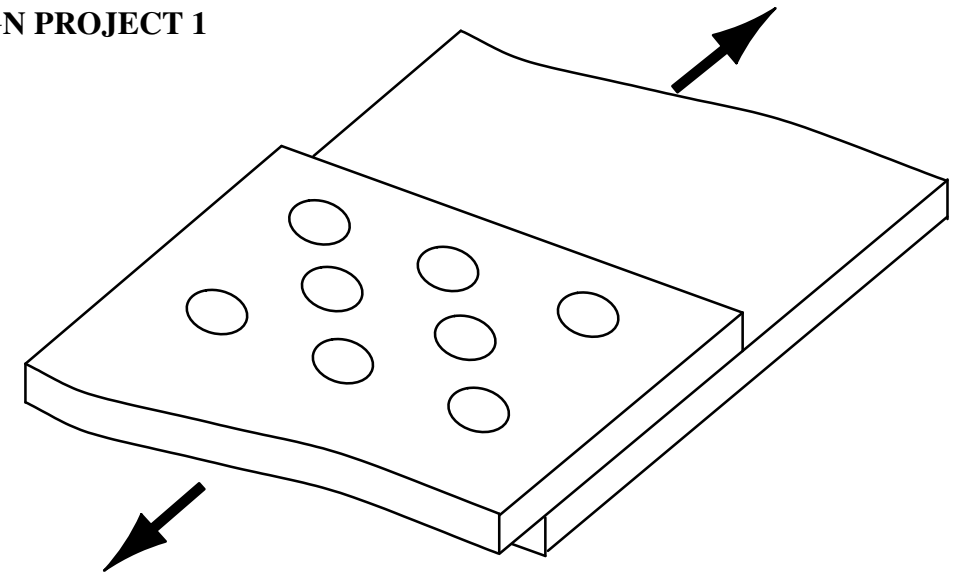
DESIGN PROJECT 1



Design a longitudinal, riveted lap joint in the skin running fore and aft along the outside of the fuselage of our commercial aircraft. The joint must withstand design pressurization hoop tension loads and contribute minimum weight to the aircraft. Attempts should be made to limit the number of fasteners to be installed during assembly.

Design Requirements

Prevent:

a. skin bearing failure
b. rivet shear failure
c. net area shear failure
d. net area tension failure
e. stress concentration failure
edge margin from plate edge ⩾ 2.0 hole diameters
distance closest hole ⩾ 4.0 hole diameters
f. rivet bearing failure

Fixed Constraints:

1. The basic design hoop tension in the skin of the pressurized passenger cabin of the a/c is 16200 psi.
2. Distance between frames is 20 in.
3. Rivets: 2017–T3 aluminum, 100 degree flush–head, solid, machine–countersunk
maximum allowable rivet diameter = 1/4 in. (to prevent feather edge limitations)
minimum allowable rivet diameter = 5/32 in. (to reduce fatigue Severity Factor)
4.Skin: 2024–T3 0.071 inch thick aluminum alloy sheet
5.Design ultimate load factor = 3.0

APPROACH

Planning

Criteria:

1. **Keep the total excess weight to a minimum**
2. **Keep number of rivets to be installed a minimum**

Physical Principles:

1. **Statically determinate forces on a free body**
2. **Material strength in shear, tension, bearing, fatigue**

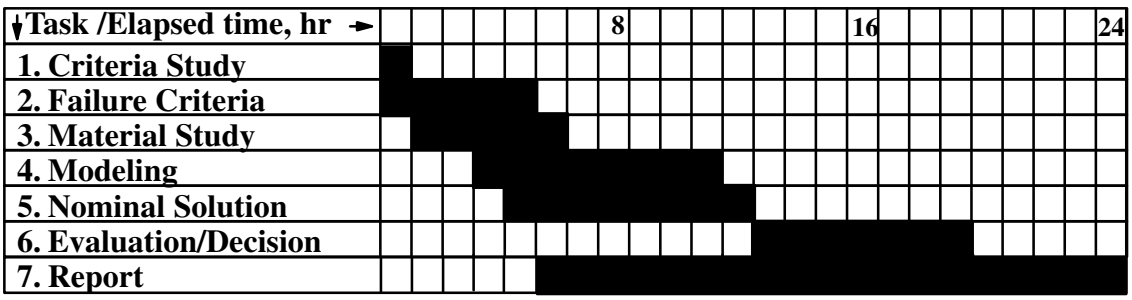
Solution Plan:

1. Decide on how to satisfy two, possibly conflicting criteria	**Task 1: Criteria Study**
2. Collect information on the 6 failure (inequality) constraints	**Task 2: Failure Criteria**
3. Collect information on rivets and sheet **a. rivet geometries** **b. material strengths**	**Task 3: Material Study**
4. Physical & mathematical modeling **draw free–body diagram** **define length of free–body** **apply equilibrium constraints** **apply expressions for inequality constraints**	**Task 4: Modeling**
5. Develop computer code for criterion function(s) **search for feasible solution solution**	**Task 5: Nominal Solution**
6. Iterate to satisfy criterion	**Task 6: Evaluation/Decision**
7. Write report	**Task 7: Report**

Data/Principle Sources:

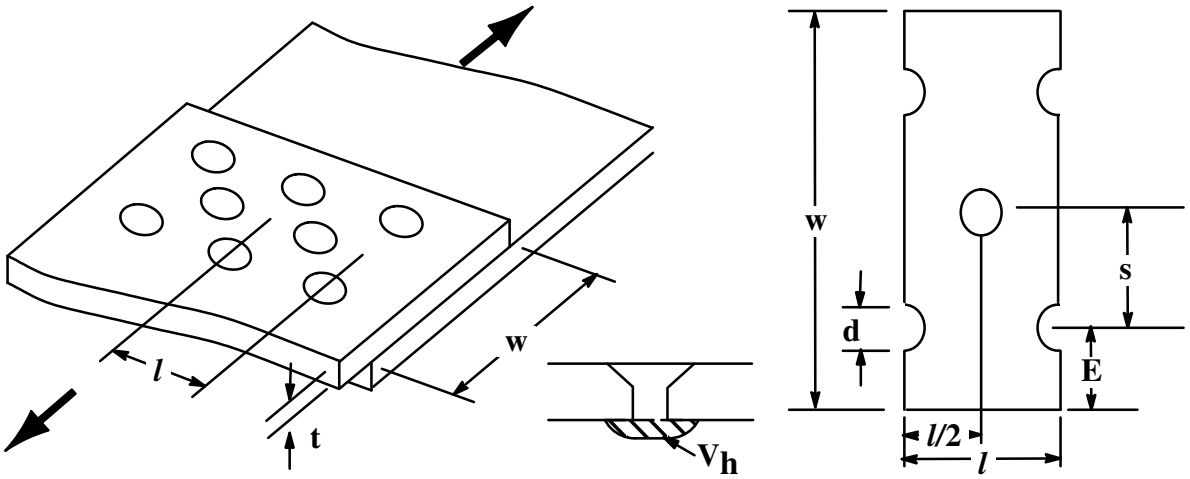
1. **Rivets: MIL–HDBK**
2. **Sheet : MIL–HDBK, spec QQ–A–250/4**
3. **Strength Considerations: Niu, Airframe Structural Design**

Scheduling



MODELING

Physical Model



Assumptions

Choose the rivet spacing to be length of the joint to be analyzed.
Distance between frames fixed at 20 in.
Rivet density equal to plate density, and rivet fills the hole
Staggered rows
Double symmetry
Same rivet throughout
Constant load/rivet
No bending

Criterion Functions

1. excess volume due to sheet overlap and rivet heads

$V_e = V_O + n_{riv} V_h$ — V_e = excess volume/unit length; V_O = sheet overlap volume; V_h = volume of rivet head; n_{riv} = number of rivets

2. total number of rivets

$n_{riv} = n_{row}\, L/l$ — l=rivet spacing along row; L = frame spacing

Use Criterion Function 1 to set up solution. Criterion Function 2 can then be determined from the same inequality constraint expressions.

Mathematical Model

t = plate thickness, in
l = rivet spacing along row
w = width of overlap
E = plate edge distance
d = rivet diameter
s = spacing between rows

Functional constraints		Fixed Constraints		State Variables	
$V_e = V_O + n_{riv} V_h$	(1)			V_e, V_O, n_{riv}, V_h	4
$V_h = 1/2\ \pi\ d^3$ (est)	(2)			d	1
$V_O = wLt$	(3)	t = .071 in	1	w	1
$n_{riv} = n_{row}\, L/l$	(4)	L=20 in	1	n_{row}, l	2
$w=2E+(n_{row}-1)s$	(5)			E,s	2
	5		1		10

Parameters

Number required = state variables – functional constraints = 10–5 = 5.
Maximum d.o.f. = 5. use geometry variables n_{row}, *l, d, E, s.*

Analysis of Inequality Constraints

Assume constant load per rivet

P = load per rivet
f = stress in original skin
f_{all} = allowable stresses
F_S = safety factor
E = edge distance from plate edge
E_c = distance from closest hole center
s = rivet spacing between rows $\equiv 0$ for $n_{row} = 1$

$Ec_r = l$ **to closest rivet along a row**

smaller of $Ec_s = \frac{s}{2}$ **&** $\sqrt{s^2 + \left(\frac{l}{2}\right)^2}$ **to closest rivet in staggered row,** $n_{row} = 3,5$

$Ec_s = s$ $\quad n_{row} = 2,4$

$w = (n_{row} - 1)s + 2E$

$P = F_S f Lt/n_{riv}$

skin bearing	$f_{br} = \frac{P}{dt}$	$MS_{br} = \frac{f_{br_{all}}}{f_{br}} - 1$
rivet shear	$f_s = \frac{4P}{\pi d^2}$	$MS_s = \frac{f_{s_{all}}}{f_s} - 1$
net area shear	$f_{as} = \frac{P}{2t(E - \frac{D}{2}cos(40))}$	$MS_{as} = \frac{f_{as_{all}}}{f_{as}} - 1$
net area tension	$f_{at} = \frac{P}{(w - d)t}$	$MS_{at} = \frac{f_{at_{all}}}{f_t} - 1$

Inequality Constraints

$MS_{br} \geq 1$

$MS_s \geq 1$

$MSas \geq 1$

$MS_{at} \geq 1$

Fixed constraints

f = 16200 psi
F_S = 3.0
$f_{br_{all}} =$ **129000 psi**
$f_{s_{all}} =$ **38000 psi**
$f_{as_{all}} =$ **39000 psi**
$f_{t_{all}} =$ **64000 psi**

hole margin $Ec_r/d \geq 4.0$

$Ec_S/d \geq 4.0$

edge margin $E/d \geq 2.0$

feather edge limit $d \geq 1/4$ **in.**

fatigue limit $d \leqslant <5/32$ **in.**

Nominal Solution

excess volume =	**2.14 in^3**	**criterion function 1**	
rivet number=	**33**	**criterion function 2**	
number of rows =	**3**		
width of overlap =	**2.0 in**		
spacing along row =	**1.8 in**	**$Ec_r/D = 7.2 \geqslant 4$**	**OK**
diameter of rivets =	**.25 in**	**$\leqslant$ 1/4 in**	**OK**
		$\geqslant$ 5/16 in	**OK**
spacing between rows =	**.5 in**	**$Ec_S/D = 4.1 \geqslant 4$**	**OK**
edge distance =	**.5 in**	**$E/D = 2. \geqslant 2$**	**OK**
bearing MS_{ult} =	**.095**	**$\geqslant$ 0**	**OK**
shear MS_{ult} =	**.784**	**$\geqslant$ 0**	**OK**
net area shear MS_{ult} =	**.070**	**$\geqslant$ 0**	**OK**
net area tension MS_{ult} =	**2.37**	**$\geqslant$ 0**	**OK**

EVALUATION & DECISION

Design Space Studies

- **Design Space Extent**

  diameter limitation $5/3 \leq d \leq 1/4$ used to limit rivet diameters

  rivet spacing limit $.5 \leq l \leq 1$.

  the lower limit is below 4 x minimum diameter

  the upper limit is an estimate, which can be increased if the optimum solution is not obtained

  number of rows $2 \leq n_{row} \leq 5$

  the lower limit taken as 2 since no feasible solution could be found for $n_{row} = 1$

  the upper limit is an estimate, which can be increased if the optimum solution is not obtained

- **Degrees of Freedom**

  According to the functional constraint $w = 2E + (n_{row} - 1)s$

  For a fixed n_r then E and s should be kept to a minimum

  $\therefore$ edge spacing is taken as $E = 2d$ **(6)**

  and $Ec_S = 4d$ giving $$s = \sqrt{(4d)^2 - \left(\frac{l}{2}\right)^2} \quad \text{for } n_{row} = 2,4 \qquad (7)$$

  $$Ecs = 2d \quad \text{for } n_{row} = 3 \qquad (7')$$

  This adds 2 more functional constraints and

  d.o.f = state variables –functional constraints = 10–7 = 3.

  Remaining design parameters are n_{row}, l, d.

- **Sensitivity Study**

parameter		sensitivity at nominal solution point	
rivet spacing	l	.24	(affects rivet weight only)
number of rows	n_{row}	.80	(directly affects skin width)
rivet diameter	d	.89	(affects width & rivet weight)

- **Parameters Choice**

  Sensitivities range over 4:1 but all parameters important. Therefore keep all 3.

- **Inequality Constraints**

  With the expressions used (6) & (7) above, the number of inequality constraints is reduced by 2, leaving the following:

  bearing MS_{ult}, shear MS_{ult}, net area shear MS_{ult}, net area tension MS_{ult},

  $d \leq 1/2$, $d \geq 5/16$, $l/d \geq Ec_r$

Search

- **Data Base**

  Carpet/contour data bases were created for each value of n_{row} with l the inner loop variable and d the outer loop variable.

  5 inequality constraints are included; the rivet diameter limitation is satisfied by the limits on the range of d.

  The data are attached as Tables 1,2 &3.

- **Search for Best Solution**

  Carpet and contour plots are attached for each value of n_{row} as Figures 1,2&3. the unfeasible side of each inequality constraint line is indicated by hatching. The best feasible solution is indicated on each.

  The following results are found:

n_{row}	excess	number of rivets	
2	2.08	50	<= best
3	2.14	60	
4	3.89	57	

	Achieved	Required
excess volume =	2.08 in^3	
rivet number =	50	
number of rows =	2	
width of overlap =	1.25 in	
spacing along row =	.8 in = 5.12 diameters	⩾ 4 diameters
diameter of rivets =	5/32 in	5/32 ⩽ diameter ⩽ 1/4 in
spacing between rows =	5/8 in = 4.0 diameters	⩾ 4 diameters
edge distance =	5/16 in = 2.0 diameters	⩾ 2 diameters
bearing MS_{ult} =	.04	⩾ 0
shear MS_{ult} =	.06	⩾ 0
net area shear MS_{ult} =	.01	⩾ 0
net area tension MS_{ult} =	1.1	⩾ 0

Discussion & Recommendations

1. Excess Weight Criterion

- **The minimum width is determined entirely by the edge constraints. Minimizing weight therefore is accomplished by minimizing the diameter and number of rivets. That is the number of rows is to be a minimum and rivet spacing is to be a maximum. These are limited in turn by the the resulting rivet shear and skin bearing loads.**
- **The final solution is limited by resulting rivet shear,skin bearing and net area shear and this occurs at the minimum allowed rivet size implying that the design is the best possible with the geometry chosen. Since the solution is with 2 rows though, improvements could apparently be achieved by staggering the second row, and this should be studied.**
- **A butt joint would double the weight if the joint. If a flush surface is required, this would be the penalty.**

2. Rivet Number Criterion

- **The solution for the weight criterion assures a minimum number of rivets, as discussed above, since the overlap width is fixed by the edge distance inequality constraints.**