Segmentation

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- Read Sections 5.1,5.2,5.3 of [1]
- Edge detection and filtering : Canny edge detection algorithm to get a contour of the object boundary
- Hough transform: looks for a particular type of curve (defined by an equation with a few unknown parameters, e.g. circle with unknown center and radius).
- Histogramming + Thresholding
- K-means clustering (special case of alternating minimization).
- Region growing, region split-merge
- Last 3 techniques return only the segmented regions. To get the boundary contour, use a "contour tracing" algorithm
- Energy minimization methods

Energy minimization: Parametric Active Contours

- Read Section 8.2 of [1].
- First work: Kass-Witkin-Terzopolous ICJV 1987
- Disadvantages: cannot deal with changes in contour length (occurs due to change in size or increase in concavities), some points come too close and some go too far away when contour length changes significantly or change in contour topology (splitting merging of contours)
- Some other solutions: Snake growing, Topological Snakes, Adaptive B-splines

Geometric Active Contours

- Read relevant parts of Chapter 1 of [2]
- Depends on the geometry of the curve, not the specific parametrization chosen
- A "geometric" quantity is a function of arc-length and derivatives of the contour w.r.t. it, e.g. curvature
- Arc-length parametrization: unit velocity parametrization, $||\frac{\partial C}{\partial s}|| = 1$
- Converting any parametrization to arc-length parametrization: $ds = ||C_p|| dp$
- Definitions of normal, tangent, curvature
- If energy is geometric, then its gradient w.r.t. the contour is also geometric: can be expressed in terms of the level-set function

Energy Minimization: Contour Evolution

- Read relevant parts of Chapter 1 of [2]
- Use gradient descent to minimize the energy: i.e. move contour in negative direction of gradient of energy functional w.r.t. the contour evaluated at current contour. Re-written as a PDE with an artificial time parameter (to denote gradient descent iteration): $\frac{\partial C}{\partial t} = -\nabla_C E(C)$, run it until RHS close to zero.
- Image based energies: Edge based or Region based or use other image features e.g. texture, intensity histograms etc
- Shape prior based energy terms

Edge based energy example

- Read Sections 2 and 3 of [3]
- Given

$$E(C) = \int_{s=0}^{L(C)} g(||\nabla I||)ds \tag{1}$$

then can show that gradient is

$$\nabla_C E = g(||\nabla I(C)||)\kappa N - (\nabla g \cdot N)N \tag{2}$$

where $L(C)$ denotes length of contour, N denotes the normal and κ denotes curvature.

- Main idea: *In the expression for E, the integral is over a region ($s = 0$ to $s = L(C)$) that depends on the contour itself. Hence cannot apply the* *standard calculus of variations formulas derived earlier. Solution: Assume an arbitrary fixed parametrization $C(p)$ with $p \in [0,1]$ and $C(0) = C(1)$. Then $ds = ||C_p||dp$ and the integral runs from $p = 0$ to $p = 1$. Then apply the standard calculus of variations formulas*

Two things to note in the above are

1. When parameterizing contours by arc-length, s, the definition of "inner product" is: For 2 vectors $h_1(s)$ and $h_2(s)$ the inner product is $h_1 \cdot h_2 = \int_{s=0}^{L(C)} h_1(s)h_2(s)ds = \int_{p=0}^{1} h_1(p)h_2(p)||C_p||dp$.
2. Also note that there is abuse of notation when I also define the inner product in $\mathbb{R}^2$ by the same "$\cdot$" notation: for e.g. $\nabla g \cdot N$ is an inner product in $\mathbb{R}^2$.

Region based energy example

- Read Appendix A.1 of [4]
- Given

$$E(C) = \int_{C_{inside}} (I(x,y) - u)^2 dxdy + \int_{C_{outside}} (I(x,y) - u)^2 dxdy + \alpha \int_{s=0}^{L(C)}$$

can show that

$$\nabla_C E = (u - v)(I(C) - \frac{u+v}{2})N + \alpha \kappa N \tag{4}$$

NOTE: There may be a minus sign in the above, please verify.

- Main idea: *Above is a special case of a general region based energy* *functional of the form:*

$$E(C) = \int_{C_{inside}} f(x,y)dxdy \tag{5}$$

The expression for E *is a region integral depending on the contour. First convert it to a boundary integral over the contour boundary (using the divergence theorem:* $\int_R (\nabla \cdot F)(x,y)dxdy = \int_{s=0}^{L(C)} (F(C) \cdot N)ds$ *where* C *is the boundary of region* $R = C_{inside}$ *and choose* F *so that* $\nabla \cdot F = f$*). Then use the same idea as described for edge based example.*

Level Set Method

- Read Section 2.2 of [2] and Section 5 of [4]
- A method of representing contours implicitly: as a zero level set of a two-dim function.
- Converting any geometric contour evolution to level set function evolution
- Signed distance function: most commonly used level set function.
- Defining a level-set function (signed distance function) about a contour
- Defining Extension velocities
- Finding zero level set of a level-set function.
- Narrowband method

References

[1] M. Sonka, V. Hlevac, and R. Boyle, *Image Processing, Analysis and Computer Vision*, Brooks/Cole, Pacific Grove, CA, 1999.

[2] G.Sapiro, *Geometric Partial Differential Equations and Image Analysis*, Cambridge University Press, 2001.

[3] Satyanad Kichenassamy, Arun Kumar, Peter J. Olver, Allen Tannenbaum, and Anthony J. Yezzi, “Gradient flows and geometric active contour models,” in *ICCV*, 1995, pp. 810–815.

[4] A Yezzi, A Tsai, and A Willsky, “A statistical approach to curve evolution for image segmentation,” Tech. Rep., MIT LIDS, January 1999.