Normal forms

- Any propositional formula can be written in either
  i) conjunctive normal form (CNF), or
  ii) Disjunctive normal form
- Literal $\equiv$ propositional variable or its negation
- **CNF** $\equiv$ conjunction of disjunctions of literals

$$f \equiv C_1 \wedge C_2 \ldots \wedge C_n$$
$$C_i \equiv l_{i1} \vee l_{i2} \ldots \vee l_{in_i}$$

- **DNF** $\equiv$ Disjunction of conjunctions of literals

$$f \equiv C_1 \vee C_2 \ldots \vee C_n$$
$$C_i \equiv l_{i1} \wedge l_{i2} \ldots \wedge l_{in_i}$$

- Example: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$$\equiv (\neg p \vee q) \wedge (\neg q \vee p) \quad \leftarrow \text{CNF}$$
$$\equiv (\neg p \wedge \neg q) \vee \underbrace{(\neg p \wedge p)}_{\text{FALSE}} \vee \underbrace{(q \wedge \neg q)}_{\text{FALSE}} \vee (q \wedge p)$$
$$\equiv (\neg p \wedge \neg q) \vee (p \wedge q) \quad \leftarrow \text{DNF}$$

- DNF can be easily read from truth table:

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

From truth table: $p \leftrightarrow q \equiv \underbrace{(p \wedge q)}_{1^{st}\text{ row}} \vee \underbrace{(\neg p \wedge \neg q)}_{4^{th}\text{ row}}$