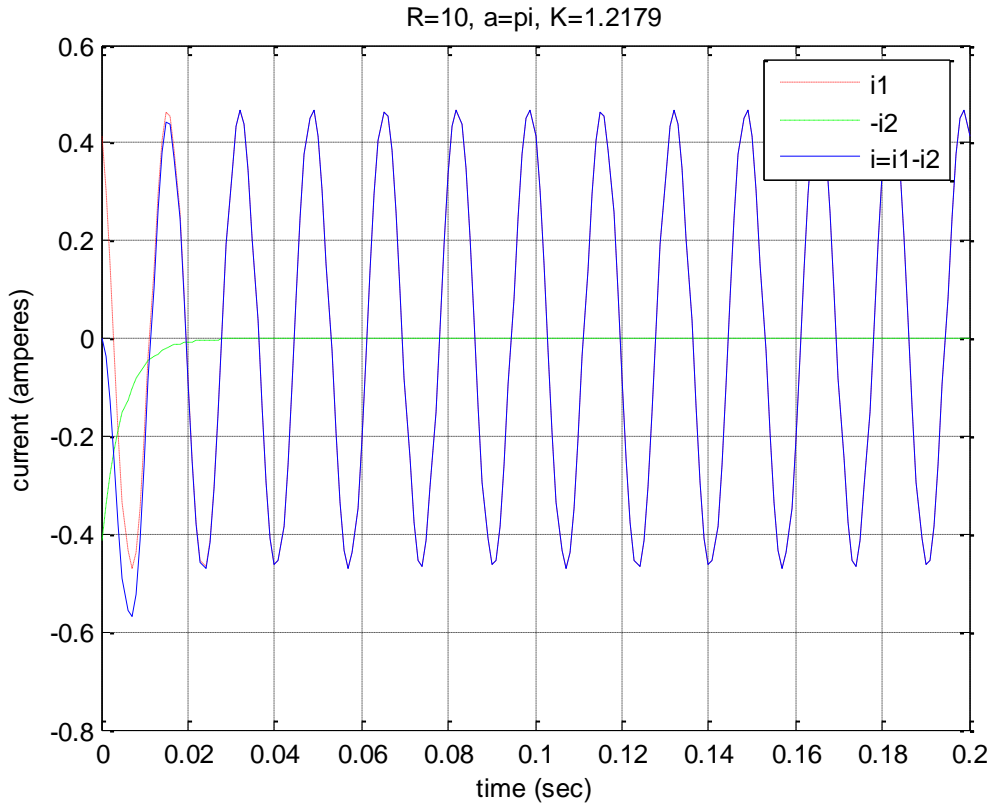


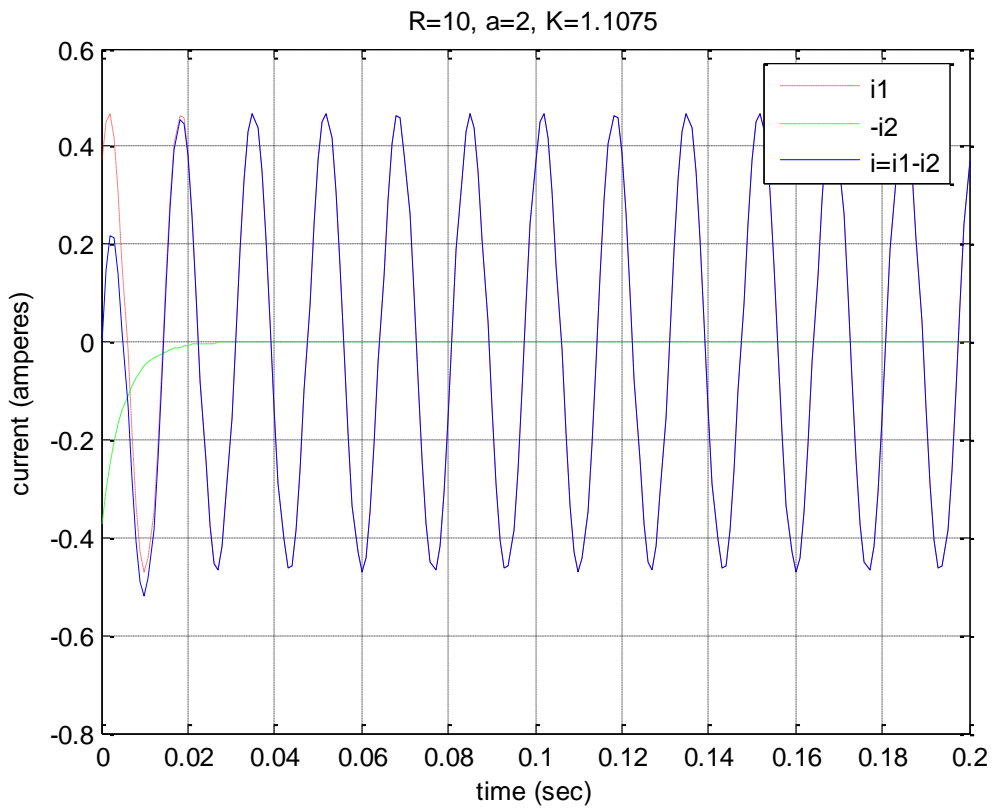
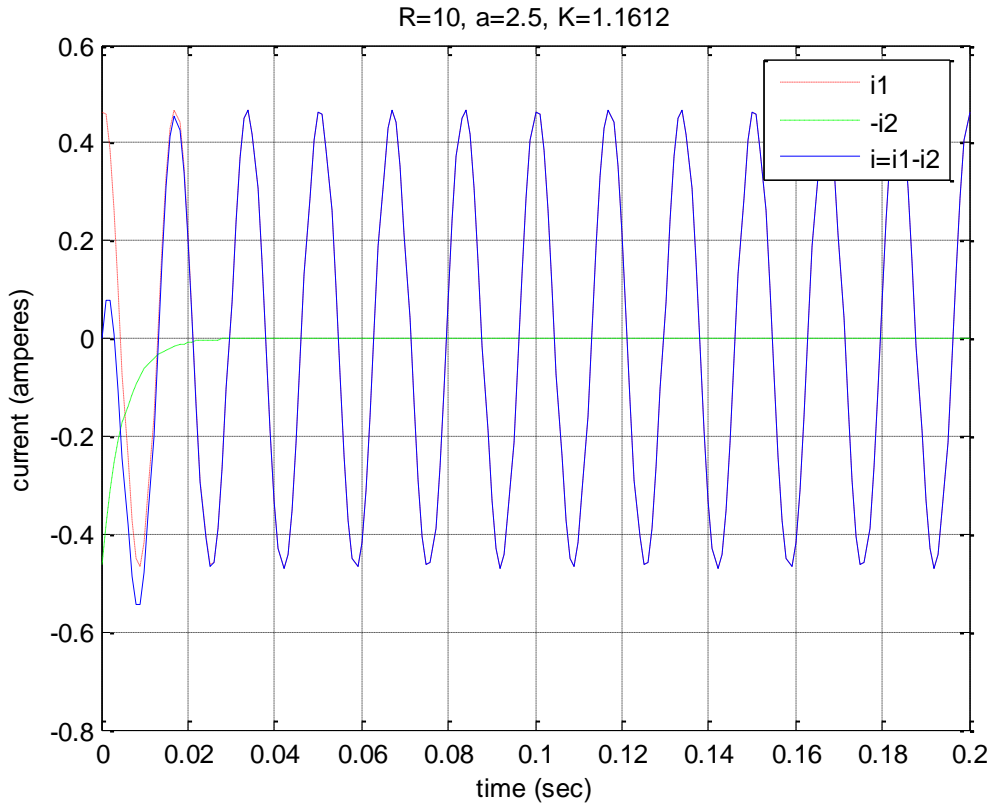


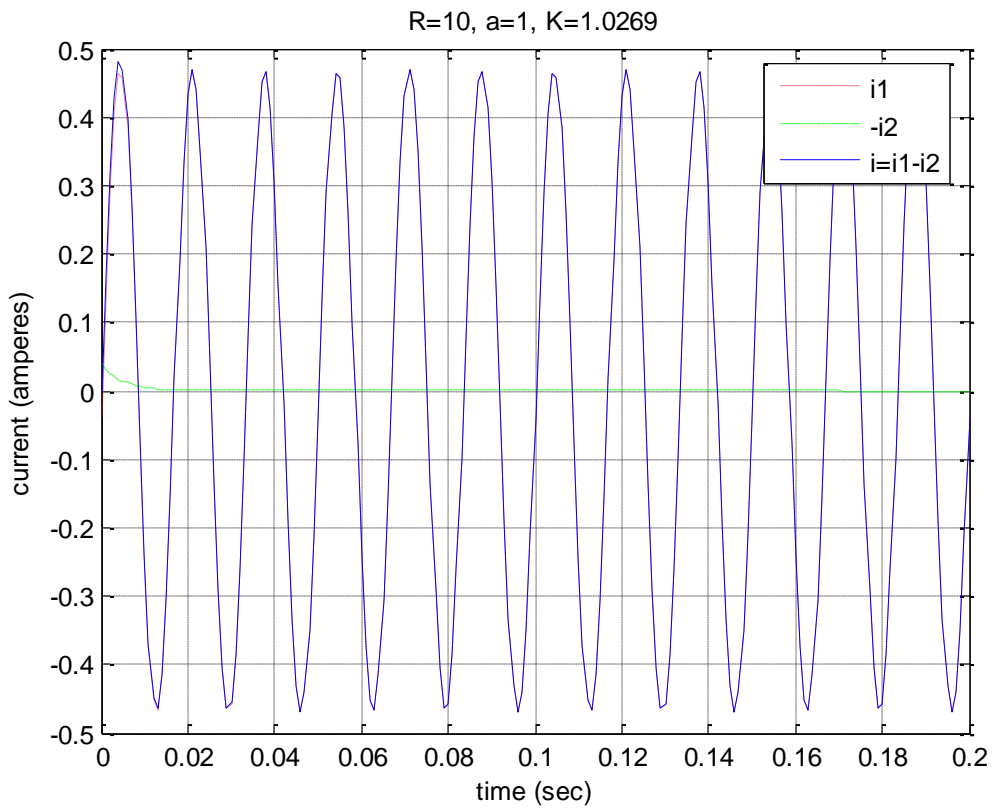
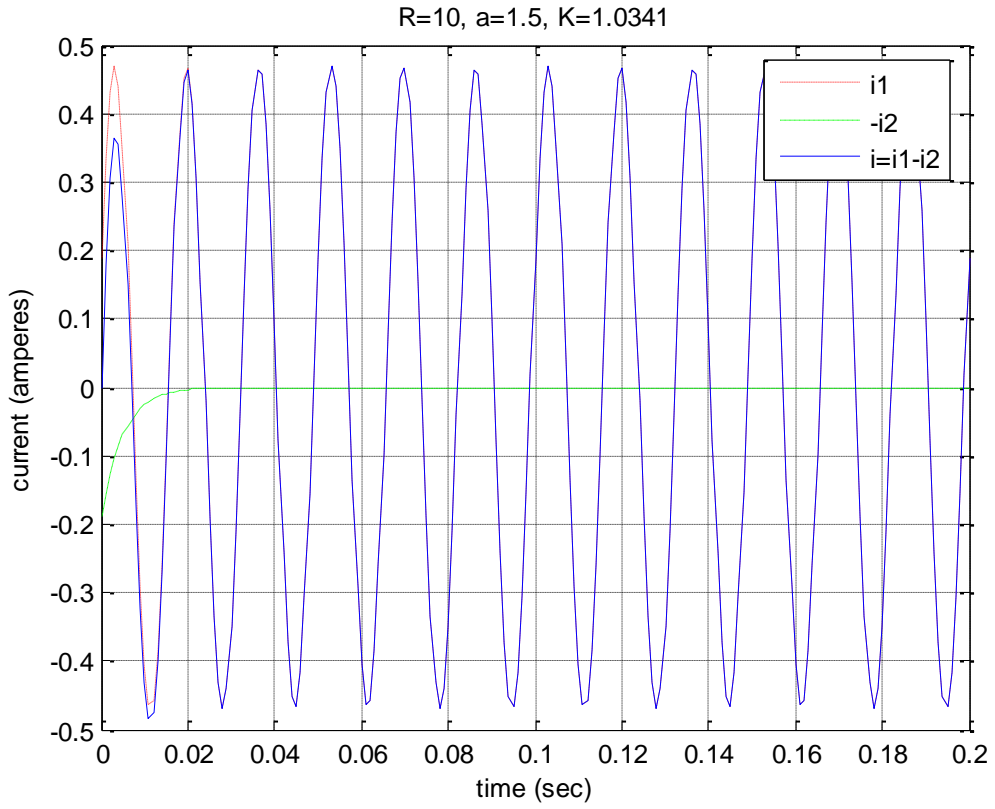


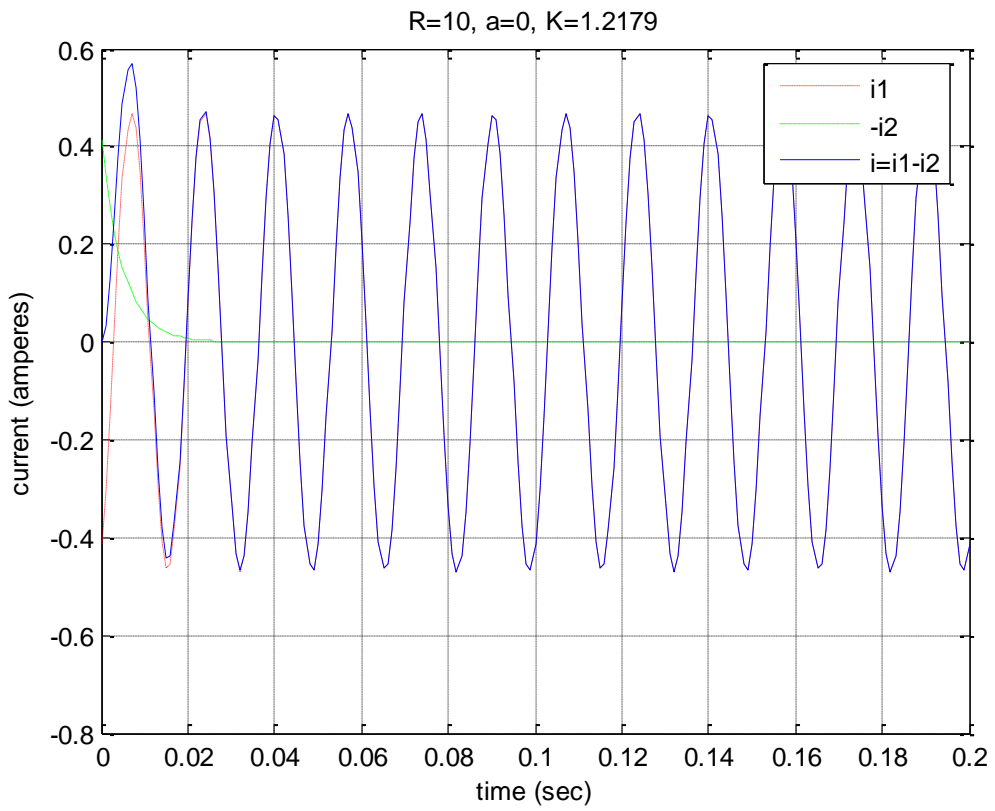
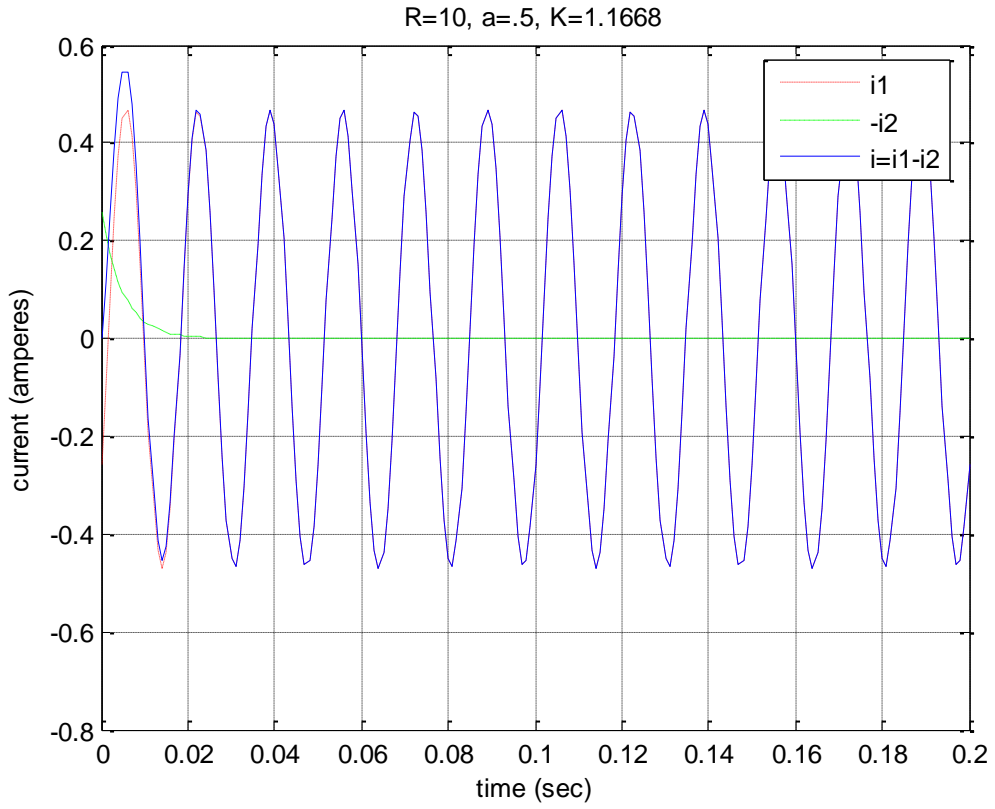










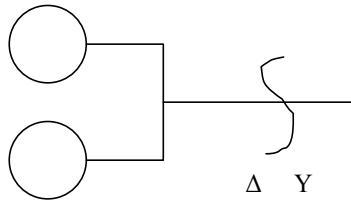


Summary

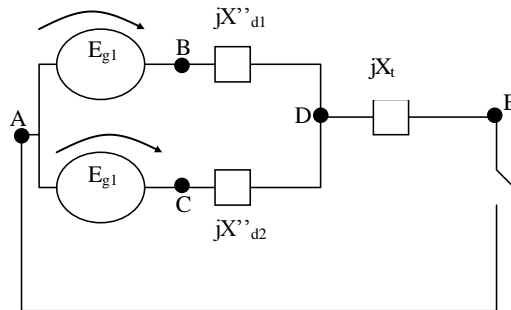
$\alpha \rightarrow$	π	3	2.5	2	1.5	1	.5	0
R=1	1.8509	1.8272	1.6815	1.3699	1.0142	1.4288	1.7407	1.8509
R=.1	1.9769	1.9666	1.7884	1.4033	1.0628	1.5216	1.8629	1.9769
R=10	1.2179	1.2203	1.1612	1.1075	1.0341	1.0269	1.1668	1.2179

Problem #2:

Two generators are connected in parallel to the low-voltage side of a three-phase Δ -Y transformer, as shown in the figure below. Generator 1 is rated 50,000 kVA, 13.8kV. Generator 2 is rated 25,000kVA, 13.8kV. Each generator has a subtransient reactance of 25% on its own base. The transformer is rated 75,000kVA, 13.8 Δ /69Y kV, with a reactance of 10%. Before the fault occurs, the voltage on the high-voltage side of the transformer is 66kV. The transformer is unloaded and there is no circulating current between the generators. Find the subtransient current in each generator when a three-phase short circuit occurs on the high-voltage side of the transformer.



Hint: The circuit to analyze should appear as below.

**Solution:**

Select 69kV, 75,000kVA as base in the high-voltage circuit. Then, the base voltage on the low-voltage side is 13.8kV.

Generator 1:

$$X''_{d1} = .25 \left(\frac{750000}{50000} \right) \left(\frac{13.8}{13.8} \right)^2 = .375 pu$$

Generator 2:

$$X''_{d2} = .25 \left(\frac{750000}{25000} \right) \left(\frac{13.8}{13.8} \right)^2 = .75 pu$$

Before the fault, since there is no circulating current flowing between the generators, then the current through X''_{d1} and X''_{d2} is zero, and the internal subtransient voltages within each generator must be equal. This is the voltage we will see just as we switch into the fault. Thus

$$E_g = E_{g1} = E_{g2}$$

Since the open circuit voltage is 66kV (when the transformer is unloaded), then the subtransient voltage is also 66kV. Thus, in per-unit:

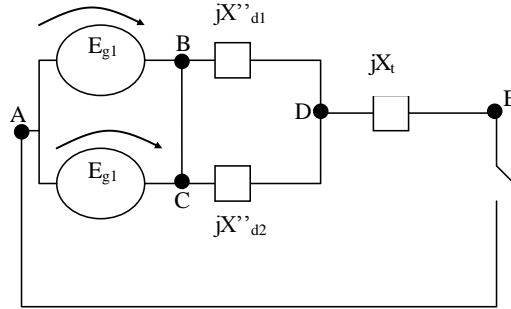
$$E_g = \frac{66}{69} = .957 pu$$

The per-unit reactance of the transformer is:

$$X_t = .10 pu$$

We could use nodal or mesh analysis to solve the circuit at this point and we will do so at the end of these notes. But here is another way...

Inspecting the circuit, one notices that, if the two voltage sources are the same (E_g), then the potential at the two points B and C must be the same (since the low side of the identical voltage sources is common at point A). This is as if the two voltage sources were in parallel, as illustrated in the figure below.



Regarding the two subtransient reactances X''_{d1} and X''_{d2} , it is now clear that they are also in parallel since their low side and high side terminals are at the same potential. The equivalent parallel subtransient reactance is:

$$X''_d = \frac{X''_{d1} * X''_{d2}}{X''_{d1} + X''_{d2}} = \frac{.375 * .75}{.375 + .75} = .25 pu$$

The circuit is now very easy to solve:

$$I'' = \frac{E_g}{jX''_d + jX_t} = \frac{.975}{j.25 + j.1} = -2.735 pu$$

We can now find the voltage on the delta side of the transformer

$$V_t = I'' * jX_t = .2735 pu$$

In Gens 1 and 2:

$$I''_1 = \frac{E_{g1} - V_t}{jX''_{d1}} = \frac{.957 - .2735}{j.375} = -j1.823 pu$$

$$I''_2 = \frac{E_{g2} - V_t}{jX''_{d2}} = \frac{.957 - .2735}{j.75} = -j.912 \text{ pu}$$

You can convert these currents back out of per-unit as follows:

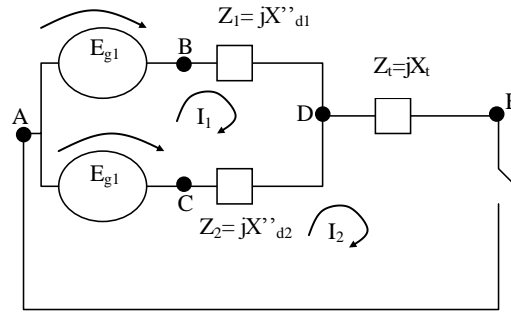
$$I_b = \frac{S_b}{\sqrt{3}V_b} = \frac{75000}{\sqrt{3} \times 13.8} = 3137.77 \text{ Amps}$$

$$|I''_1| = 5720.16 \text{ Amps}$$

$$|I''_2| = 2861.65 \text{ Amps}$$

Alternative approach using mesh analysis:

Define loop currents I_1 and I_2 as indicated below. Note that $I_1 = I''_1$ used above but I_2 is not the same as I''_2 used above.



Writing KVL around loop 2 results in

$$E_g - I_1 Z_1 - Z_2 (I_1 - I_2) - E_g = 0$$

and manipulation results

$$I_2 = \frac{I_1 (Z_1 + Z_2)}{Z_2} \quad (*)$$

Then, writing KVL around loop 1 results in

$$E_g + Z_2 (I_1 - I_2) - I_2 Z_t = 0$$

Manipulation results in

$$E_g + Z_2 I_1 - I_2 (Z_2 + Z_t) = 0$$

Substitution of (*) into the last equation, and manipulating, yields

$$I_1 = \frac{E_g Z_2}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t}$$

Substitution of the last equation into (*) yields

$$I_2 = \frac{E_g (Z_1 + Z_2)}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t}$$

The current through Z_2 is $I_2 - I_1$. Call this current I_3 , which is

$$\begin{aligned} I_3 = I_2 - I_1 &= \frac{E_g (Z_1 + Z_2)}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t} - \frac{E_g Z_2}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t} \\ &= \frac{E_g Z_1}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t} \end{aligned}$$

Thus, the two currents can be computed according to

$$\begin{aligned} I_1 &= \frac{E_g Z_2}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t} \\ &= \frac{0.957(j0.75)}{(j0.375)(j0.75) + (j0.375)(j0.1) + (j0.75)(j0.1)} \\ &= \frac{j0.71775}{j^2 0.39375} = -j1.823 \end{aligned}$$

$$\begin{aligned} I_3 &= \frac{E_g Z_1}{Z_1 Z_2 + Z_1 Z_t + Z_2 Z_t} \\ &= \frac{0.957(0.375)}{(j0.375)(j0.75) + (j0.375)(j0.1) + (j0.75)(j0.1)} \\ &= \frac{j0.358875}{j^2 0.39375} = -j0.912 \end{aligned}$$