

Examples of expressions in predicate logic

1. No books are gaseous. Dictionaries are books. Therefore, no dictionary is gaseous.

$B(x)$: x is book; $G(x)$: x is gaseous; $D(x)$: x is dictionary.

$$\neg \exists (B(x) \wedge G(x)), \forall x (D(x) \rightarrow B(x))$$

$$\underline{\neg \exists x (D(x) \wedge G(x))}$$

2. Every child is younger than its mother.

$C(x)$: x is child; $M(y, x)$: y is x 's mother, $Y(x, y)$: x younger than y .

$$\forall x \forall y (C(x) \wedge M(y, x) \rightarrow Y(x, y))$$

3. Andy & Paul have same maternal grandmother.

prop. variables 'a' for Andy, 'p' for Paul

$$\forall x \forall y \forall u \forall v (M(x, a) \wedge M(y, x) \wedge M(u, p) \wedge M(v, u) \rightarrow y = v)$$

'Functions' (instead of predicates) can be used to simplify.

$m(x)$ returns mother of x .

$$2.' \forall x (C(x) \rightarrow Y(x, m(x)))$$

$$3.' m(m(a)) = m(m(p)).$$

Functions can be used if the "return-value" is unique.

($m(x)$ fine for "mother", $b(x)$ won't work for "brother")

For "brother" use a predicate/relation, $B(x, y)$: x brother of y .

4. Andy likes one of Paul's brother.

$$\exists x (B(x, p) \wedge L(a, x)) \quad L(x, y): x \text{ likes } y.$$

5. Andy likes all of Paul's brothers.

$$\forall x (B(x, p) \wedge L(a, x)).$$

6. Every son of my mother is my brother.

$$\forall x \forall y (M(x, i) \wedge S(y, x) \rightarrow B(y, i))$$

$S(y, x)$: y son of x ; i : proposition 'me' or 'I'.

$$\forall x (S(x, m(i)) \rightarrow B(x, i))$$