LEVEL SET METHOD

Define a 2 D fn $\psi(x,y)$ s.t. the

Contour points ~~form~~ are

given by $\{x,y : \psi(x,y) = 0\}$.

(or $\psi(x,y) = k$ in general)

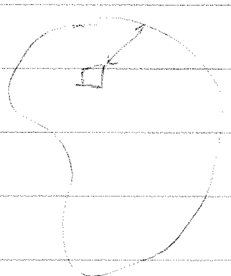
Eg circle : C

$$\boxed{\psi(x,y) = x^2+y^2-1 = 0} \rightarrow C$$

(parametric : $C(p) = \begin{matrix}\cos 2\pi p \\ \sin 2\pi p\end{matrix}$)

$$\psi(x,y) = x^4 + y^4 + 2x^2y^2 - 1 = 0.$$

$\psi(x,y)$ → Signed Dist. Fn.



Min Euclid dist from Contour, sign "−" inside "+" outside

$\boxed{\text{Evolution}}$ $\psi(C(p,t), t) = 0$

LSF is always zero at the current contour position

$$\psi\left(c^{x,y}(p,t),\ t\right) = K \quad \forall\ t$$

Total derivative

$$\frac{d\psi}{dt} = 0$$

$$\frac{\partial \psi}{\partial t} + \frac{\partial \psi}{\partial x} c_t^x + \frac{\partial \psi}{\partial y} c_t^y = 0$$

$$\frac{\partial \psi}{\partial t} = -\nabla_{(x,y)}\psi \cdot \underbrace{\boxed{c_t}}_{\frac{\partial c}{\partial t}}$$

$\boxed{\text{Define Normal}}$: $\frac{\partial \psi}{\partial \vec{T}} = \nabla\psi \cdot \vec{T}$

$\frac{\partial \psi}{\partial \vec{T}} = 0$; ψ constant along contour

$$\nabla\psi \cdot \vec{T} = 0$$

$$\Rightarrow \frac{\nabla\psi}{\|\nabla\psi\|} = \vec{N}$$

$\boxed{\text{Geometric Flows}}$: $\boxed{\text{Always along } \vec{N}}$

Any tangential flow: reparametrized to get zero flow along tangent

$$C_t = \beta \vec{N}.$$

$\Downarrow$

Convert to LSF

$$\Psi_t = -\beta \|\nabla \Psi\|$$

NEXT CLASS / (1) $(I-u)^2 - (I-v)^2$.

$\Downarrow$

Also update $\hat{u}$, $\hat{v}$ as you go along.

(2) Relation to Alternating Maximization (a K means).

(3) E-M Algo — $\boxed{\text{Classification \& Clustering}}$ on any feature space

(4) Level Set Method.
(5) Deformation for Registration.

LEVEL SET METHOD (LSM) → Sethian, Osher; early 1990's. UCLA Berkeley

How do I implement

$C_t = \kappa N$.

- parametrize: change param. at every t; move "very" slowly"

- handle singularities: 

- topology change.

implicitly. $C(p,t)$ ↔

2D fn $\phi(x,y)$ s.t.

$C = \{(x,y) \in \mathbb{R}^2 : \phi(x,y) = \gamma\}$.

$\boxed{\gamma = 0}$. – zero level set.

$\phi(x,y) \triangleq x^2 + y^2 - r^2 = 0$. circle

$x^4 + y^4 + 2x^2y^2 - r^4 = 0$

param eg $C(p) = \begin{matrix}\cos 2\pi p \\ \sin 2\pi p\end{matrix}$

Evolution of zero level set

$$\phi(c(p,t), t) \equiv 0 \quad \forall t.$$

"t" : artificial time param

governs "curve evolution"

[moving curve along $-\nabla E$ to min E].

Total derivative.

$$\phi_t + \frac{\partial \phi}{\partial x} \cdot C_t^x + \frac{\partial \phi}{\partial y} C_t^y = 0.$$

$$\phi_t = -\nabla\phi \cdot C_t.$$

$$\boxed{C_t = \alpha \vec{T} + \beta \vec{N}}$$

Note

$$\vec{N} = -\frac{\nabla\phi}{\|\nabla\phi\|}.$$

$$\phi_t = \|C_t\| \, \|\nabla\phi\|.$$
$$= \beta \|\nabla\phi\|.$$

$$\boxed{\nabla\phi \cdot \vec{T} = 0.}$$

$$\therefore \frac{\partial \phi}{\partial T} = 0.$$

ϕ constant along the curve.

"Geometric flow" : no tangential component

"coming from geom energy fn"

$C_t = \frac{\partial c}{\partial t}(p,t) = \alpha \vec{T}$: reparametrize

If β is geometric then no tang flow

If β is geometric then no : can be tangential flow. shown

Tang. flow : reparametrization of the contour

Numerical (Need to know to implement LSM) : $\|\nabla\phi\| = 1$

1. - Constructing S.D.F

2. - zero LS : passes through every pixel having diff signs at its 4 corners : similar idea to "border tracing"

   Interpolation to find exact location

3. Extension velocity : extend along the normal

   $\nabla\beta \cdot \nabla\phi = 0 \rightarrow \frac{\partial\beta}{\partial t} = \text{sign}(\phi)\,(\nabla\beta \cdot \nabla\phi)$

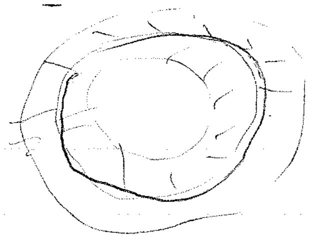
4. SDF : Maintain SDF

   in theory if β Vext constant along normal SDF stays SDF.

   practice : problems : Level sets come together — Reinitialize or steady state of $\frac{\partial\phi}{\partial t} = \text{sign}(u)\,(1 - \|\nabla u\|)$

(5) Narrowband evolution.

3 pixels
on either side



When curve is
"near boundary", re-initialize

(6) CFL condition

$$\left(\max_{\Omega} \beta\right) \Delta t < \Delta x$$

front not cross more than one
grid cell at every t.

NB method : Alive : all NB pts
LandMine : near boundary
Far : outside.

~~Pag~~ & Solve LS eq until LandMine
hit — then reinitialize.