Registration and Landmark Shape Analysis

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Registration

- Given a set of corresponding points in 2 frames, find a "global map" (usually a group action) that takes one set of points to the other

- If given two segmented object regions, find the corresponding points at the segmentation boundaries using optical flow (Least Squares optical flow method on 3×3 blocks will suffice). Or use curvature vertices (extrema and discontinuities) of the contour as the corresponding points.

- Global map can be affine, scaled Euclidean, Euclidean or translation group depending on the camera model and motion assumptions.

- Solve for the group action parameters using Least Squares Estimation (or its various modifications).

- When corresponding points not available, use an alternating

minimization algorithm: use initial guess of group action to guess location of the corresponding points in the second frame. Find the corresponding point in the vicinity of the guessed location (or find the new segmentation boundary in the vicinity of the guessed boundary). Iterate a few times. One such solution is given in [1].

- Affine registration: 6 parameter group. Need at least 3 corresponding points to get AN estimate. Usually need many more to get a robust least squares estimate.

- Scaled Euclidean registration: defines "landmark shape"

  - Use theory of landmark shape analysis to solve the least squares registration problem (find scale, rotation angle and x and y translation).

  - Represent the x and y coordinates of all the points in a frame as a complex vector with x coordinates forming the real part and y coordinates forming the imaginary part: compact notation, use results from complex algebra to get closed form solutions

Landmark Shape Analysis

- Read from [2]: Sections 2.1,2.2, 3.1,3.2,3.3,3.4, 4.1 (only 4.1.1 - 4.1.4, only centered configuration, pre-shape, shape), 4.3 (only 4.3.1)

- Definitions of configuration, centered configuration, pre-shape, shape-1 (full Procrustes shape) and shape-2 (partial Procrustes shape).

- Translation, scale, rotation normalization.

- Procrustes distance: full (shape-1) and partial (shape-2)

- Definition of "mean shape" ("mean" of a set of configurations modulo scale, rotation and translation): minimizes the sum of full Procrustes distances of the set of configurations from it.

- Easy to show that "mean shape" is given by the largest eigenvector of $S = \sum_{i=1}^{n} w_i w_i^*$ where w_i is the pre-shape (translation and scale

normalized configuration C_i). *Exercise: show this.*

- Analogously can also define an "affine mean shape" for a set of configurations, but no closed form exact solution for the mean in that case (to the best of my knowledge). *Exercise: define the affine mean shape. Instead of complex notation, just write the configuration of K points as a $2 \times K$ matrix with x components forming the first row and y components forming the second row.*

- A "mean shape" characterizes the mean over variations in the data, other than global scaling, rotation, translation (or for affine mean shape, other than affine motion). Useful to represent mean shapes of animals such as mice, which may be of different sizes or their images may be rotated or translated. Also useful to mod out variations due to motion of a weak-perspective camera.

- Application: classification (define probability densities on the shape

space or in the tangent space at the mean).

- Tangent space: linearization of the non-linear shape space about a certain point (usually the mean of the dataset). Meaningful only if all dataset points lie close to each other on the shape space. Use: can use tools from linear multivariate statistics in the tangent space, e.g. can do PCA or can define linear regressions to model shape dynamics.

- Partial Procrustes Tangent space coordinate of shape z about μ:

$$v = [I - \mu\mu^*]ze^{j\hat{\theta}}, \quad \hat{\theta} = angle(z^*\mu), \quad j = \sqrt{-1} \tag{1}$$

If z is a partial Procrustes shape w.r.t μ (rotated to align with μ) then $\hat{\theta} = 0$

References

[1] A. Yezzi and S. Soatto, "Deformotion: Deforming motion, shape average and the joint registration and approximation of structures in images," *International Journal of Computer Vision*, vol. 53, no. 2, pp. 153–167, 2003.

[2] I.L. Dryden and K.V. Mardia, *Statistical Shape Analysis*, John Wiley and Sons, 1998.