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# If you want to use an inertial measurement system...

*... which technical data you should analyse and compare before making your decision*

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In fact, for unskilled users as well as for advanced users of inertial technology, it is often very difficult to get the right "feeling" which one of all the different provided inertial measurement systems or inertial navigation systems or attitude heading reference systems or inertial measurement units or inertial sensors will best and most economically meet your application requirements.

With this article we will help you to understand the physics behind those inertial navigation or inertial measurement systems and sensors and also to validate the datasheets of the vendors by yourself.

Using this information we hope we can help you making the best technical and economical decision – please don't hesitate to contact us for any further questions!

## Introduction into Inertial Measurement Technology:

Inertial guidance systems were originally developed for navigating rockets, today they are used in many applications from horizontal directional drilling up to space vehicle navigation. Today everybody is daily in touch with inertial technology, for example every modern car contains at least one gyro and two accelerometers for ESP (electronic stability program or more complex advanced driver assistance systems [ADAS]) or for the airbag control to make travelling as safe as possible even in difficult environment.

A typical inertial navigation system uses a combination of roll, pitch and azimuth gyroscopes, to stabilize the x, y and z accelerometers to solve a large set of differential equations to convert these readings into estimates of velocities, position and attitude, starting off from a known initial position of latitude and longitude.

Today's implementation of inertial navigation systems (INS) is typically in so-called strap-down technology, where all inertial sensors (gyros and accelerometers) are stiff mounted (strapped down) on the vehicle. In the past the systems had been designed in so-called gimballed technology, where the gyros had been used to stabilise the accelerometers mechanically in space. In strap-down systems the stabilisation is done methematically, and therefore all inertial sensors suffer the full vehicle's dynamics. Due to missing mechanical gimbals the strap-down systems are much more robust in operation than the gimballed systems.

All inertial navigation systems suffer from integration drift, as small errors in measurement are integrated into progressively larger errors in velocity and especially position. This is a problem that is inherent in every open loop control system.

Inertial navigation may also be used to supplement other navigation systems, providing a higher degree of accuracy than is possible with the use of any single navigation system. For example, if, in terrestrial use, the inertially tracked velocity is intermittently updated to zero by stopping, the position will remain precise for a much longer time, a so-called "zero velocity update" (ZUPT).

Control theory in general and Kalman filtering in particular, provide a theoretical framework for combining of the information from various sensors. One of the most common alternative sensors is a satellite navigation system such as GPS/GNSS.

**Dynamical Environment:** It is a big difference to operate an inertial measurement system in static or low dynamic environment or in the "real-world". Check the performance of the IMS (IMS = **i**nertial **m**easurement **s**ystem) for the environment you want to operate the system in. Will it be used on an aircraft (transportation aircraft, helicopter, drone or fighter?), on a rail vehicle, a passenger car or a truck or a tank, on a naval ship, a ferry or a speed boat or on an underwater surveying vehicle or inside of a missile or a torpedo? Or will it be used in a drilling application or in pipeline surveying or in machinery guidance?

Compare the conditions of the data sheet of the system and the conditions in your application:

E.g. will GPS be available in the way as it is assumed for the data in the data sheet of the system? What is the behavior of the system under coning motion, which is e.g. the typical motion for ship applications? How does the system's parameters influence the desired performance? What operation mode is required (free inertial navigation, aided navigation, surveying, ZUPT operation, control and guidance or something else...)?

Take into mind that, also if you only want to know the motion of one single axis (e.g. only roll angle), under dynamic conditions in general a three axes measuring system (3 angular rate sensors and 3 accelerometers) is required to achieve the specification of the application. In general it is not possible to calculate a single axis motion in multi-axes excitation (solution of a non-linear transformation differential equation based on quaternions or direction cosine matrix) with sufficient accuracy using a single axes gyro or using one high accurate gyro and two lower grade gyros. The motion error due to scalefactor errors of the inertial sensors is always dominated by the lowest performance gyro installed.

Take into consideration that a MEMS gyro (working on coriolis law using vibratory excitation) and mechanical gyros (DTG) show a so-called g-dependent drift, i.e. they produce a drift (angular rate offset) dependent on linear acceleration and environmental vibration influence. Optical gyros (FOG = fiber optic gyros, RLG = ring laser gyros) do not show such g-dependent drift.

**Gyro Bias:** If the system operates unaided (without odometer/velocity or GPS or magnetometer aiding), the gyro bias indicates the increase of angular error over time (in deg/h or deg/s). If the system is aided with speed information (e.g. odometer or Doppler log), the roll and pitch gyro drift can be compensated in the measurement system and the gyro drift mainly affects the heading accuracy over time. If the system consists of low drift gyros also the true heading can be estimated using gravity and earth rate information (so-called north-seeking).

If the system is aided with position information (e.g. GPS or GLONASS or GALILEO), also heading drift can be corrected and true heading can be provided (even with medium grade gyros). But of course the smaller the gyro drift the better all possible angular corrections and the longer the allowed time where the aiding information may be not present (e.g. GPS in urban canyons)!

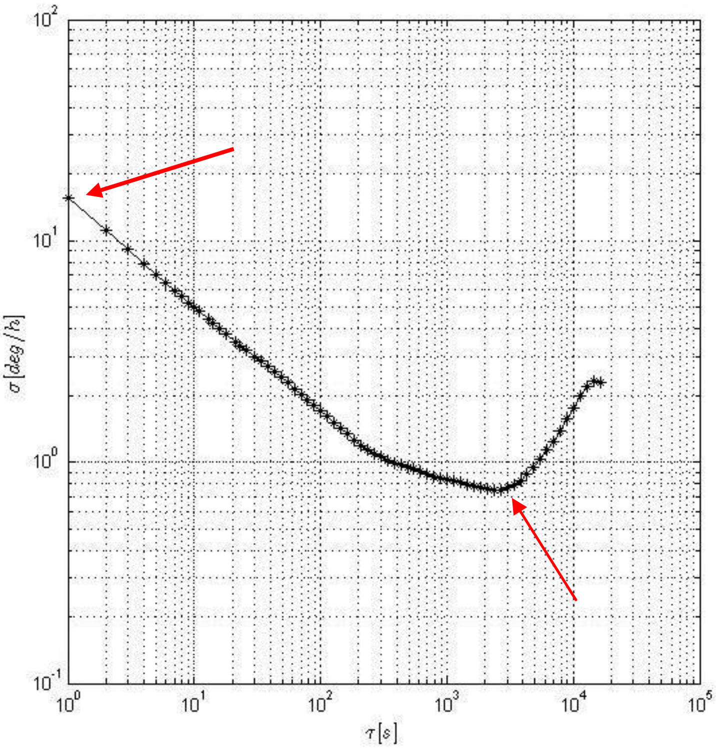
If the system is operated in free navigation mode, the gyro bias is responsible for the position and velocity error over time (so-called Schuler oscillation).

**Gyro Scale Factor Error:** This is an indication of the angular error which occurs during rotation. E.g. with 300 ppm scale factor error (=0.03%) the angular error is in the area of 0.1 degree after one revolution. With a laser gyro or high performance fiber optical gyro system with 10 ppm scale factor error the angular error is less than 1 arcsec (0.0003 deg) if the rotation angle is 30 deg.

**Misalignment:** A misalignment between the gyro axes (or accelerometer axes) causes a cross-coupling between the measurement axes. A misalignment of 0.1 mrad inside of the system (e.g. residual calibration mismatch) leads to a roll error of 0.036 degree during one revolution around the yaw axis (if the system is unaided). The smaller the required misalignment the higher the requirements to sensor performance and calibration equipment (e.g. iMAR's three-axes turn-tables).

**Accelerometer Offset:** An offset on the accelerometer leads to an error during alignment, i.e. determination of initial roll and pitch angle. An offset of 0.1 mg leads to approx. 0.006 degree angular error (attitude error). The sensor offsets can be estimated during operation by the system integrated Kalman filter, using GPS or DGPS data or ZUPT (zero velocity update procedure).

**Gyro Random Walk:** This value, given in deg/sqrt(hr), shows the noise of the used gyro. The higher the noise the more noise is measured on the angular rates and on the angles. Some manufacturers also specify it as the noise density in deg/h/sqrt(Hz). Both values are equivalent - if the second value is divided by 60, you get it in deg/sqrt(hr). An angular random walk of 0.003 deg/sqrt(hr) indicates, that the angular error (incertainty) due to random walk is e.g. 0.001 deg after 6 minutes (unaided) or 0.0004 deg after 1 minute (all values one sigma). The angular random walk is very important for the



accuracy of north seeking, because if the random walk decreases times 2 then the needed duration for north seeking decreases by times four (if the resolution of the gyro is high enough).

The plot of the Allan Variance shows the square-root ARW of a MEMS gyro graphically (take the value at 1 sec and divide it by sixty to obtain the ARW in [deg/sqrt(hr)]).

At 1 sec the value of the square-root of the AllanVariance is 15 deg/hr. This leads to a value of the Angular Random Walk (ARW) of 15/60 deg/sqrt(hr) = 0.25 deg/sqrt(hr) = 0.0042 deg/s/sqrt(Hz) = 15 deg/hr/sqrt(Hz) [white gyro noise assumed]. The bias stability (minimum point of the graph) is 0.8 deg/hr at a correlation time of 3'000 seconds. So it is really quite a good MEMS gyro which we have in use.

**Bandwidth:** In general the dynamic performance of an inertial measurement system is as better as higher the internal sampling rate and the bandwidth of the inertial sensors is. Also the proper internal data synchronisation is very important for accurate signal processing if the IMS is operated under difficult dynamical environment. A high precision internal time reference therefore is very important to be available inside of the IMS.

**Position error of an *unaided* INS:** We have to distinguish between short-time accuracy and long-time accuracy of an **i**nertial **n**avigation **s**ystem (INS).

### *Long-time accuracy of an unaided INS:*

This value (e.g. given in nm/hr i.e. nautical miles per hour) gives the global position error of an INS due to accelerometer errors and gyro errors, if the system is driven in a so-called Schuler loop operation. Then the position error oscillates with a period duration of approx 84 minutes. The amplitude of oscillation depends on the accelerometer offset and the "shift" (avarage of position drift) depends on gyro drift (simple model assumption; details can be seen from the inertial differential equations!).

To improve the long-time performance of position determination without aiding (no GPS, no odometer!), the system can be set to zero-velocity all x minutes (ZUPT, zero velocity update). During this stand-still period, which may take 10 seconds all 3 minutes (example), the Kalman filter is able to estimate the internal sensor errors of the gyros and accelerometers and can improve the position performance dramatically (e.g. position error over 70 km distance with iNAV-RQH-0018 has been shown to be 3 meters as an example).

### *Short-time accuracy of an unaided INS (free inertial navigation):*

This value (given in m or m/s) is important for measuring durations less than approx. 20...40 minutes, because Schuler oscillation is not really relevant for short time measurements. An accelerometer offset leads to an position error increasing quadratically over time

$$\text{delta\_s} = 0.5 \times \text{delta\_a} \times T^2 \quad [\text{m}] \qquad \text{(a)}$$

with delta_a = accelerometer offset and T = measuring time.

Example for a medium accurate system:

$$\text{delta\_a} = 1\ \text{mg} \approx 0.01\ \text{m/s}^2,\ T = 100\ \text{sec} \Rightarrow \text{delta\_s} = 50\ \text{m}$$

The gyro drift delta_omega affects the position error corresponding to the equation

$$\text{delta\_s} = g/6 \times \text{delta\_omega} \times T^3 \quad [\text{m}] \qquad \text{(b)}$$

with delta_omega in [rad/s] and $g = 9.81\ \text{m/s}^2$ .

An attitude (roll/pitch) error of e.g. delta_attitude affects the position error due to a wrong compensation of the gravity on the horizontal IMS axes:

$$\text{delta\_s} = 0.5 \times g \times \sin(\text{delta\_attitude}) \times T^2 \quad [\text{m}] \qquad \text{(c)}$$

### Example, how you can validate manufacturer's statements (with data from a vendor's datasheet):

If someone promotes an IMS with 0.005 deg roll/pitch accuracy and advertises a horizontal position error of 0.7 m (and a vertical position error of only 0.5 m) after 300 seconds in free inertial navigation mode (i.e. without odometer aiding, without ZUPT; without internal vibration isolators), you can just check and calculate two things with the simple thumb rule equations given above:

- Position error due to 0.005 deg roll or pitch error after 300 sec:
  $0.5 \times 9.81\ \text{m/s}^2 \times \sin(0.005°) \times (300\ \text{sec})^2 = 38\ \text{m}$ (from equ. (c))

- What must be the accelerometer accuracy to achieve 0.7 m after 300 sec?
  $0.7\ \text{m} / (0.5 \times (300\ \text{sec})^2) = 1.5\ \mu\text{g}$ (!!) absolute accuracy over 300 sec
  (from equ. (a))

The easy calculation shows the mismatch of the announced performance data (i.e. position error must be much worse or attitude error must be much smaller to achieve the advertised performance). For information: An absolute accuracy of accelerometers of 1.5 µg is close to gravimeter accuracy but not reliable available in industrial or military land navigation systems. Consider, that already the gravity by itself changes by 0.3 µg per meter of height !

**Position error of an *aided* INS:** If the INS is aided, we have to distinguish between position aiding (e.g. by GPS/GLONASS) and velocity aiding (e.g. by Doppler velocity log or odometer/wheel sensor or GPS Doppler velocity).

### *Position aiding:*

The INS provides accurate measurements of high dynamic position changes while it shows significant position drift over long-time measurements. GPS e.g. provides position information with high noise and low data rate, but the position error does not increase over measuring time.
Therefore, using a Kalman filter approach, the short-time accurate INS can be coupled with a long time accurate position reference system (e.g. GPS). The Kalman filter typically is adapted to the application (e.g. number of

states, setting of covariances, stability control and supervision technics). In such solution high dynamics will be provided with excellent so-called neighborhood accuracy, but the global position error can never be better than the global position error of the position aiding system. E.g. if GPS shows a constant position error over a longer time, also the INS/GPS solution will follow those position error. But using different sources of aiding (GPS, ZUPT, odometer) the total position error can be minimized.

***Velocity aiding:***
If velocity is provided for aiding (e.g. from Doppler velocity log) instead of position, the position error of the total Kalman filter solution will grow with the scale factor error of the velocity aiding sensor.

**Open Interfaces:** Open interfaces are very important for the user to have highest flexibility in using the system. Interfaces are user-interfaces as well as interfaces to external sensors like GPS, odometer, depth/altitude sensor etc. The system's architecture should also provide custom specific interfaces if required. For higher volume markets the system shall be designed directly to these applications to meet the economical demands of those applications.

Also a lot of other features have important influence on the performance of an inertial measurement system. If you have additional questions please do not hesitate to contact us for further information.

Additional information can be found on our download site at www.imar-navigation.de

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