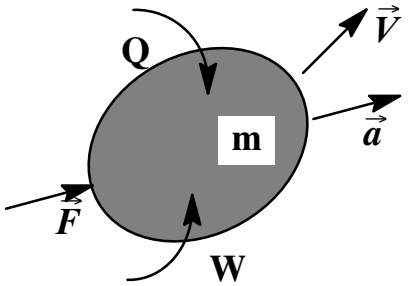
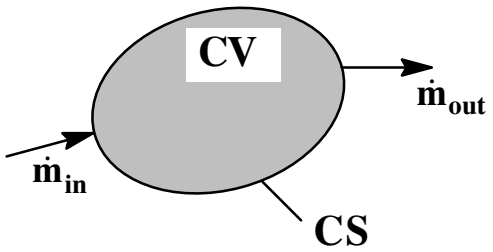
Name	Word Statement	Mathematical Statement
Mass Conservation	**Mass cannot be created or destroyed**	$m = \text{constant}$
Momentum Conservation (Newton's 2nd Law of Motion)	**Force applied to a body increases its momentum at a rate proportional to the force**	$\vec{F} = k\frac{d(m\vec{V})}{dt}$
Energy Conservation (First Law of Thermodynamics)	**The internal (thermal) energy of a body is increased by the flow of energy (heat) across its surface and by the work done on it**	$\Delta U = Q + \frac{W}{J}$



Control Mass



Control Volume

Conserved Quantity	Word Statement
Mass	**All fluid flowing into the system flows out at the same rate.**
Momentum	**A force *on* a control volume *increases* the net rate of momentum flow out of the system relative to that flowing in. The rate of increase is proportional the force applied.**
Energy	**Heat added *to*, and work done *on*, a contol volume *increase* the net rate of energy flow out of the system, relative to that flowing in. The increase is proportional to the sum of the rate at which heat is added and work is done at the control surface.**

Assumptions

Steady Flow Assumption(s)
Incompressible Fluid, Constant Density Assumption (inc)
One−Dimensional Flow Assumption (1D)
Inviscid Flow Assumption (inv)

Control Volume

Mass Conservation or Continuity Equation

$$\Sigma(\dot{m})_{in} = \Sigma(\dot{m})_{out} \qquad \textbf{1.1 (s)}$$

$$\dot{m} = \left\{\frac{dm}{dt}\right\}_{CS}$$

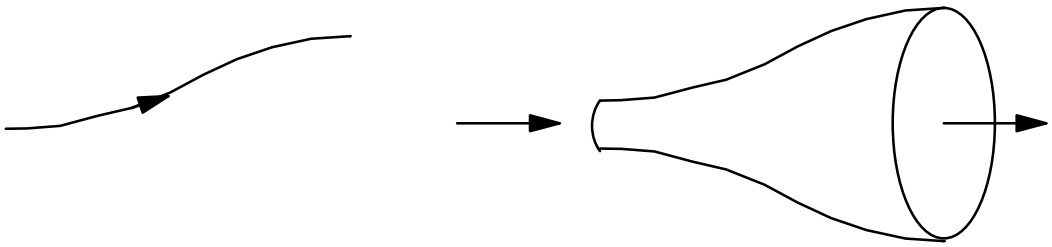
Momentum Equation

Similarly,
$$\vec{F} = \Sigma\left\{\frac{d(m\vec{V})}{dt}\right\}_{out} - \Sigma\left\{\frac{d(m\vec{V})}{dt}\right\}_{in}$$

since from the steady state assumption $\frac{d\vec{V}}{dt} = 0$ **on the CS**

$$\therefore \vec{F} = \Sigma(\dot{m}\vec{V})_{out} - \Sigma(\dot{m}\vec{V})_{in} \qquad \textbf{1.2 (s)}$$

Streamlines and Streamtubes



streamline **streamtube**

1−D Continuity Equation

$$\frac{dm}{dt} = \rho VA \equiv \dot{m} = \text{constant} \qquad \textbf{2.1 (s,1D)}$$

$$VA = \text{constant} \qquad \textbf{2.1a (s,1D,inc)}$$

1−D Momentum Equation

$$\vec{F} = \dot{m}_{out}\vec{V}_{out} - \dot{m}_{in}\vec{V}_{in}$$

single inlet and single outlet

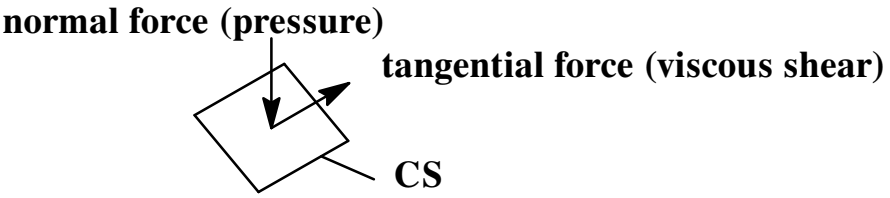
$$\vec{F} = \dot{m}(\vec{V}_{out} - \vec{V}_{in}) \qquad \textbf{2.2 (s,1D)}$$

$$\Sigma F_x = \dot{m}(V_{out_x} - V_{in_x}) \qquad \textbf{2.3a (s,1D)}$$

$$\Sigma F_y = \dot{m}(V_{out_y} - V_{in_y}) \qquad \textbf{2.3b (s,1D)}$$

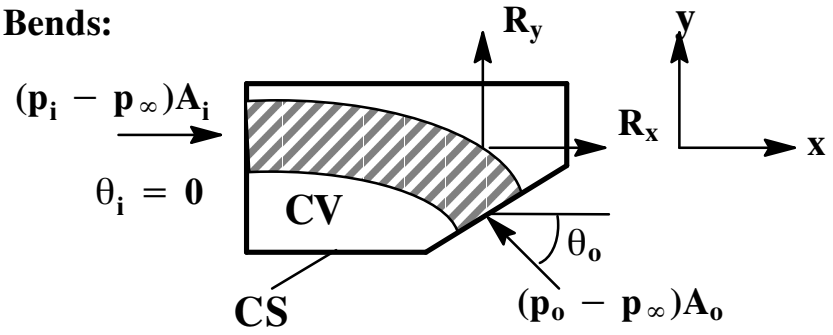
$$\Sigma F_z = \dot{m}(V_{out_z} - V_{in_z}) \qquad \textbf{2.3c (s,1D)}$$

Aerodynamic forces



Momentum Equation Applications

Pipe Bends:



$$\sum F_x = (p_i - p_\infty)A_i \cos\theta_i - (p_o - p_\infty)A_o \cos\theta_o + R_x = \dot{m}(V_{o_x} - V_{i_x})$$

$$\sum F_y = (p_i - p_\infty)A_i \sin\theta_i + (p_o - p_\infty)A_o \sin\theta_o + R_y = \dot{m}(-V_{o_y} - 0)$$

Jet Engines:

$T = \dot{m}\Delta V$ **: on design**

Hydrostatic Equation

$$dp = -\varrho g dh$$

manometer equation

$$\Delta p = -\varrho g \Delta h$$

Inviscid DifferentialStreamline Equations

$$\frac{d\varrho}{\varrho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \textbf{3.1a} \quad \textbf{(s,1D)}$$

$$\frac{dV}{V} + \frac{dA}{A} = 0 \qquad \textbf{3.1b} \quad \textbf{(s,1D,inc)}$$

$$dp + \varrho V dV = 0 \qquad \textbf{3.2a} \quad \textbf{(s,1D,inv)}$$

Bernoulli Equation

$$p + \frac{\varrho}{2}V^2 = \text{constant} = p_o \qquad \textbf{3.2b} \quad \textbf{(s,1D,inc,inv)}$$

$p \equiv$ static pressure

$p_o \equiv$ stagnation pressure

$q = \frac{\varrho}{2}V^2 \equiv$ dynamic pressure

Pitot−static probes, Wind tunnel applications

Airspeed Relations

$$V_E = \sqrt{\sigma} V_T \qquad \sigma = \frac{\varrho}{\varrho_s} = \frac{\varrho(h)}{\varrho(0)}$$

Non−dimensional Parameters

$$C_L = \frac{L}{\frac{\varrho}{2}V^2 S} \qquad RN = \frac{\varrho V \bar{c}}{\mu}$$

$$C_D = \frac{D}{\frac{\varrho}{2}V^2 S} \qquad M = \frac{V}{a}$$

$$C_M = \frac{M}{\frac{\varrho}{2}V^2 S\bar{c}} \qquad AR = \frac{b^2}{S}$$