

where  $\mu$  is the specific growth rate coefficient ( $\text{hr}^{-1}$ ). It is referred to as a specific rate coefficient because it defines the rate of biomass growth in terms of the concentration of active biomass present, i.e., the mass of biomass COD formed per unit time per unit of active biomass COD present. Equation 3.35 holds for any type of bacterial growth, regardless of the nature of the electron donor or acceptor, although much of the following is written in terms of heterotrophic biomass growth on an organic substrate. Consequently, subscripts are not used at this point to distinguish between heterotrophic and autotrophic biomass, although they will be used later when it is necessary to make that distinction. Substitution of Eq. 3.35 into Eq. 3.34 defines the rates of substrate removal and oxygen (electron acceptor) utilization associated with biomass growth. It is important to note that the equation for oxygen utilization is also true for other electron acceptors, such as nitrate, as long as the quantity is expressed in oxygen equivalents.

### 3.2.7 Effect of Substrate Concentration on $\mu$

*The Monod Equation.* Originally, exponential growth of bacteria was considered to be possible only when all nutrients, including the substrate, were present in high concentration. In the early 1940s, however, it was found that bacteria grow exponentially even when one nutrient is present only in limited amount.<sup>83</sup> Furthermore, the value of the specific growth rate coefficient,  $\mu$ , was found to depend on the concentration of that limiting nutrient, which can be the carbon source, the electron donor, the electron acceptor, nitrogen, or any other factor needed by the organisms for growth. Since that time, the generality of this observation has been substantiated often, so that it can now be considered to be a basic concept of microbial kinetics.<sup>34</sup> Let us first consider the situation when only an organic substrate is growth limiting.

Figure 3.1 illustrates the relationship that is obtained when  $\mu$  is measured as a function of a single limiting substrate concentration. A number of different types of experiments can be performed to develop such a relationship and they are discussed in Chapter 8. The important thing to note at this time is that  $\mu$  initially rises rapidly as the substrate concentration is increased, but then asymptotically approaches a maximum, which is called the maximum specific growth rate,  $\hat{\mu}$ .

The question of the best mathematical formula to express the relationship shown in Figure 3.1 has been the subject of much debate. No one yet knows enough about the mechanisms of biomass growth to propose a mechanistic equation that will characterize growth exactly. Instead, experimenters have observed the effects of various factors on growth and have then attempted to fit empirical equations to their observations. Consequently, all equations that have been proposed are curve-fits and the only valid arguments for use of one over another are goodness of fit, mathematical utility, and broad acceptance.

The equation with historical precedence and greatest acceptance is the one proposed by Monod (mo nō').<sup>83</sup> Although his original work was done in batch reactors, it was later extended and refined by workers using continuous cultures of single bacterial species growing on defined media and it was concluded that the curve could be approximated adequately by the equation for a rectangular hyperbola.<sup>34</sup> Consequently, Monod proposed the equation:

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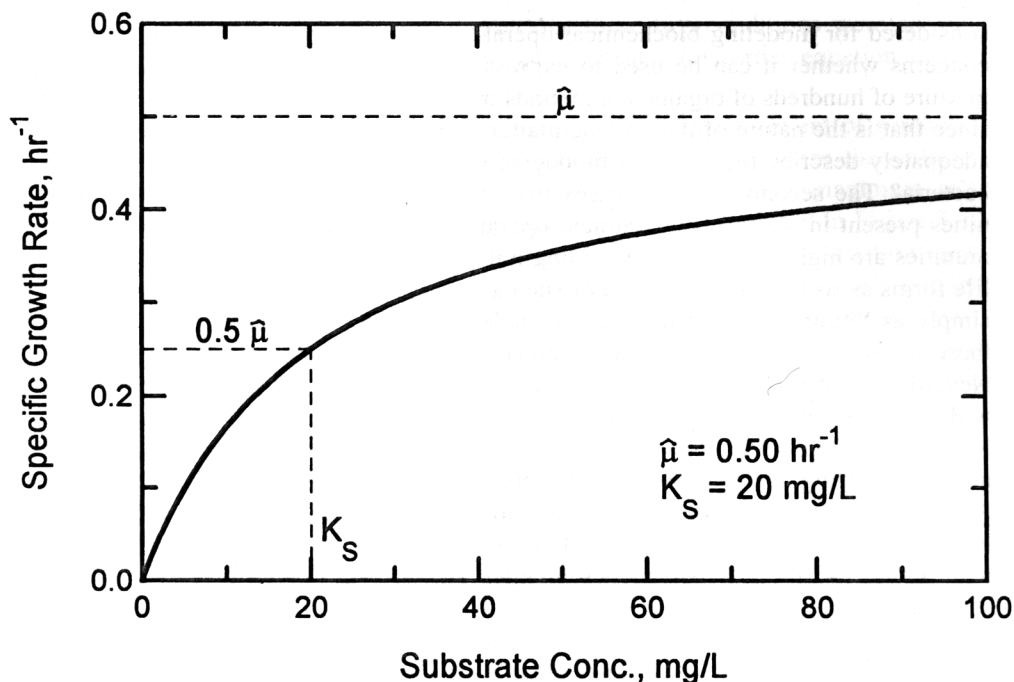
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**Figure 3.1** Typical plot of the relationship between the specific growth rate coefficient and the concentration of a noninhibitory substrate. The parameter values given were used to construct the curve with the Monod equation (3.36).

$$\mu = \hat{\mu} \frac{S_s}{K_s + S_s} \quad (3.36)$$

where  $K_s$  is the half-saturation coefficient.  $K_s$  determines how rapidly  $\mu$  approaches  $\hat{\mu}$  and is defined as the substrate concentration at which  $\mu$  is equal to half of  $\hat{\mu}$ , as shown in Figure 3.1. The smaller it is, the lower the substrate concentration at which  $\mu$  approaches  $\hat{\mu}$ . Because of his pioneering efforts in defining the kinetics of microbial growth, Eq. 3.36 is generally referred to as the Monod equation.

Because of the similarity of Eq. 3.36 to the Michaelis–Menten equation in enzyme kinetics, many people have erroneously concluded that Monod proposed it on mechanistic grounds. While the Michaelis–Menten equation can be derived from consideration of the rates of chemical reactions catalyzed by enzymes, and has a mechanistic basis, the Monod equation is strictly empirical. In fact, Monod himself emphasized its empirical nature.<sup>83</sup>

The Monod equation has been found to fit the data for many pure cultures growing on single substrates, both organic and inorganic, and has been used extensively in the development of models describing the continuous cultivation of microorganisms. It has not been blindly accepted, however, and other workers have proposed alternative equations that fit their data better.<sup>84,97,108</sup> Nevertheless, it is still the most widely used equation.

Because the Monod equation was developed for pure cultures of bacteria growing on single organic substrates, two significant questions arise when its adoption is

considered for modeling biochemical operations for wastewater treatment. The first concerns whether it can be used to express removal of a substrate that is really a mixture of hundreds of organic compounds measured by a nonspecific test like COD, since that is the nature of the organic matter in wastewater. Can the Monod equation adequately describe the effect of biodegradable COD on the specific growth rate of bacteria? The second question arises from consideration of the microbial communities present in wastewater treatment operations. As seen in Chapter 2, those communities are highly complex, containing not only many bacterial species, but higher life forms as well. Can the growth of such a heterogeneous assemblage be expressed simply as "biomass" by the Monod equation? Many researchers have investigated these questions, and it is generally agreed that the answer to both is yes.<sup>3,18,31,36,71</sup> Nevertheless, it should be recognized that the manner in which the culture is grown will have a strong impact on its community structure, and that the values of  $\hat{\mu}$  and  $K_s$  obtained from mixed culture systems are in reality average values resulting from many interacting species.<sup>19,36,38</sup> Consequently, it has been recommended that  $\hat{\mu}$  and  $K_s$  be characterized by ranges, rather than by single values, just as was recommended for  $Y$ . It can be concluded, however, that the Monod equation is a reasonable model with which to describe the kinetics of microbial growth on complex organic substrates in wastewater treatment systems, and consequently, it is widely used. There are situations, however, in which it would be desirable to model the effects on microbial growth rates of individual organic compounds in complex mixtures. This situation is very complicated,<sup>72</sup> however, and will be covered in Chapter 22.

*Simplifications of the Monod Equation.* Examination of Eq. 3.36 reveals that two simplifications can be made, and this is often done in the modeling of wastewater treatment systems. First, it can be seen that if  $S_s$  is much larger than  $K_s$ , the equation may be approximated as:

$$\mu \approx \hat{\mu} \quad (3.37)$$

This is called the zero-order approximation because under that condition the specific growth rate coefficient is independent of the substrate concentration, i.e., it is zero order with respect to  $S_s$ , and equal to the maximum specific growth rate coefficient. In other words, the bacteria will be growing as rapidly as possible. Second, if  $S_s$  is much smaller than  $K_s$ , the term in the denominator may be approximated as  $K_s$  and the equation becomes:

$$\mu \approx \frac{\hat{\mu}}{K_s} S_s \quad (3.38)$$

This is called the first-order approximation because  $\mu$  is first order with respect to  $S_s$ . Although Eq. 3.38 is often easier to use than the Monod equation, care should be exercised in its use because serious error can result if  $S_s$  is not small relative to  $K_s$ . When COD is used as a measure of the total quantity of biodegradable organic matter,  $K_s$  can be relatively large, with the result that  $S_s$  in activated sludge reactors is often less than  $K_s$ . Consequently, Eq. 3.38 is sometimes used to model such systems.

Garrett and Sawyer<sup>35</sup> were the first to propose the use of Eqs. 3.37 and 3.38 because they had observed that the specific growth rate coefficient for bacteria was directly proportional to the substrate concentration at low values and independent of



it at high values. Although they recognized that these two conditions were special cases of the Monod equation, others who adopted their first-order equation incorrectly considered it to be an alternative expression.

**Inhibitory Substrates.** On occasion, particularly in the treatment of synthetic (xenobiotic) organic compounds in industrial wastewaters, situations are encountered in which the specific growth rate of the microorganisms reaches a maximum and then declines as the substrate concentration is increased, as illustrated in Figure 3.2. Obviously, the Monod equation is not adequate for depicting this situation, and consequently, considerable effort has been expended to determine an appropriate equation.<sup>32,86,104</sup> As with normal, naturally-occurring, noninhibitory (biogenic) substrate, many different models could be used to represent the observed relationship between the substrate concentration and  $\mu$ , and from a statistical point of view there is little to recommend one over another.<sup>32,104</sup> Consequently, as with the Monod equation, it has been argued that model selection should be based on familiarity and ease of use, leading to a recommendation that an equation based on the enzymatic model of Haldane<sup>46</sup> should be used. Andrews<sup>2</sup> was the first to propose general use of such a function for depicting the effects of inhibitory organic substrates on bacterial growth rates, and thus it will be called the Andrews equation herein. Its form is:

$$\mu = \hat{\mu} \frac{S_s}{K_s + S_s + S_s^2/K_i} \quad (3.39)$$

Examination of Eq. 3.39 reveals that it is similar to the Monod equation, containing only one additional parameter,  $K_i$ , the inhibition coefficient. Note that when  $K_i$  is very large the Andrews equation simplifies to the Monod equation, demonstrating that  $\hat{\mu}$  and  $K_s$  have the same meaning in both equations. Unlike the situation for a noninhibitory substrate, however,  $\hat{\mu}$  cannot actually be observed and is a hypothetical maximum specific growth rate that would be attained if the substrate were not inhibitory. Furthermore, since  $\hat{\mu}$  cannot be observed,  $K_s$  also takes on a hypothetical meaning. The most outstanding characteristic of the curve in Figure 3.2 is that  $\mu$  passes through a maximum,  $\mu^*$ , at substrate concentration  $S_s^*$ , where

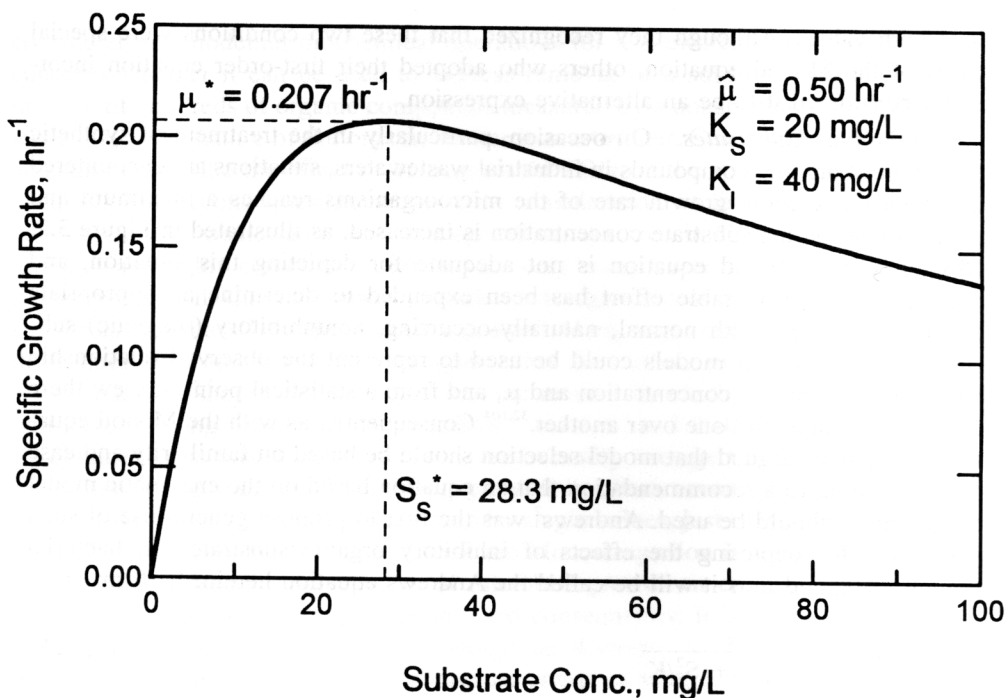
$$\mu^* = \frac{\hat{\mu}}{2(K_s/K_i)^{0.5} + 1} \quad (3.40)$$

and

$$S_s^* = (K_s \cdot K_i)^{0.5} \quad (3.41)$$

Equation 3.40 is important because it demonstrates that the degree of inhibition is determined by  $K_s/K_i$ , and not just by  $K_i$  alone. The larger  $K_s/K_i$ , the smaller  $\mu^*$  is relative to  $\hat{\mu}$ , and thus, the greater the degree of inhibition. Furthermore, because they are measurable,  $\mu^*$  and  $S_s^*$  are important in the determination of the kinetic parameters for inhibitory substrates. Equation 3.39 has been used widely in the modeling of various wastewater treatment systems, and will be adopted herein for depicting the effect of an inhibitory substrate on the specific growth rate of bacteria degrading it.

**Effects of Other Inhibitors.** Sometimes one compound may act to inhibit microbial growth on another compound. For example, some organic chemicals are known to inhibit the growth of nitrifying bacteria,<sup>59,122</sup> whereas others inhibit the



**Figure 3.2** Typical plot of the relationship between the specific growth rate coefficient and the concentration of an inhibitory substrate. The parameter values given were used to construct the curve with the Andrews equation (3.39). Note that the values of  $\hat{\mu}$  and  $K_s$  are the same as in Figure 3.1.

growth of heterotrophic bacteria on biogenic organic matter.<sup>126</sup> In those cases it is necessary for the kinetic expression to depict the effect of the concentration of the inhibitor ( $S_i$ ) on the relationship between  $\mu$  and  $S_s$ . If the Monod equation can be used to relate  $\mu$  to  $S_s$  in the absence of the inhibitor, then the effect of the inhibitor can be expressed as an effect on  $\hat{\mu}$  and/or  $K_s$ .<sup>50,125</sup> Several types of inhibitors have been defined by analogy to enzyme inhibition, but all can be modeled by an extension of the Monod model proposed by Han and Levenspiel:<sup>48</sup>

$$\mu = \hat{\mu} \left( 1 - \frac{S_i}{S_i^*} \right)^n \left[ \frac{S_s}{S_s + K_s(1 - S_i/S_i^*)^m} \right] \quad (3.42)$$

where  $S_i^*$  is the inhibitor concentration that causes all microbial activity to cease and  $m$  and  $n$  are exponents that reflect the impact of increasing inhibitor concentrations on  $K_s$  and  $\hat{\mu}$ , respectively. Equation 3.42 has been used successfully to model the effects of various xenobiotic compounds on the removal of biogenic organic matter.<sup>126</sup> Its use will be discussed in Chapter 22.

### 3.2.8 Specific Substrate Removal Rate

In earlier sections it was stated that the basis for writing stoichiometric equations was arbitrary and that the reference component was the choice of the investigator.

Thus, it is not surprising that many investigators<sup>71,82,124</sup> have selected substrate removal, rather than biomass growth, as their basic event and have written their rate equations accordingly. Combining Eqs. 3.34 and 3.35 yields:

$$r_{ss} = -(\mu/Y)X_B \quad (3.43)$$

The term  $\mu/Y$  has been called the specific substrate removal rate and given the symbol  $q$ .<sup>43</sup> (Note that the subscript H has been dropped from Y and  $X_B$  to emphasize the general nature of Eq. 3.43.) Obviously,  $q$  will be influenced by  $S_s$  in exactly the same way as  $\mu$ , and Eqs. 3.37 through 3.42 can be written in terms of it. When this is done, the maximum specific substrate removal rate,  $\hat{q}$ , is used in place of  $\hat{\mu}$ , where:

$$\hat{q} = \hat{\mu}/Y \quad (3.44)$$

Both first- and zero-order approximations have been used for the relationship between  $q$  and  $S_s$ , just as they have for  $\mu$ . In fact, the ratio of  $\hat{q}$  over  $K_s$  has been called the mean reaction rate coefficient and given the symbol  $k_e$ .<sup>29</sup>

$$k_e = \hat{q}/K_s \quad (3.45)$$

All restrictions that apply to the approximate expressions for the effect of  $S_s$  on  $\mu$  also apply to  $q$ .

### 3.2.9 Multiple Limiting Nutrients

In the broad sense, nutrients can be divided into two categories: complementary and substitutable.<sup>9</sup> Complementary nutrients are those that meet entirely different needs by growing microorganisms. For example, ammonia provides the nitrogen needed for protein synthesis while glucose provides carbon and energy. If either was missing from the growth medium and no substitute was provided, no growth would occur. Substitutable nutrients, on the other hand, are those that meet the same need. For example, ammonia and nitrate can both provide nitrogen whereas glucose and phenol can both provide carbon and energy. Thus, ammonia and nitrate are substitutable for each other, as are glucose and phenol. In this section, we will consider simultaneous limitation of specific growth rate by two complementary nutrients. As stated previously, consideration of the effects of multiple carbon sources, i.e., multiple substitutable nutrients, is very complex,<sup>72</sup> and will be covered in Chapter 22.

In spite of its potential importance in the environment, relatively little is known about how microorganisms respond to simultaneous limitation by two or more complementary nutrients.<sup>9</sup> Because the uncertainty increases greatly as the number of nutrients involved increases, we will limit our considerations to only two.

**Interactive and Noninteractive Relationships.** Consider two complementary nutrients,  $S_{s1}$  and  $S_{s2}$ . Both are required for biomass growth and are present at low concentration in the environment in which the biomass is growing. Which will control the specific growth rate? Two different philosophies have been developed to answer this question, and the models representing them have been classified as interactive and noninteractive.<sup>6</sup>

An interactive model is based on the assumption that two complementary nutrients can both influence the specific growth rate at the same time. If both are

required for growth and are present at concentrations equal to their half-saturation coefficients, then each alone can reduce  $\mu$  to one-half of  $\hat{\mu}$ . However, since both effects are occurring simultaneously, the result would be to reduce  $\mu$  to one-fourth of  $\hat{\mu}$ . The most common type of interactive model in use is the multiple Monod equation:<sup>6,113</sup>

$$\mu = \hat{\mu} \left( \frac{S_{S1}}{K_{S1} + S_{S1}} \right) \left( \frac{S_{S2}}{K_{S2} + S_{S2}} \right) \quad (3.46)$$

Any time the concentrations of  $S_{S1}$  and  $S_{S2}$  are such that both  $S_{S1}/(K_{S1} + S_{S1})$  and  $S_{S2}/(K_{S2} + S_{S2})$  are less than one, they both act to reduce  $\mu$  below  $\hat{\mu}$ . This has two impacts. First, for a given value of  $S_{S1}$ ,  $\mu$  will be lower when  $S_{S2}$  is also limiting than it would be if  $S_{S2}$  were present in excess. Second, there is not a unique value of  $\mu$  associated with a given value of  $S_{S1}$  or  $S_{S2}$ , as there was with Eq. 3.36. Rather, it depends on both.

A noninteractive model is based on the assumption that the specific growth rate of a microbial culture can only be limited by one nutrient at a time. Therefore,  $\mu$  will be equal to the lowest value predicted from the separate single-substrate models:<sup>119</sup>

$$\mu = \min \left( \frac{\hat{\mu} S_{S1}}{K_{S1} + S_{S1}}, \frac{\hat{\mu} S_{S2}}{K_{S2} + S_{S2}} \right) \quad (3.47)$$

If  $S_{S1}/(K_{S1} + S_{S1}) < S_{S2}/(K_{S2} + S_{S2})$ , nutrient  $S_{S1}$  is rate limiting, and vice versa. If  $S_{S1}/(K_{S1} + S_{S1}) = S_{S2}/(K_{S2} + S_{S2})$ , then both are rate limiting, but that occurs only under special conditions. In the noninteractive conceptualization, the normal Monod equation (Eq. 3.36) would apply for whichever nutrient was rate limiting and the concentration of the other would have no impact on  $\mu$ .

Only limited experimental evidence is available to support one model over the other. Bae and Rittmann<sup>7</sup> have shown both theoretically and experimentally that the interactive model is more appropriate when the two limiting constituents are the electron donor and acceptor. Furthermore, Bader<sup>6</sup> has compared the mathematical characteristics of the two expressions. The noninteractive model, by its very nature, causes a discontinuity at the transition from one nutrient limitation to another. It also predicts significantly higher growth rates in the region where  $S_{S1}/K_{S1}$  and  $S_{S2}/K_{S2}$  are small. The interactive model does not cause discontinuities, but may err on the side of predicting lower growth rates when  $S_{S1}/K_{S1}$  and  $S_{S2}/K_{S2}$  are both small. Both functions become asymptotically the same if either nutrient is present in excess. Finally, the interactive model is mathematically preferable for modeling dynamic situations because it is continuous.

Equation 3.46, the interactive model, will be adopted for use herein. There are three reasons for this choice. First is the evidence provided by Bae and Rittmann.<sup>7</sup> Second, for the type of situation likely to be encountered in biochemical operations for wastewater treatment, the interactive model is more conservative. Third, it works well when one nutrient is the electron donor (the substrate) and the other is the electron acceptor (oxygen or nitrate),<sup>105,113</sup> a common occurrence in wastewater treatment systems.

A special case of multiple nutrients occurs when an increase in the concentration of one nutrient acts to diminish microbial activity. For example, consider the

growth of heterotrophic bacteria under anoxic conditions. Because nitrate reduction can serve as an alternative to aerobic respiration, the enzymes involved in the transfer of electrons to nitrate and its reduced products are influenced negatively by the dissolved oxygen concentration and consideration must be given to this fact when expressing the kinetics of growth under anoxic conditions. Oxygen can have two effects: (1) it can repress the synthesis of denitrifying enzymes, and (2) it can inhibit their activity.<sup>24,67,94,118</sup> Although there are exceptions, as a general rule the presence of oxygen in the medium (and/or its active utilization as the terminal electron acceptor) represses the synthesis of the nitrate reducing enzyme system. When oxygen is absent, or is present in amounts that are insufficient to meet the needs of the culture, derepression occurs and the enzymes are synthesized. Complications occur, however, when the biomass is cycled between aerobic and anoxic conditions and this appears to alter the regulatory system so that some enzyme synthesis can continue at diminished rates even in the presence of dissolved oxygen.<sup>112</sup> The effect of oxygen on the activity of the enzymes depends on the bacterial species involved. In some, the activities are diminished in the presence of oxygen, whereas in others they are not. Nevertheless, it appears that inhibition of enzyme activity by oxygen is the primary mechanism influencing nitrate reduction rates in systems in which the bacteria are continually cycled between aerobic and anoxic conditions,<sup>112</sup> and that prior growth under anoxic conditions will provide enzymes which can function at diminished rate even in the presence of dissolved oxygen. One factor complicating the determination of the effects of oxygen on nitrate reduction in wastewater treatment systems is the necessity to grow the bacteria as flocculent cultures or as biofilms. Because diffusion is the only mechanism supplying oxygen to the bacteria in the interior of a floc particle or biofilm, some bacteria may be in an environment completely devoid of oxygen even when oxygen is present in the bulk liquid.<sup>68</sup>

Because of the complexity associated with the effects of dissolved oxygen on anoxic growth of heterotrophic bacteria, and because all effects have not been clearly defined, relatively simple models have been used to express them.<sup>11,54,55</sup> A popular approach has been to use Eq. 3.46 to depict the simultaneous effects of organic substrate ( $S_s$ ), and nitrate ( $S_{NO}$ ) on  $\mu$  and to add a third term which diminishes  $\mu$  as the dissolved oxygen concentration,  $S_o$ , increases:

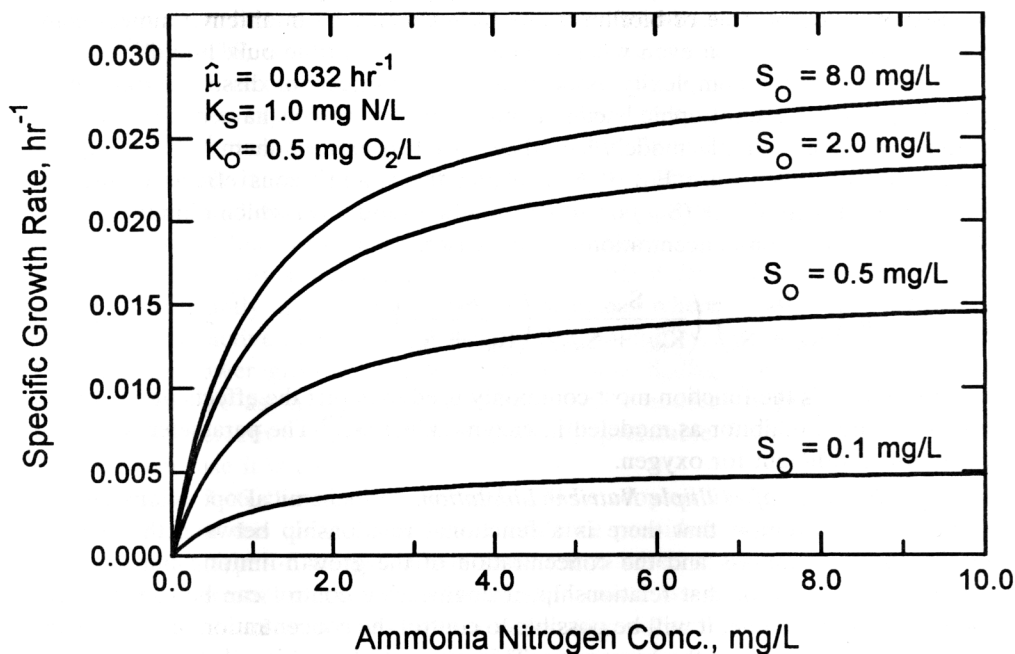
$$\mu = \hat{\mu} \left( \frac{S_s}{K_s + S_s} \right) \left( \frac{S_{NO}}{K_{NO} + S_{NO}} \right) \left( \frac{K_{IO}}{K_{IO} + S_o} \right) \quad (3.48)$$

The third term is the function most commonly used to depict the effects of a classical noncompetitive inhibitor as modeled in enzyme kinetics.<sup>125</sup> The parameter  $K_{IO}$  is the inhibition coefficient for oxygen.

**Implications of Multiple Nutrient Limitation.** Biochemical operations are designed on the premise that there is a functional relationship between the specific growth rate of biomass and the concentration of the growth-limiting nutrient in a bioreactor. Because of that relationship, if engineering control can be exerted over the specific growth rate, it will be possible to control the concentration of the growth-limiting nutrient leaving the bioreactor. This can only be achieved, however, if the nutrient the engineer wishes to control is the growth-limiting one. If the design objective is the removal of soluble organic matter, then all other nutrients must be supplied in excess. Or, if the goal is to remove nitrate-N by allowing it to serve as

the terminal electron acceptor, then it should be made rate limiting at the appropriate place in the process. A clear definition of the objective to be met must be combined with knowledge of the concentrations of the various constituents in the wastewater to ensure that the resultant biochemical operation can indeed meet that objective.

Because oxygen is a gas of very low solubility, it must be supplied continuously to aerobic systems and the concentration in solution will depend on the relative rates of supply and utilization. Furthermore, because oxygen transfer is one of the major costs associated with aerobic wastewater treatment, it is uneconomic to oversize the oxygen delivery system. As a consequence, it is not uncommon for the oxygen concentration to decrease sufficiently to make  $S_o/(K_o + S_o) < 1.0$  (where  $K_o$  is the half-saturation coefficient for dissolved oxygen). Thus, it would be instructive to examine the impact of this occurrence. Figure 3.3 illustrates the simultaneous limitation of the specific growth rate of autotrophic nitrifying bacteria by ammonia (the electron donor) and oxygen (the electron acceptor) using typical parameter values. These bacteria were chosen because they are more sensitive to dissolved oxygen concentration than heterotrophic bacteria, ( $K_{O,H} < K_{O,A}$ , where the subscripts H and A signify heterotrophic and autotrophic bacteria, respectively). Examination of Figure 3.3 reveals two things. First, if we could operate a bioreactor in a way that maintained a constant specific growth rate, decreasing the oxygen concentration in the bioreactor would cause the ammonia concentration to increase. Second, decreasing the oxygen concentration is analogous to decreasing  $\hat{\mu}$  for the bacteria. This can also be seen



**Figure 3.3** Double Monod plot showing the effects of both ammonia nitrogen and dissolved oxygen concentrations on the specific growth rate of autotrophic nitrifying bacteria. The parameter values given were used to construct the curves with Eq. 3.46.



by examining Eq. 3.46. The consequence of this is discussed in more detail in Chapter 6, but suffice it to say now that a decrease in  $\hat{\mu}$  makes it more difficult for the autotrophic bacteria to compete for space in the bioreactor.

Both nitrogen and phosphorus are required for the synthesis of new biomass. If those proper quantities are not present, balanced biomass growth cannot occur and treatment performance will be impaired. Thus, care must be exercised to provide sufficient quantities. We have just seen, however, that if the concentrations of essential nutrients are very low in a bioreactor they can become rate limiting, which is undesirable when the treatment objective is removal of organic matter. This means that the concentration of nitrogen or phosphorus supplied to a bioreactor must be sufficiently high to meet the synthesis needs of the biomass as defined by stoichiometry while leaving enough residual in solution to prevent their concentrations from being rate limiting. Goel and Gaudy<sup>39</sup> determined that  $K_s$  for ammonia nitrogen during normal heterotrophic growth lies between 1.5 and 4.0 mg/L as N. Using  $0.50 \text{ hr}^{-1}$  as a representative value for  $\hat{\mu}$ , it can be shown that if the influent nitrogen concentration exceeds the stoichiometric requirement by 1.0 mg/L as N, nitrogen will not be rate limiting to heterotrophic biomass at the specific growth rates normally employed in wastewater treatment. Although some work has been done on kinetic limitation of heterotrophs by phosphorus, the results are not as clear as those with nitrogen. Attempts to measure the limiting phosphorus concentration in both pure and mixed microbial cultures found it to be too low to detect with the techniques available at the time.<sup>107</sup> Consequently, if the concentration of phosphorus in the influent exceeds the stoichiometric amount by a few tenths of a mg/L as P, phosphorus should not be rate limiting. In some biochemical operations, the microorganisms pass through a growth cycle, and nutrients will be taken up in one phase and released in another. To prevent nutrient limitation during the phase of nutrient uptake, the amounts presented above should be in excess of the maximum quantity removed, not the net amount as determined by the final effluent.

### 3.2.10 Representative Kinetic Parameter Values for Major Microbial Groups

*Aerobic Growth of Heterotrophic Bacteria.* The values of the parameters  $\hat{\mu}_H$  and  $K_s$  are very dependent on the organism and substrate employed. If an axenic bacterial culture is grown on each of several substrates under fixed environmental conditions, the values of  $\hat{\mu}_H$  and  $K_s$  will vary from substrate to substrate. Likewise, if the same substrate is fed to each of several pure cultures, the values of  $\hat{\mu}_H$  and  $K_s$  will depend on the species of organism. This makes it very difficult to generalize about parameter values and care should be exercised in the use of values considered to be typical. It can be stated, however, that readily biodegradable substrates are characterized by high values of  $\hat{\mu}_H$  and low values of  $K_s$ , whereas slowly biodegradable substrates have low  $\hat{\mu}_H$  values and high  $K_s$  values. For example, benzoic acid had  $\hat{\mu}_H$  values between  $0.61$  and  $0.64 \text{ hr}^{-1}$  and  $K_s$  values between  $4.2$  and  $5.8 \text{ mg/L as COD}$ , whereas 2-chlorophenol had values of  $0.020$ – $0.025 \text{ hr}^{-1}$  and  $16$ – $17 \text{ mg/L as COD}$  for the two parameters.<sup>23</sup> Even lower  $K_s$  values have been reported for very easily degradable substrates, such as biogenic materials like carbohydrates and amino acids, with values as low as  $0.2 \text{ mg/L}$  for galactose and  $0.5 \text{ mg/L}$  for