EDC3

1.0 Introduction

In the last set of notes (EDC2), we saw how to use penalty factors in solving the EDC problem with losses. In this set of notes, we want to address two closely related issues.

- What are, exactly, penalty factors?
- How to obtain the penalty factors in practice?

2.0 What are penalty factors?

Recall the definition:

$$L_i = \frac{1}{\left[1 - \frac{\partial P_L(P_{G2}, \ldots, P_{Gm})}{\partial P_{Gi}}\right]} \quad (1)$$

In order to gain intuitive insight into what is a penalty factor, let's replace the numerator and denominator of the partial derivative in (1) with the approximation of $\Delta P_L/\Delta P_{Gi}$, so:

$$L_i = \frac{1}{\left[1 - \frac{\Delta P_L}{\Delta P_{Gi}}\right]} \qquad (2)$$

Multiplying top and bottom by ΔP_{Gi}, we get:

$$L_i = \frac{\Delta P_{Gi}}{\left[\Delta P_{Gi} - \Delta P_L\right]} \qquad (3)$$

What is ΔP_{Gi}?

It is a small change in generation.

But that cannot be all, because if you make a change in generation, then there must be a change in injection at, at least, one other bus. Let's assume that a compensating change is equally distributed throughout all other load buses. By doing so, we are embracing the so-called "*conforming load*" assumption, which indicates that all loads change proportionally.

Therefore we have that $\Delta P_{Gi}=\Delta P_D$. But this will also cause a change in losses of ΔP_L, which will be offset by a compensating change in generation at the swing bus by ΔP_1. Therefore we will have

$$\Delta P_{Gi} + \Delta P_{G1} = \Delta P_D + \Delta P_L \quad (4)$$

where we see generation changes are on the left and load & loss changes are on the right. Solving for ΔP_{Gi}-ΔP_L (because it is in the denominator of (3)), we get

$$\Delta P_{Gi} - \Delta P_L = \Delta P_D - \Delta P_{G1} \quad (5)$$

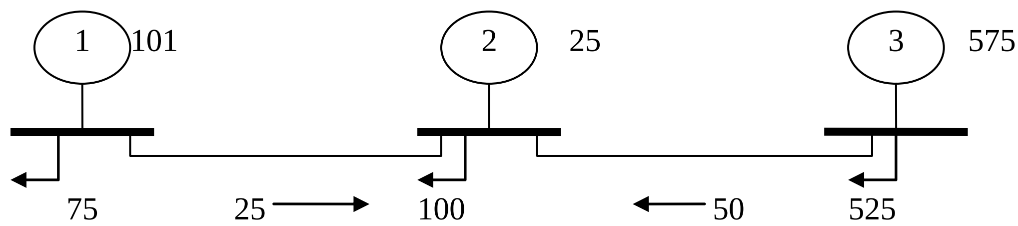
Substituting (5) into (3), we obtain:

$$L_i = \frac{\Delta P_{Gi}}{\Delta P_D - \Delta P_{G1}} \quad (6)$$

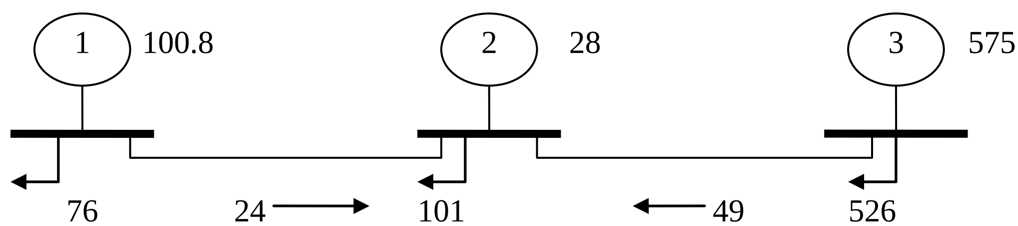
So from (6), we can see that the penalty factor indicates the amount of generation at unit i necessary to supply a change in load of ΔP_D. Clearly this is going to depend on how the load is changed, which is why we must have the *conforming load* assumption.

A simple example, similar to the one we worked in class last time, will illustrate the significance of (6). Consider Fig. 1.

Basecase

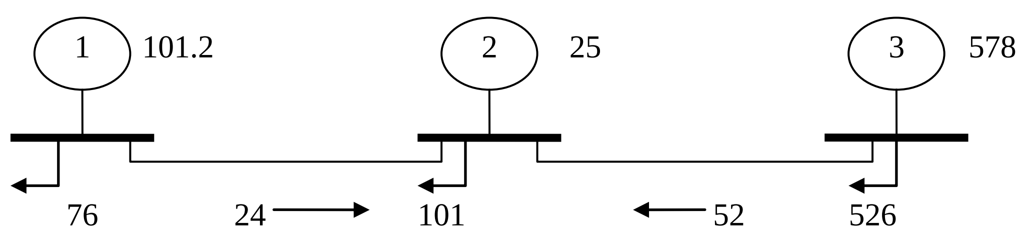


Increase load by 1 MW at each bus, compensated by gen increase at bus 2



$$L_2 = \frac{3}{3-(-0.2)} = \frac{3}{3.2} = 0.9375$$

Increase load by 1 MW at each bus, compensated by gen increase at bus 3



$$L_3 = \frac{3}{3-(+0.2)} = \frac{3}{2.8} = 1.074$$

Fig. 1

One observes that $L_2<1$. This is because a load change compensated by a gen change at bus 2 decreases the losses as indicated by the fact that the bus 1 generation decreased by 0.2 MW.

On the other hand, L_3>1. This is because a load change compensated by a gen change at bus 3 increases the losses as indicated by the fact that the bus 1 generation increases by 0.2. MW.

Why does the bus 2 generation reduce losses whereas the bus 3 generation increases losses?

<u>Answer</u>: Because increasing bus 2 tends to reduce the line flow.

So we see that in general, generators on the receiving end of flows will tend to have lower penalty factors (below 1.0), and generators on the sending end of flows will tend to have higher penalty factors (above 1.0).

Because transmission systems are in fact relatively efficient, with reasonably small losses in the circuits, the amount of generation necessary to supply a load change tends to be very close to that load change. Therefore penalty factors tend to be relatively close to 1.0.

A list of typical penalty factors for the power system in northern California is illustrated in Fig. 2. The generators marked to the right are units in the San Francisco Bay Area, which is a relatively high import area for the Northern California system. Most of the penalty factors for these units are below 1.0.

HALTY FACTORS FOR BASE-CASE GENERATION AND LOAD LEVEL

1985 SPRING PENALTY FACTORS   RANCHO SECO DOWN

IS	BUS	GENERATOR AXIS NAME	AXIS MW	PENALTY FACTOR
1	1	MALIN 500. INPUT FROM NORTHWEST.	2499.9995	1.142489
2	2	MIDWAY 500. INPUT FROM S. C. E.	-1199.9997	0.995580
3	3	SIERRA PACIFIC INTERTIE FROM SIERRA	0.8000	1.164796
4	4	SHASTA 230. SHT, KSW, CARR, SP CK, TRH	739.9998	1.117241
5	5	HUMBOLDT 115. HUM. P. P. 1-3.	30.0000	0.916989
6	6	ROUND MT. 230. PIT 3-7, BLACK	629.9999	1.125850
7	7	COTTONWD 230. PIT 1	60.0000	1.104298
8	8	CARIBOU 230. CARIBOU UNIT 4&5, BELDEN	120.0000	1.141188
9	9	MID/TID - INTERCHANGE FR PARKER & WALNUT	-36.4000	1.000940
10	10	POE 230: POE,CRESTA,BUCKS&ROK CRK,BELDEN	474.9999	1.185117
11	11	RANCHO SECO 230.	0.1000	1.021453
12	12	TABLE MT 230. INPUT FR STATE O/T AT TM	397.9999	1.094720
13	13	PALERMO 115. FORBSTOWN , WOODLEAF	80.0000	1.131894
14	14	DRUM 115. DRUM,DTCH FLT 1&2,CHICAGO PK	148.0000	1.163768
15	15	GOLD HILL 230. MID FK,FR MEADWS,RALSTON	197.0000	1.069882
16	16	CARIBOU 115. CARIBOU UNIT 1-3, BUTT VLY	65.0000	1.152115
17	17	FOLSOM 230. FOLSOM 1-3, NIMBUS	128.0000	1.043291
18	18	COLGATE 230. COLGATE, NARROWS 1&2	344.9999	1.116326
19	19	TRACY P. 230. INPUT FR TRACY PUMP & CCID	-75.0000	1.003675
20	20	WOKL. EQ 230. ELECTRA, SLT. SPG. AND TG	157.0000	1.045539
21	21	WRNRVILE 230. INTERCHANGE FROM CITY S.F.	23.9000	1.038416
22	22	NEWARK 115. INTERCHANGE FROM CITY S.F.	72.7000	0.977999
23	23	STANISLAS 115. STANISLAUS G	55.0000	1.088110
24	24	MELONES 115. DONNELLS,BEARDSLEY,TULLOCH	76.0000	1.075023
25	25	CON. CSTA 230. CCPP 1-7	200.0000	1.006337
26	26	PITTSBRG 230. PTSB PP 3-7	654.7870	0.984111
27	27	PITTSBRG 115. PTSB PP 1 & 2	0.1000	0.981577
28	28	MARTINEZ 115. AVON AND MARTINEZ	7.0000	0.960786
29	29	OLEUM 115. OLEUM 1&2	10.0000	0.968778
30	30	HNTRS. PT. 115. HUNTERS POINT PP 1-4	185.0000	0.949433
31	31	POTRERO 115. POTRERO PP 1-6	200.0000	0.947466
32	32	MOSS LDG. 500. MOSS LANDING PP 6 & 7	699.9999	1.007338
33	33	MOSS LDG. 230. MOSS LANDING PP 1-5	0.1000	1.004706
34	34	AMES 115. INTERCHANGE FROM AMES	-80.0000	0.969340
35	35	SLAC 230. INTERCHANGE FROM SLAC	-55.0000	0.979686
36	36	MORRO BAY 230. MORRO BAY PP 1-4	499.9999	1.015732
37	37	PIEDRA SW. 115. KINGS RIVER	40.0000	1.009973
38	38	KERCKHOFF 115. KERCKHOFF GEN	120.0000	1.086544
39	39	EXCHEQUER 115. EXCHEQUER GEN	70.0000	1.119342
40	40	BALCH EQ. 230. BALCH 2, HAAS & PINE FLT	324.9999	1.045456
41	41	HELMS PP 230KV	0.1000	0.988100
42	42	UARP-SMUD HYDRO	439.9999	1.042280
43	43	OAKLAND STA C 115, STA C GAS TURB GEN	0.1000	0.966901
44	44	NEW MELONES 230. (LOOPED)	200.0000	1.046535
45	45	DELTA P. 230. INTERCHANGE FR DELTA PUMP	-57.0000	1.001290
46	46	DS AMIGS 230.INTERCHANGE FR DS AMIGOS P.	-17.0000	1.004139
47	47	LS BANOS 230.INTERCHANGE FR SAN LUIS GEN	0.1000	1.006157
48	48	GEYSERS 230. GEYSER UNITS ON 230KV	1089.9998	1.089819
49	49	GEYSERS 115. GEYSER UNITS ON 115KV	145.0000	1.091903
50	50	DIABLO 500. DIABLO 1&2	999.9998	1.003612

Figure 13-3

Note: A generation level of 0.1MW is equivalent to the unit(s) being shut down.

Fig. 2

But why do we actually call them penalty factors? Consider the criterion for optimality in the EDC with losses:

$$\lambda = L_i \frac{\partial C_i(P_{Gi})}{\partial P_{Gi}} \qquad \forall i = 1,\ldots m \qquad (7)$$

This says that all units (or all regulating units) must be at a generation level such that the product of their incremental cost and their penalty factor must be equal to the system incremental cost λ.

Let's do an experiment to see what this means. Consider that we have three identical units such that their incremental cost-rate curves are identical, given by $IC(P_G)=45+0.02P_G$.

Now consider the three units are so located such that unit 1 has penalty factor of 0.98, unit 2 has penalty factor of 1.0, and unit 3 has penalty factor of 1.02, and the demand is 300 MW.

Without accounting for losses, this problem would be very simple in that each unit would carry 100 MW.

But with losses, the problem is as follows:

λ=0.98(45+0.02P_{G1})=44.1+0.196P_{G1}
λ=1.0(45+0.02P_{G2})=45+0.02P_{G2}
λ=1.02(45+0.02P_{G3})=45.9+0.0204P_{G3}

Putting these three equations into matrix form results in:

$$\begin{bmatrix} 0.0196 & 0 & 0 & -1 \\ 0 & 0.02 & 0 & -1 \\ 0 & 0 & 0.0204 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ P_{G3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -44.1 \\ -45 \\ -45.9 \\ 300 \end{bmatrix}$$

Solving in Matlab yields:

$$\begin{bmatrix} P_{G1} \\ P_{G2} \\ P_{G3} \\ \lambda \end{bmatrix} = \begin{bmatrix} 147.32 \\ 99.37 \\ 53.31 \\ 46.9875 \end{bmatrix}$$

One notes that the unit with the lower penalty (unit 1) was "turned up" and the unit with the higher penalty (unit 3) was "turned down." The reason for this is that unit 1 has a better effect on losses.

3.0 Calculation of penalty factors

Consider a power system with total of n buses of which bus 1 is the swing bus, buses 1…m are the PV buses, and buses m+1…n are the PQ buses.

Consider that losses must be equal to the difference between the total system generation and the total system demand:

$$P_L = P_G - P_D \tag{8}$$

Recall the definition for bus injections, which is

$$P_i = P_{Gi} - P_{Di} \tag{9}$$

Now sum the injections over all buses to get:

$$\sum_{i=1}^{n} P_i = \sum_{i=1}^{n} (P_{Gi} - P_{Di})$$

$$= \sum_{i=1}^{n} P_{Gi} - \sum_{i=1}^{n} P_{Di} = P_G - P_D \tag{10}$$

Therefore,

$$P_L = \sum_{i=1}^{n} P_i \qquad (11)$$

which is eq. (11.46) in the text.

Now differentiate with respect to a particular bus angle θ_k (where k is any bus number except 1) to obtain:

$$\frac{\partial P_L}{\partial \theta_k} = \frac{\partial P_1}{\partial \theta_k} + \frac{\partial P_2}{\partial \theta_k} \ldots + \frac{\partial P_m}{\partial \theta_k} + \frac{\partial P_{m+1}}{\partial \theta_k} + \ldots + \frac{\partial P_n}{\partial \theta_k}, k = 2, \ldots, n \qquad (12)$$

Assumption to the above: All voltages are fixed at 1.0 (this relieves us from accounting for the variation in power with angle through the voltage magnitude term).

Now let's assume that we have an expression for losses P_L as a function of generation P_{G2}, P_{G3},…,P_{Gm}, i.e.,

$$P_L = P_L(P_{G2}, P_{G3}, \ldots, P_{Gm}) \qquad (13)$$

Then we can use the chain rule of differentiation to express that

$$\frac{\partial P_L}{\partial \theta_k} = \frac{\partial P_L(\underline{P}_G)}{\partial P_{G2}} \frac{\partial P_2}{\partial \theta_k} + \ldots + \frac{\partial P_L(\underline{P}_G)}{\partial P_{Gm}} \frac{\partial P_m}{\partial \theta_k}, k = 2,...,n \tag{14}$$

Subtracting eq. (12) from eq. (14), we obtain, for k=2,…,n:

$$\frac{\partial P_L}{\partial \theta_k} = \frac{\partial P_1}{\partial \theta_k} + \frac{\partial P_2}{\partial \theta_k} \ldots + \frac{\partial P_m}{\partial \theta_k} + \frac{\partial P_{m+1}}{\partial \theta_k} + \ldots + \frac{\partial P_n}{\partial \theta_k}$$

$$-\frac{\partial P_L}{\partial \theta_k} = -\left(\frac{\partial P_L(\underline{P}_G)}{\partial P_{G2}} \frac{\partial P_2}{\partial \theta_k} + \ldots + \frac{\partial P_L(\underline{P}_G)}{\partial P_{Gm}} \frac{\partial P_m}{\partial \theta_k} \right)$$

$$0 = \frac{\partial P_1}{\partial \theta_k} + \frac{\partial P_2}{\partial \theta_k}\left(1 - \frac{\partial P_L(\underline{P}_G)}{\partial P_{G2}}\right) + \ldots + \frac{\partial P_m}{\partial \theta_k}\left(1 - \frac{\partial P_L(\underline{P}_G)}{\partial P_{Gm}}\right)$$

$$+ \frac{\partial P_{m+1}}{\partial \theta_k} + \ldots + \frac{\partial P_n}{\partial \theta_k}$$

Now bring the first term to the left-hand-side, for k=2,…,n

Writing the above

$$-\frac{\partial P_1}{\partial \theta_k} = \frac{\partial P_2}{\partial \theta_k}\left(1-\frac{\partial P_L(\underline{P}_G)}{\partial P_{G2}}\right)+\ldots+\frac{\partial P_m}{\partial \theta_k}\left(1-\frac{\partial P_L(\underline{P}_G)}{\partial P_{Gm}}\right)$$

$$+\frac{\partial P_{m+1}}{\partial \theta_k}+\ldots+\frac{\partial P_n}{\partial \theta_k}$$

The above equation, when written for k=2,…,n, can be expressed in matrix form as

$$\begin{bmatrix} \frac{\partial P_2}{\partial \theta_2} & \cdots & \frac{\partial P_m}{\partial \theta_2} & \cdots & \frac{\partial P_n}{\partial \theta_2} \\ \vdots & \cdots & \vdots & \cdots & \vdots \\ \frac{\partial P_2}{\partial \theta_n} & \cdots & \frac{\partial P_m}{\partial \theta_n} & \cdots & \frac{\partial P_n}{\partial \theta_n} \end{bmatrix} \begin{bmatrix} 1-\frac{\partial P_L(\underline{P}_G)}{\partial P_{G2}} \\ \vdots \\ 1-\frac{\partial P_L(\underline{P}_G)}{\partial P_{Gm}} \\ 1 \\ \vdots \\ 1 \end{bmatrix} = -\begin{bmatrix} \frac{\partial P_1}{\partial \theta_2} \\ \vdots \\ \frac{\partial P_1}{\partial \theta_2} \end{bmatrix}$$