Examples of proofs in 1st order logic

- $\forall x: (Q(x) \to R(x)),\ \exists x: (P(x) \land Q(x)) \to \exists x: (P(x) \land R(x))$

1.	$\forall x: (Q(x) \to R(x))$		premise
2.	$\exists x: (P(x) \land Q(x))$		premise
3.		$[P(a) \land Q(a)]$	assume $x = a$ in 2
4.		$Q(a) \to R(a)$	$\forall$- elimination in 1
5.		$P(a)$	$\land$ - elimination in 3
6.		$Q(a)$	$\land$- elimination in 3
7.		$R(a)$	MP in 4 & 6
8.		$P(a) \land R(a)$	$\land$- intro in 5 & 7
9.	$\exists x: (P(x) \land R(x))$		$\exists$ - intro. in 8

("a" in $P(a) \land R(a)$ is assumed.)

- $\exists x: P(x),\ \forall x \forall y: (P(x) \to Q(y)) \to \forall y: Q(y)$

1.	$\exists x: P(x)$		premise
2.	$\forall x \forall y: (P(x) \to Q(y))$		premise
3.		$[P(a)]$	assume $x = a$ in 1
4.		$\forall y: P(a) \to Q(y)$	$\forall$ elimination in 2
5.		$P(a) \to Q(b)$	$\forall$ elimination in 4.
6.		$Q(b)$	MP 3 & 5
7.	$\forall y: Q(y)$		$\forall$- intro. in 6

("b" in $Q(b)$ is arbitrary.)

- $\neg \forall x: P(x) \to \exists x: \neg P(x)$

1.	$\neg \forall x: P(x)$			premise
2.		$[\neg \exists x: \neg P(x)]$		assume
3.			$[\neg P(a)]$	assume
4.			$\exists x: \neg P(x)$	$\exists$ - intro in 3
5.		$P(a)$		2, 3 & 4 (reduction to absurd)
6.		$\forall x: P(x)$		$\forall$- intro
7.	$\exists x: \neg P(x)$			1, 2, 6 (reduction to absurd)