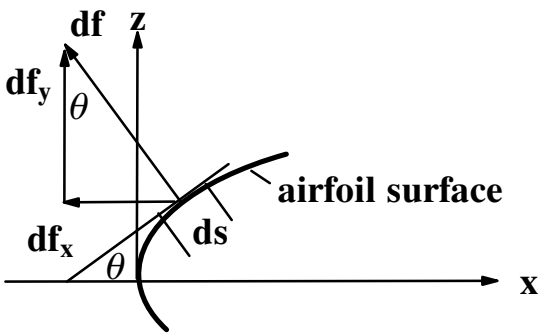
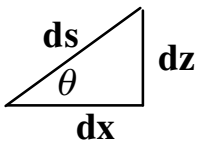
7. PRESSURE FORCES AND MOMENTS ON AN AIRFOIL

Application of the momentum equation analysis to an airfoil or wing results only in expressions for the total lift and drag. It tells us nothing about the detailed distribution of forces over the surface and as a consequence also does not tell us anything about moments on the body. This type of information must be obtained by a more sophisticated analysis or by resort to experimental measurements. Whichever way is chosen, the pressure distribution over the surface is obtained, and from this the lift, drag and moments can be obtained. If we deal only with the pressure forces of course we will not account for viscous forces properly, and for that reason our results will not be accurate for drag prediction unless drag is dominated by pressure. In other words this approach does not give good drag results for slender airfoils at small angles of attack. Experimentally, we can measure w_∞ far behind a wing model in a wind tunnel and use the analysis of Chapter 7 to determine α_i and $C_{D_i} = C_L \alpha_i$

7.1 Forces



$df = -pds$



$dx = ds \cos\theta$; $dz = ds \sin\theta$

Consider now the airfoil (2−dimensional case) as sketched above

$$df_x = -df\sin\theta = +p\sin\theta\,ds = p\,dz \qquad df_z = df\cos\theta = -p\cos\theta\,ds = -p\,dx$$

To find the total force, we integrate around the surface (ccw +'ve)

$$f_x = \oint p\,dz \qquad f_z = -\oint p\,dx$$

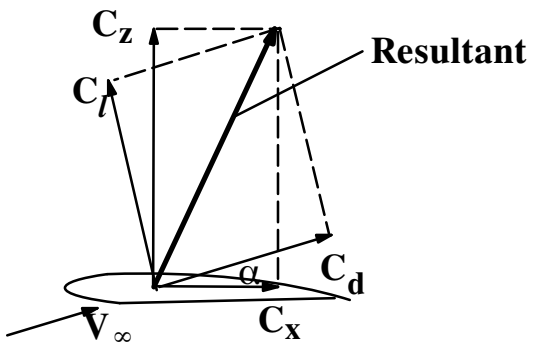
Using $\oint p_\infty dz \equiv 0$ $\quad\oint p_\infty dx \equiv 0;\ p_\infty =$ **free−stream pressure**

$$f_x = \oint (p - p_\infty)dz \qquad f_x = -\oint (p - p_\infty)dz$$

$$C_x = \frac{f_x}{qc} = \oint C_p d\left(\frac{z}{c}\right) \qquad C_z = \frac{f_x}{qc} = -\oint C_p d\left(\frac{x}{c}\right)$$

$$\text{where } C_p = \frac{p - p_\infty}{qc}$$

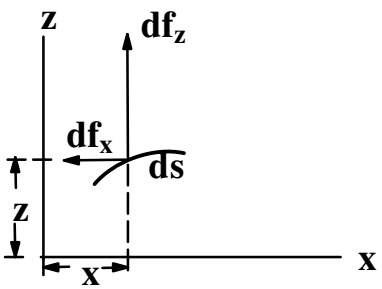
The force coefficients above are parallel and perpendicular to the chord direction, whereas lift and drag coefficients are defined as parallel and perpendicular to the free−stream velocity direction. We must therefore do a resolution of the forces into the desired direction.



$$C_l = C_z\cos\alpha - C_x\sin\alpha \simeq C_z\ (>0)$$
$$C_d = C_z\sin\alpha + C_x\cos\alpha \simeq C_z\alpha + C_x\ (>0)$$

7.2 Moments

Now, taking moments about the leading edge of the airfoil, i.e. about the point (0,0).



$$dm_{le} = -z\,df_x - x\,df_z$$

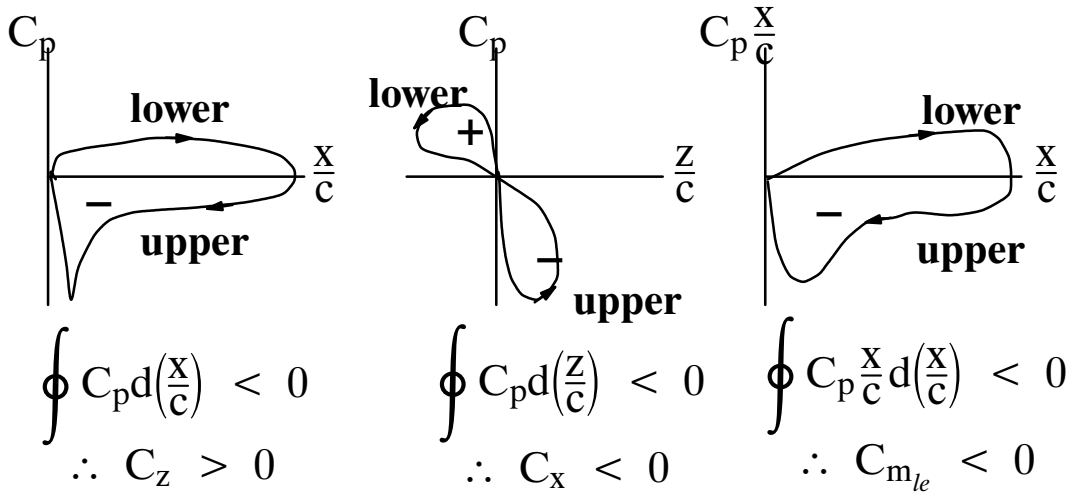
(pitch−up moments are +'ve, clockwise here)

$$m_{le} = -\oint zp\,dz + \oint xp\,dx = -\oint C_p \frac{z}{c} d\left(\frac{z}{c}\right) + \oint C_p \frac{x}{c} d\left(\frac{x}{c}\right)$$

$$C_{m_{le}} = -\oint C_p \frac{z}{c} d\left(\frac{z}{c}\right) + \oint C_p \frac{x}{c} d\left(\frac{x}{c}\right)$$

$$\text{or} \quad C_{m_{le}} \simeq \oint C_p \frac{x}{c} d\left(\frac{x}{c}\right)$$

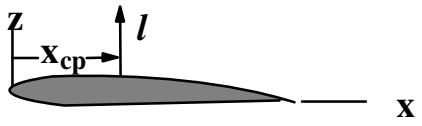
In the previous equation, we have neglected the moment due to the axial force, because the moment arm, $z << c$ and $f_x << f_z$. Plotting the integrands of the equations for C_x, C_z,, and $C_{m_{le}}$ against the independent variable in each case, for example from wind tunnel measurements, we get curves like those shown below for $\alpha > 0$.



$$\oint C_p d\left(\frac{x}{c}\right) < 0 \qquad \oint C_p d\left(\frac{z}{c}\right) < 0 \qquad \oint C_p \frac{x}{c} d\left(\frac{x}{c}\right) < 0$$

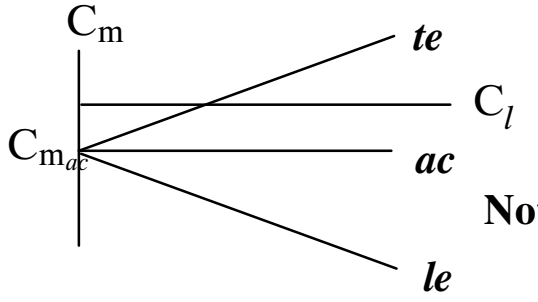
$$\therefore\ C_z > 0 \qquad \therefore\ C_x < 0 \qquad \therefore\ C_{m_{le}} < 0$$

We define the effective point of application of the lift force as acting at the centroid of the pressure distribution. Calling this point the center of pressure location, x_{cp}, we obtain



$$m_{le} = -x_{cp}\, l \quad \text{or} \quad C_{m_{le}} = -\frac{x_{cp}}{c} C_l$$

A plot of this as C_m vs C_l would appear as the *le* line in the sketch below.



Note: $C_{m_{ac}} = 0$ **for no camber**

If we now take moments about the trailing edge we would get a curve (*te* above) with a +'ve (pitch−up) slope. At any point along the chord the curve would lie between these two limits and, in particular, there is a point on the chord where the slope is zero. This point is called the aerodynamic center, and is defined as the point

where the moment is independent of the lift, i.e. is independent of angle of attack.

Transferring moments to a point x along the chord, we would obtain:

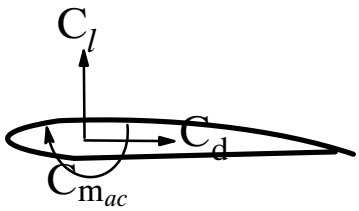
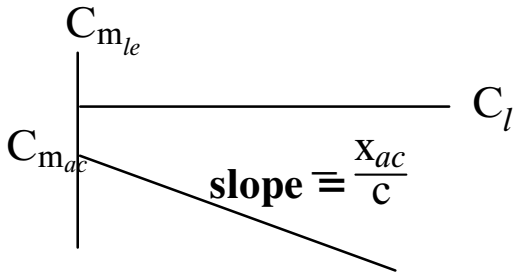
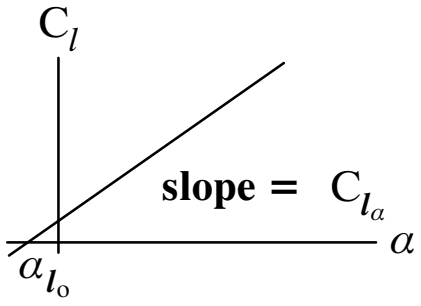
$$m_x = -(x_{cp} - x)l = m_{le} + lx \quad \text{since} \quad m_{le} = -x_{cp}l$$

$$\text{or } C_{m_x} = C_{m_{le}} + \frac{x}{c}C_l$$

Applying this at the aerodynamic center, $x = x_{ac}$,

$$C_{m_{ac}} = C_{m_{le}} + \frac{x_{ac}}{c}C_l$$

$$\text{or} \quad C_{m_{le}} = C_{m_{ac}} - \frac{x_{ac}}{c}C_l$$

From the slope of a plot of $C_{m_{le}}$ vs C_l as shown below, we can then obtain x_{ac}/c and from the intercept we can find $C_{m_{ac}}$. For a flat plate and thin airfoils, $x_{ac}/c = .25$

$C_{m_{ac}}$ and x_{ac}/c along with C_{l_α} and α_{l_o} are used to give the characteristics of airfoil sections.