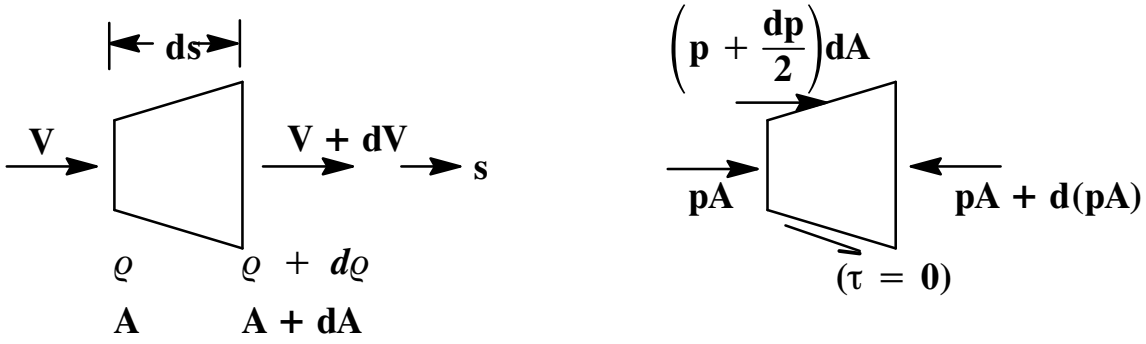
3. THE DIFFERENTIAL EQUATIONS OF MOTION − Steady, 1 D



3.1 Continuity

Consider as a control volume, a differential stream tube of length ds, inlet area A, and outlet area A + dA as sketched above. Equation 2.1 now becomes:

$$d(\varrho VA) = 0 \quad \text{or} \quad \frac{d\varrho}{\varrho} + \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \text{3.1a (s,1D)}$$

If we assume constant density, this becomes:

$$d(VA) = 0 \quad \text{or} \quad \frac{dV}{V} + \frac{dA}{A} = 0 \qquad \text{3.1b (s,1D,inc)}$$

3.2 Momentum − Eulers Equation, Bernoulli's Equation

3.2.1 Inviscid Flow Assumption (inv)

If there are no large velocity gradients in the flow, within the CV or on the CS, we can assume there are negligible viscous forces present. This is called the inviscid assumption (inv) and we commonly speak about inviscid flows and treat viscous effects as a separate subject. In these notes we will deal only with inviscid flows and will treat viscous flows separately in class.

3.2.2 Streamline Equations

Having made the assumption of inviscid flow, the only force acting on our differential volume is pressure force. The pressure force on the inlet is pA, and the force on the outlet is pA + d(pA). There is also an average pressure (p+dp/2) acting on the total sloping surface whose area component normal to s is dA. This gives a further force in the s−direction of (p+dp/2)dA. Equation 2.3a now becomes:

$$pA + \left(p + \frac{dp}{2}\right)dA - pA - d(pA) = \varrho VA(V + dV - V)$$

$$-Adp = \varrho VAdV \quad \text{neglecting dpdA}$$

$$dp + \varrho VdV = 0 \qquad \text{3.2a (s,1D,inv)}$$

Equation 3.2a is known as the Euler Equation, and applies along a streamtube or streamline in steady, inviscid flows. If we now also assume constant density, equation 3.2a can be written:

$$dp + d\left(\frac{\varrho V^2}{2}\right) = 0$$

$$p + \frac{\varrho}{2}V^2 = \text{constant} = p_o \qquad \text{3.2b (s,1D,inc,inv)}$$

Equation 3.2b is called the Bernoulli Equation, and the constant, known as Bernoulli's constant, can be determined from the stagnation pressure p_o , the value of the pressure where V = 0. The Bernoulli equation applies along a streamline in steady, incompressible, inviscid flows. The various terms in the equation are identified as follows:

$$p \equiv \text{static pressure}$$

$$p_o \equiv \text{stagnation pressure}$$

$$q = \frac{\varrho}{2}V^2 \equiv \text{dynamic pressure}$$

Stagnation pressure at a point in a flow is defined as the pressure that would occur *if* the flow at that point were brought to rest without any viscous losses. A stagnation streamline is any streamline which originates or terminates at a stagnation point.