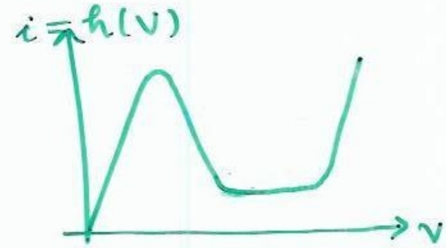
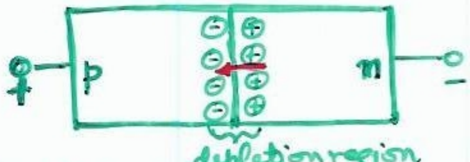
# Tunnel Diode Circuit



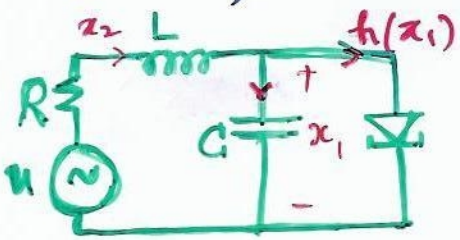

Voltage vs. current characteristics of a tunnel diode is as shown in figure. Note there is portion with -ve slope $\Rightarrow$ -ve resistance. Tunnel diodes created by heavily doping p-n junction, which creates a narrow depletion region.




{ holes from p side move to n side
{ electrons from n side move to p side

p-type impurity (group 3 elements, B, In, Al) on one side and n-type impurity (group 5 elements, P, As, Sb) on other create step discontinuity in hole-electron concentration. Diffusion move holes/electrons across boundary, which sets up electric field against diffusion current, eventually stopping it.

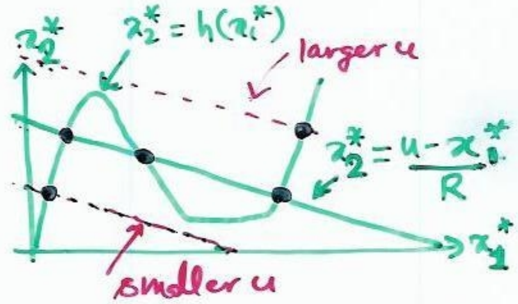
A -ve potential adds to this in-built electric field and so no current flows under "reverse bias". +ve potential has to first overcome this in-built electric field before current starts to flow in "forward bias".
Normally this forward-bias current increases as applied voltage is increased, but in tunnel diode there exists a voltage range over which current starts to decrease, called tunneling effect.




States: voltage across capacitor, $x_1$
current through inductor, $x_2$

$$C\frac{dx_1}{dt} = x_2 - h(x_1)$$

$$L\frac{dx_2}{dt} = u - Rx_2 - x_1$$

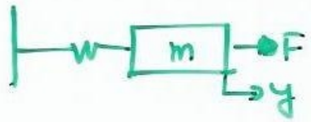



At equilibrium, $\left.\begin{aligned} x_2^* - h(x_1^*) &= 0 \\ u - Rx_2^* - x_1^* &= 0 \end{aligned}\right\} \Rightarrow$

At certain u, 3 equilibrium points, and at others only one eq. point.

How to decide which eq. point system would occupy when there are multiple isolate eq. points? Depends on initial condition, input, and nature of equilibrium.

# Mass- spring system




$$m\ddot{y} = F - F_{sp} - F_f$$

spring force $F_{sp} = g(y) = \begin{cases} ky & \text{for small } y \\ k(1-a^2y^2)y & \text{softening spring} \\ k(1+a^2y^2)y & \text{hardening spring} \end{cases}$

Friction force $F_f = F_{static} + F_{coulomb} + F_{viscous}$


$|F_{static}| \le \mu_s mg$ $(\dot{y}=0)$ $\quad F_{coulomb} = -\mu_k mg\, sgn(\dot{y})$ $(\dot{y}\neq 0)$ $\quad F_{viscous} = c\dot{y}$

$$m\ddot{y} = (F - ky) - c\dot{y} - \eta(y,\dot{y}), \qquad \eta(y,\dot{y}) = \begin{cases} \mu_k mg\, sgn(\dot{y}) & \dot{y}\neq 0 \\ F-ky & \dot{y}=0,\ |F-ky|\le \mu_s mg \\ \mu_s mg\, sgn(F-ky) & \dot{y}=0,\ |F-ky| > \mu_s mg \end{cases}$$

$$x_1 = y,\ x_2 = \dot{y},\ F=0 \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}\left[-kx_1 - cx_2 - \eta(x_1,x_2)\right] \end{cases}$$

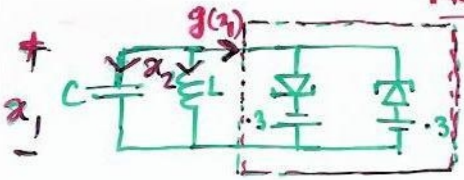
$$\begin{matrix} x_2 = 0 \\ |kx_1| \le \mu_s mg \end{matrix} \Rightarrow \begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = 0 \end{cases} \qquad \begin{matrix} x_2 = 0 \\ |kx_1| > \mu_s mg \end{matrix} \Rightarrow \begin{cases} \dot{x}_1 = 0 \\ \dot{x}_2 = \frac{1}{m}\left[-kx_1 + \mu_s mg\, sgn(x_1)\right] \end{cases}$$

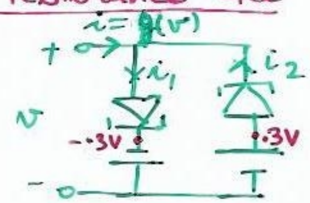
(two subcases: $x_1 > 0$ or $x_1 < 0$)

$$x_2 \neq 0 \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{m}\left[-kx_1 - cx_2 - \mu_k mg\, sgn(x_2)\right] \end{cases}$$

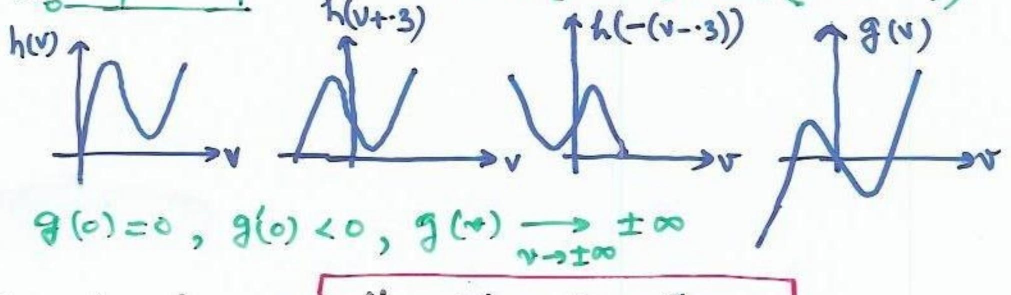
(two subcases: $x_2 > 0$ or $x_2 < 0$)

# Negative resistance ckt




$$i = i_1 - i_2$$
$$= h(v + .3) - h(.3 - v)$$
$$= h(v + .3) - h(-(v - .3))$$




$$g(0) = 0, \quad g'(0) < 0, \quad g(v) \xrightarrow[v\to\pm\infty]{} \pm\infty$$

$$C\frac{dx_1}{dt} + x_2 + g(x_1) = 0$$

$$L\frac{dx_2}{dt} = x_1$$

$$\Rightarrow C\ddot{x}_1 = -\dot{x}_2 - g'(x_1)\dot{x}_1 = -\frac{x_1}{L} - g'(x_1)\dot{x}_1 \Rightarrow \boxed{C\ddot{x}_1 + g'(x_1)\dot{x}_1 + \frac{x_1}{L} = 0}$$

## Negative resistance Ckt (cont'd.)

$$C\ddot{x}_1 + g'(x_1)\dot{x}_1 + \frac{x_1}{L} = 0$$

Set $t = \tau\sqrt{LC}$ $\Rightarrow \left\{ \begin{array}{l} \dfrac{dx}{dt} = \dfrac{dx}{d\tau}\cdot\dfrac{d\tau}{dt} = \dfrac{dx}{d\tau}\cdot\dfrac{1}{\sqrt{LC}} \\ \dfrac{d^2x}{dt^2} = \dfrac{d^2x}{d\tau^2}\cdot\dfrac{1}{LC} \end{array} \right\}$

With this rescaling of time, $\dfrac{C}{LC}\dfrac{d^2x_1}{d\tau^2} + g'(x_1)\dfrac{dx}{d\tau}\cdot\dfrac{1}{\sqrt{LC}} + \dfrac{x_1}{L} = 0$

$$\Rightarrow \frac{d^2x_1}{d\tau^2} + g'(x_1)\frac{dx}{d\tau}\sqrt{\frac{L}{C}} + x_1 = 0$$

$$\Rightarrow \ddot{x}_1 + \varepsilon g'(x_1)\dot{x}_1 + x_1 = 0 \qquad \left(\varepsilon = \sqrt{\tfrac{L}{C}}\right)$$

$g(x_1) = -x_1 + \frac{1}{3}x_1^3 \Rightarrow g'(x_1) = -1 + x_1^2$

$$\Rightarrow \boxed{\ddot{x}_1 - \varepsilon(1-x_1^2)\dot{x}_1 + x_1 = 0} \quad \text{Van der Pol equation}$$

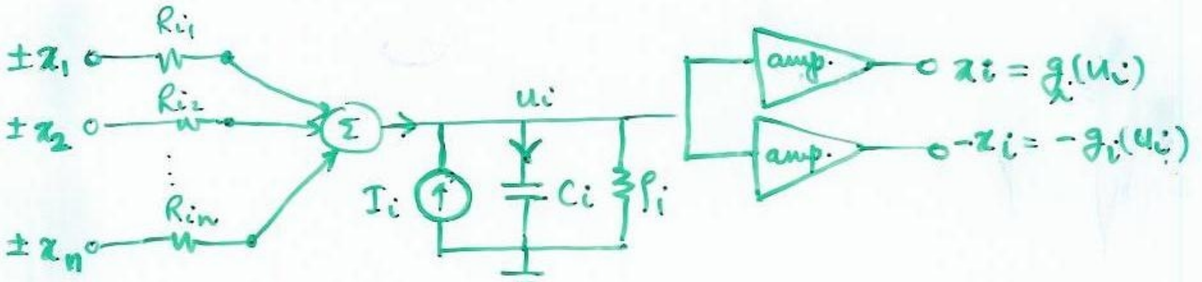
Van der Pol used above equation to study oscillations in vacuum tubes.

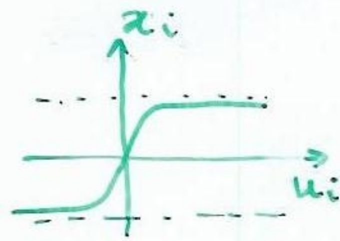
Equilibrium point: $\left.\begin{array}{l} 0 = -x_2^* - g(x_1^*) \\ 0 = x_1^* \end{array}\right\} \Rightarrow x_1^* = 0 \Rightarrow g(x_1^*) = 0 \Rightarrow x_2^* = 0.$

Van der Pol eq. possesses a unique eq. point. It also possesses a periodic solution (limit cycle) that attracts every other solution except the zero solution.

## Hopfield Network

Hopfield network is a type of artificial neural network, which are biologically inspired model of distributed processing.




$C_i, \rho_i$ : capacitance, resistance at input of amplifiers.
$I_i$ : constant input current.

$$C_i\frac{du_i}{dt} = \sum_j \frac{1}{R_{ij}}(\pm x_j - u_i) + I_i - \frac{u_i}{\rho_i} = \sum_j \frac{\pm x_j}{R_{ij}} + I_i - \frac{u_i}{R_i}$$

$$\left(\frac{1}{R_i} = \frac{1}{\rho_i} + \sum_j \frac{1}{R_{ij}}\right)$$

# Hopfield Network (ctnd.)

$$x_i = g_i(u_i) \Rightarrow \dot{x}_i = g_i' \dot{u}_i$$

$$= \frac{1}{C} g_i' \left[ \sum_j \frac{\pm x_j}{R_{ij}} - \frac{u_i}{R_i} + I_i \right]$$

Let $g_i'(u_i)\Big|_{u_i = g_i^{-1}(x_i)} = h_i(x_i)$, then

$$\boxed{\dot{x}_i = \frac{1}{C} h_i(x_i) \left[ \sum_j \frac{\pm x_j}{R_{ij}} - \frac{g_i^{-1}(x_i)}{R_i} + I_i \right]}$$ State-eq.

Due to nature of $g_i$, $g_i' > 0 \Rightarrow h_i > 0$. So equilibruim is given by,

$$0 = \sum_j \pm \frac{x_j}{R_{ij}} - \frac{g_i^{-1}(x_i)}{R_i} + I_i$$