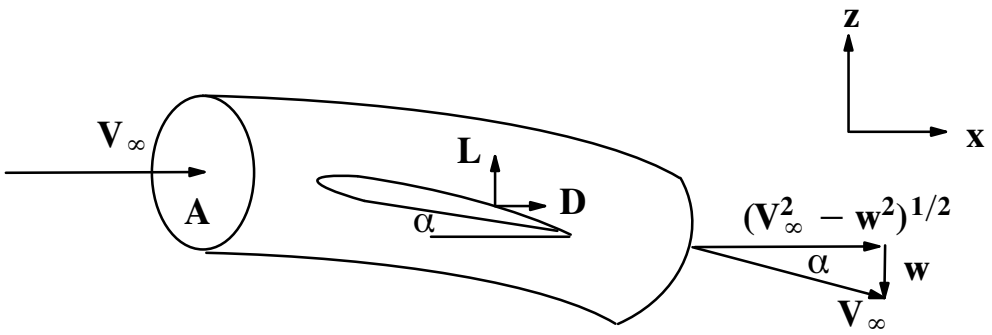
4. THE GENERATION OF LIFT (Incompressible Fluids)

4.1 The Origin of Lift Forces



If we use our first inclination to apply the momentum equation to a Control Volume (CV, above) which includes the body, we would assume that the approaching flow is deflected by the body through an angle (approximately equal to the angle of attack α) and thereby give a downward velocity w at the exit plane of the CV. With the usual assumptions that the pressure equals p_∞ everywhere on the edges of the CV, and that no viscous forces are present there, we would get:

$$F_x = \rho V_\infty A\left[\left(V_\infty^2 - w^2\right)^{1/2} - V_\infty\right] \qquad 4.1$$

$$\equiv -D, \text{ the force parallel to } V_\infty \text{ called Drag} \qquad 4.2$$

$$F_z = \rho V_\infty A(-w - 0) = -\rho V_\infty A w$$

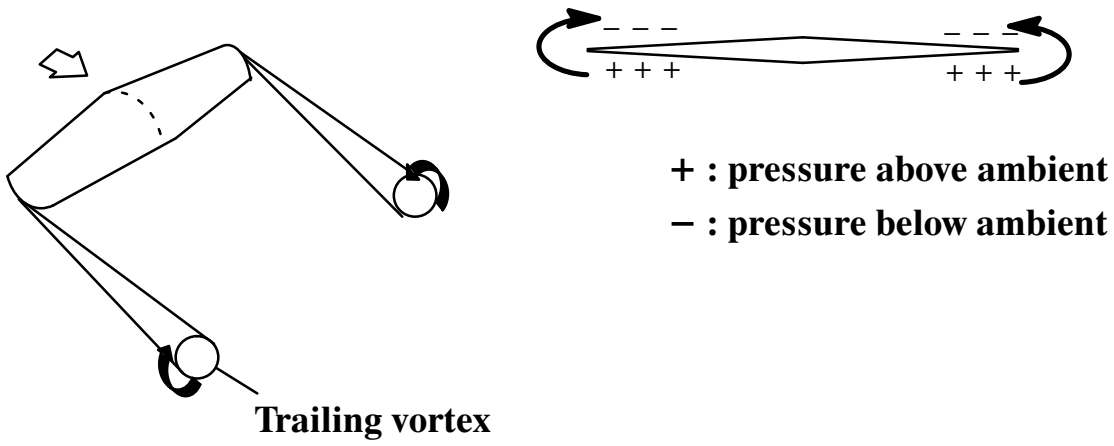
$$\equiv -L, \text{ the force normal to } V_\infty \text{ called Lift}$$

F_x and F_z are the forces acting *on* the fluid within the CV (i.e., the pressure forces exerted *on* the fluid *by* the body), and L and D are the fluid forces acting *on* the body.

We have found that in order to obtain a force in the z−direction, we must develop a downward flow velocity, w, behind the body. Where does this flow go? It cannot keep flowing downward indefinitely (to $z = -\infty$) since that would require moving an infinite volume of fluid, an obvious impossibility.

The fluid has been assumed to be incompressible so that it is not just a matter of compressing a finite volume of fluid below the body. (The behavior for compressible flows is markedly different, and we will discuss them later in the course when considering flows with M > 1) This leaves us with only one possibility for an incompressible flow, and that is the one postulated by Lanchester, namely, the flow cannot continue to go downward, but must come back up again somewhere so that the net displacement of fluid is zero. The way this occurs is different in the three−dimensional and two−dimensional cases, i.e. the finite wing and the airfoil section.

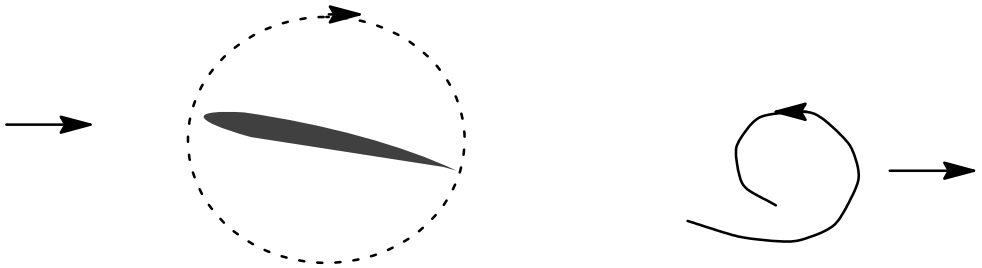
4.1.1 Finite Wing (3−D)



In the case of a finite wing, the flow deflected downward just behind the wing comes up again behind the wing tips, and circulates back down again forming a spiraling motion (vortex) trailing behind each wing tip as illustrated above. The flow has no net angular momentum before passage of the wing, and it therefore cannot have any after passage since no external torque is applied and angular momentum must be conserved. Thus the two trailing vortices must spin in opposite directions and be of equal strength. We can see physically why this happens by reference to the sketch above, which shows a wing viewed from the downstream direction. If there is lift generated, the pressure on the upper surface (−) is less than the pressure on the lower surface (+), so that the fluid tends to flow around the tip from high to low pressure. Equations 4.1 and 4.2 can then be used to determine the forces

exerted by a finite wing if we limit our CV to the region enclosing the wing and the two trailing vortices. We will do this in a later section.

4.1.2 Airfoil (2−D)

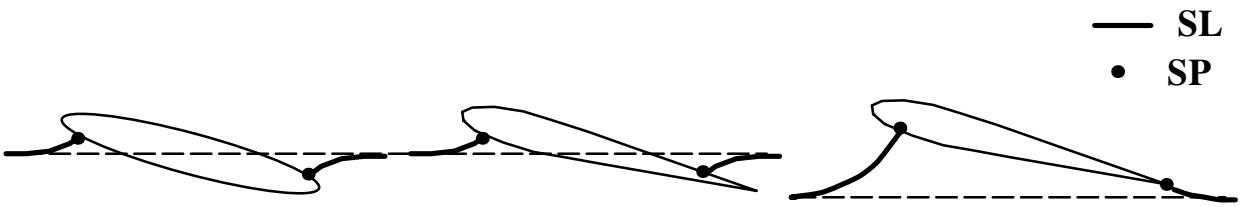


Now consider what happens if the flow is 2−dimensional or has an infinite span. In this case the downward flow cannot come back up at wing tips since they no longer exist, but can only do so upstream or downstream. In fact it must do both since angular momentum must be conserved in this case too. The flow is deflected downwards behind the airfoil and swirls back up again forming a vortex that is swept downstream. The equal and opposite vortex formed upstream as required by conservation of angular momentum, remains bound to the body. The upstream vortex is presumably bound to the body after the flow is started, giving a continual circulating flow, called the circulation, around the airfoil. The vortex attached to the body is called a "bound vortex"(sketched as a dotted line) , and the one swept downstream is called a "shed" vortex. If the velocity of the airfoil changes, or if its angle of attack changes, a further vortex is shed, and the strength of the bound vortex changes accordingly. You have probably seen this vortex behavior when a paddle is pulled smoothly through the water.

4.2 Circulation

4.2.1 Airfoil Section: The Kutta Condition

What gives rise to the circulation around an airfoil? Some force is necessary to start the flow swirling. The physical explanation is not so straightforward as it was for the finite wing where the pressure difference between the upper and lower surfaces supplied the necessary force. Now we must look to something more subtle.



a) Bluff Body **b) Airfoil Shape** **c) Airfoil Shape**

INVISCID **VISCOUS**

The other source of aerodynamic force other than pressure, is surface friction due to viscosity, and we have assumed an inviscid flow field. If we must appeal to viscosity for an explanation, we must admit that a truly inviscid, incompressible flow will not generate a lifting force on a 2−dimensional body. Rather, the stagnation streamline (SL) for the body would appear as sketched in a) above, for a blunted body, and as in b) for a body with a sharp trailing edge. The stagnation points (SP) on the upper and lower surfaces are shown, and we see not only no net deflection of SL at $\pm\infty$ but nearer the body, upstream deflection is equal to down stream deflection. If viscosity were allowed, then the flow would still appear much the same for the case in a), where there are no large velocity gradients, but the large changes required to manipulate the sharp trailing edge of b) would be prevented by viscous shearing forces. (Remember that viscous stress is proportional to velocity gradients.) Instead, the stagnation point moves down to the trailing edge under the action of combined viscous and pressure forces, and now leaves the upper and lower surfaces smoothly there, as illustrated in c) thereby making viscous forces there small. This condition at the trailing edge is known as the "Kutta Condition". In c) there is still no net deflection of SL at $\pm\infty$, but now the upstream and downstream deflections near the body are in opposite directions as we would expect for the circulating flow of a bound vor-

tex. Note too, that there is a greater upward deflection of the forward SP than in cases a) and b).

Once steady flow is established and the Kutta condition is satisfied, viscosity is no longer required an we can make the assumption that the flow is inviscid. Note however, that if the angle of attack or the free−stream velocity changes, a further vortex will be shed, and the strength of the bound vortex will change until steady conditions are achieved once again, i.e. until the shed vortex is swept far behind the airfoil. Viscous forces would of course be active during the transition period.

The viscous shear force which acts on the fluid imparts a torque to the flow give it a spinning motion, and thereby moves the rear stagnation point down to the trailing edge.

Recall that $T = \frac{d(\varrho v_\theta r)}{dt}$ **4.3**

for the relation between the torque on unit volume, T, applied by the viscous shear, and the angular momentum $\varrho v_\theta r$ **, which results. A tangential velocity** v_θ**, therefore, will result if the torque is applied for the proper amount of time and** v_θ **will be just sufficient to move the rear stagnation point to the trailing edge. Once that occurs the torque T becomes zero and no further viscous forces are required. The flow field is then left with a constant angular momentum around the airfoil of**

$$\varrho v_\theta r = \text{constant} \qquad \textbf{4.4}$$

For constant density flows, then

$$v_\theta r = \text{constant} \equiv \frac{\Gamma}{2\pi} \qquad \textbf{4.5}$$

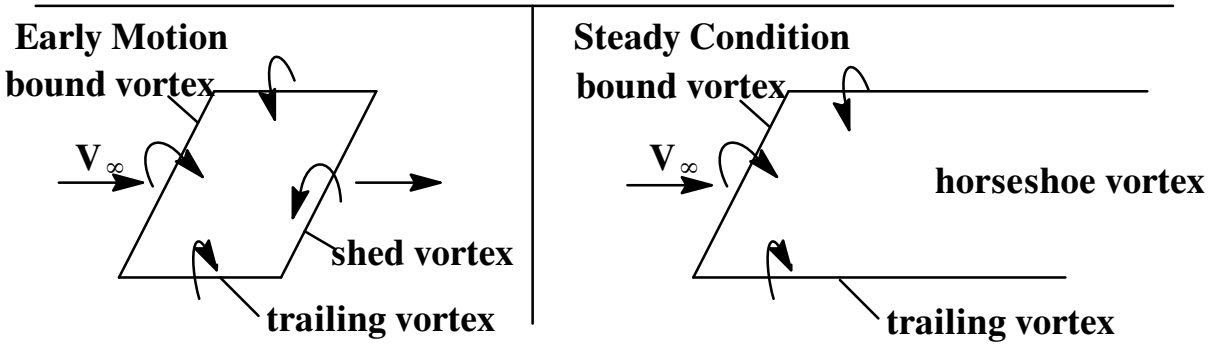
where Γ **is called the "circulation"**

(Actually circulation is defined by $\Gamma = -\oint \vec{V}\cdot d\vec{s}$ **4.6**

where the integration is a line integral around any closed path which surrounds the body. When the path of integration is a circle, then it is easy to show that Equation 4.6 gives $\Gamma = 2\pi r v_\theta$**.)**

Note that we have departed from convention and have taken v_θ positive downwards. This is done to make Γ positive clockwise.

4.2.2 Wing Vortex System



Even for a finite wing, a bound vortex and a shed vortex are formed along with trailing vortices. Actually trailing vortices are formed, generally, all along the span of the wing, but these coalesce (see a) above) to form two trailing tip vortices (see c) above). During the starting motion a vortex is shed from the trailing edge and is swept downstream as in the 2−D case, but since vortices cannot end in space, the trailing vortices are developed at the same time (see b) above). Once the shed vortex is swept far downstream the steady structure appears in a form we call a "horseshoe vortex".

4.3 SUMMARY

Two−Dimensional, Incompressible Flows

1. No lift is generated without viscosity.
2. If the effects of viscosity are included, then the flow leaves the trailing edge smoothly. This condition is known as the Kutta Condition.
3. A body with a rounded trialing edge produces little lift, a sharp trailing edge is required for efficient lift production.
4. Although the effects of viscosity are essential in establishing a flow, or when changes of speed or angle of attack occur, to a good approximation viscous effects may be neglected once steady state is achieved.
5. The starting viscous forces leave the flow field with a steady circulating motion described by $2\pi r v_\theta = \Gamma$, the circulation, constant at large distance from the body. The circulating flow is called a bound vortex of circulation Γ.

Three−Dimensional, Incompressible Flows

6. The bound vortex trails off from the wing and forms two trailing vortices.
7. The trailing vortices and the bound vortex combine to form a horseshoe vortex which extends far behind the wing.
8. The horseshoe vortex generates a downward flow in the region behind the wing (the wake) as well as at the wing itself.
9. The pressure difference between the upper and lower wing surfaces causes flow around the wing tips and generates the trailing vortices.