CNF - Satisfiability is hard problem

- It is known that 3-SATISFIABILITY is NP-Complete.
  (3-SATISFIABILITY $\Rightarrow$ All clauses have exactly 3-literals).
- NP-complete $\equiv$ { Known exp. algorithm ;
  NP { No known polynomial algorithm
  Given a guess solution can verify validity of it polynomially
  Any NP problem polynomially transformable to this problem
- Piegeon-hole principle : $(n+1)$-pigeons cannot be placed in n-holes.
- $P_{ij} = 1$ iff i^{th} pigeon placed in j^{th} hole
- For each $1 \le i \le n+1$, following clause should hold

  $c_i = p_{i1} \vee p_{i2} \cdots \vee p_{in}$ (pigeon i placed in one of the holes)

- For each $1 \le i, j \le n+1$, $1 \le k \le n$, following clause should hold

  $\bar{c}_{ijk} = \neg p_{ik} \vee \neg p_{jk}$ (pigeon i and j not in hole k together)

- $f = (\wedge c_i \quad \wedge \bar{c}_{ijk})$ has $(n+1) + \binom{n+1}{2} n = O(n^3)$ clauses
- Clearly f not satisfiable. But if we omit any single clause from f, it becomes satisfiable!