

Flatten & split operations of PVS

• Notation:

$$\frac{\begin{array}{c} \alpha_1 \\ \vdots \\ \alpha_n \\ \hline \beta_1 \\ \vdots \\ \beta_m \end{array}}{\quad} \equiv \underbrace{\alpha_1 \wedge \dots \wedge \alpha_n}_{\text{premise, CNF}} \rightarrow \underbrace{\beta_1 \vee \dots \vee \beta_m}_{\text{conclusion, DNF}}$$

- Flatten and split are used to simplify individual clauses of premise as well as conclusion.
- Flatten applied to simplify conjunction appearing in premise, disjunction in conclusion and negation either place.
- Split applied to simplify disjunction in premise, conjunction in conclusion.

• Flattening rules:

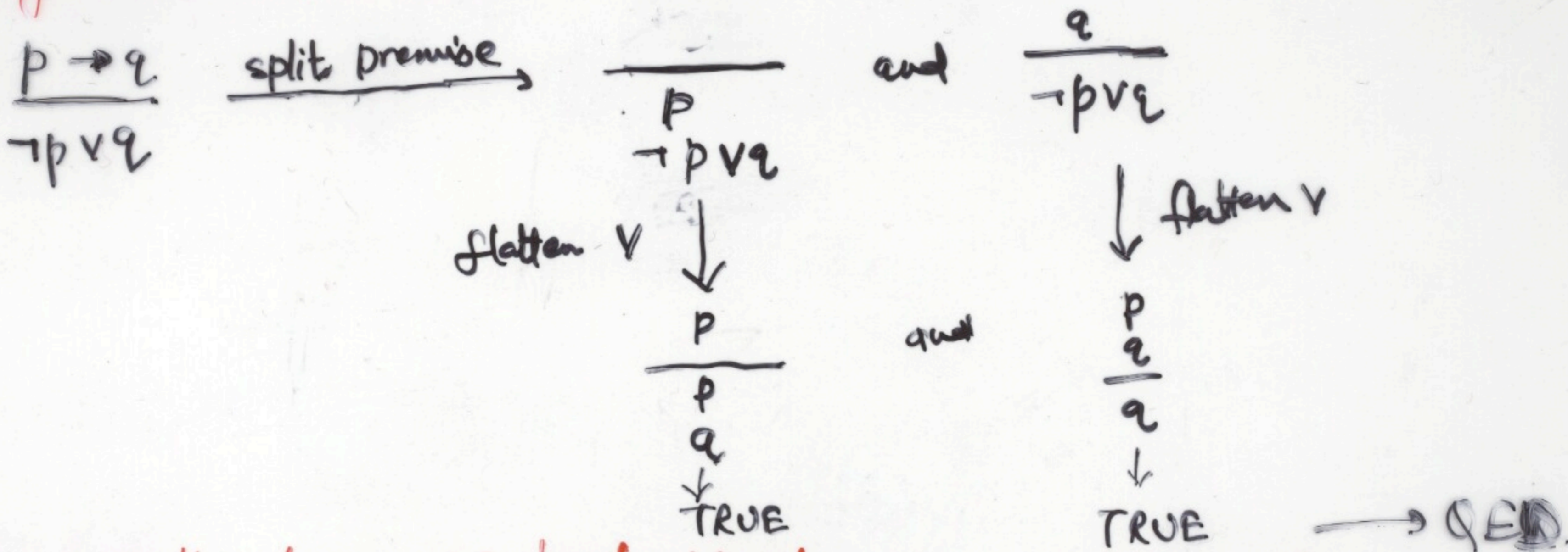
$$\begin{array}{l} \textcircled{1} \frac{\neg \alpha}{\quad} \equiv \alpha \quad \textcircled{2} \frac{\neg \neg \alpha}{\quad} \equiv \alpha \\ \textcircled{3} \frac{\alpha_1 \wedge \alpha_2}{\quad} \equiv \frac{\alpha_1}{\quad} \quad \textcircled{4} \left\{ \begin{array}{l} \frac{\beta_1 \vee \beta_2}{\quad} \equiv \frac{\beta_1}{\quad} \\ \frac{\beta_1}{\quad} \equiv \frac{\beta_1}{\quad} \\ \frac{\beta_1 \rightarrow \beta_2}{\quad} \equiv \frac{\beta_1}{\quad} \\ \frac{\beta_2}{\quad} \end{array} \right. \end{array}$$

• Splitting rules:

$$\textcircled{1} \left\{ \begin{array}{l} \frac{\alpha_1 \vee \alpha_2}{\quad} \equiv \frac{\alpha_1}{\quad} \text{ and } \frac{\alpha_2}{\quad} \\ \frac{\alpha_1 \rightarrow \alpha_2}{\quad} \equiv \frac{\alpha_1}{\quad} \text{ and } \frac{\alpha_2}{\quad} \end{array} \right. \quad \textcircled{2} \frac{\neg(\alpha_1 \wedge \alpha_2)}{\quad} \equiv \frac{\neg \alpha_1}{\quad} \text{ and } \frac{\neg \alpha_2}{\quad}$$

• ~~Using~~ a sequence of flatten and split operations can be used to prove any Boolean formula, which is what "prop" does automatically.

Example:



- Split will present the two new proof obligations in a certain order. If we rather do the 2nd one first, then use the (postpone) command on 1st one.