

Minimal DNF's (For digital ckt realization)

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• $f \equiv C_1 \vee C_2 \vee \dots \vee C_n$

$C_i \equiv (l_{i1} \wedge l_{i2} \dots \wedge l_{in})$

• Pick any pair of clauses that differ in one literal and simplify:

(i) $(g \wedge l) \vee (g \wedge \neg l)$ (differ in literal l)
 $\equiv g \wedge (l \vee \neg l) \equiv g \wedge \text{TRUE} \equiv g$

(ii) $(g \wedge l) \vee g \equiv g$ (again differ in literal l)

• Repeat above step recursively until no more simplification possible

• Pick minimal no. of clauses that "cover" original formula f .

Example: $wxy\bar{z} + w\bar{x}y\bar{z} + w\bar{x}y\bar{z} + \bar{w}x\bar{y}z + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}\bar{y}z$

• pairs to combine: $(1,3), (2,3), (2,6), (4,5), (4,6), (5,7), (6,7)$
 $\equiv w\bar{y}\bar{z} + w\bar{x}y + \bar{x}y\bar{z} + \bar{w}xz + \bar{w}y\bar{z} + \bar{w}\bar{y}z + \bar{w}\bar{x}z$

• pairs to combine: $((4,5), (6,7)), ((4,6), (5,7))$
 $\equiv w\bar{y}\bar{z} + w\bar{x}y + \bar{x}y\bar{z} + \bar{w}z + \bar{w}z$

$\equiv w\bar{y}\bar{z} + w\bar{x}y + \bar{x}y\bar{z} + \bar{w}z$ (no more reduction possible)

• Covers: $(1,3) \quad (2,6) \quad (2,6) \quad (4,5,6,7)$

• Minimal covers: $\begin{cases} w\bar{y}\bar{z} + \bar{x}y\bar{z} + \bar{w}z \\ w\bar{y}\bar{z} + w\bar{x}y + \bar{w}z \end{cases}$

Same technique can be used to obtain minimal CNF:

• Pick a pair of clauses that differ in one literal and simplify:

(i) $(g \vee l) \wedge (g \vee \neg l) \equiv g \vee (l \wedge \neg l) \equiv g \vee \text{FALSE} \equiv g$

(ii) $(g \vee l) \wedge g \equiv g$

Start DNF off truth table has less no. of TRUE entries than FALSE.