Effect of parantheses/functional view/Equivalence

- Parantheses can change value of propositional formula

$$(p \lor q) \land r \qquad \text{vs.} \qquad p \lor (q \land r)$$

When $p = TRUE$, then $p \lor (q \land r)$ is TRUE   (regardless of q, r)

When $r = FALSE$, then $(p \lor q) \land r$ is FALSE   (regardless of p, r)

Thus the values of $(p \lor q) \land r$ and $p \lor (q \land r)$ are different

for example when $p = TRUE$ and $r = FALSE$.

Since q can be arbitrarily chosen in above

- Viewing Boolean operators $(\land, \lor, \neg)$ as functions:

  Let $B = \{TRUE, FALSE\}$, then

$$\land : B \times B \longrightarrow B$$

$$\lor : B \times B \longrightarrow B$$

$$\neg : B \longrightarrow B$$

- Equivalence: Propositional formulae f_1 and f_2 are equivalent if they take the same value for same evaluations:

Recall, $\quad f := T \mid F \mid p \mid \neg f \mid f_1 \land f_2 \mid f_1 \lor f_2$

$$f, g : B^n \longrightarrow B \qquad \text{(for some } n\text{)}$$

Then $\quad f \equiv g \quad$ iff $\quad f(\vec{e}) = g(\vec{e}) \quad$ for all $\vec{e} \in B^n$.