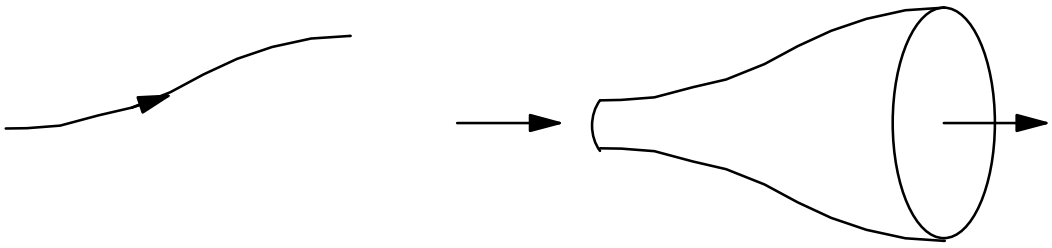
2. APPLICATIONS TO ONE−DIMENSIONAL FLOWS

2.1 One−Dimensional Flows

2.1.1 Streamlines and Streamtubes



streamline **streamtube**

A streamline is a line in a flow which is parallel to the local velocity vector. In other words, it is a line across which there is no flow. In steady flows, this is the same as the "particle path", the path followed by individual small parcels of fluid.

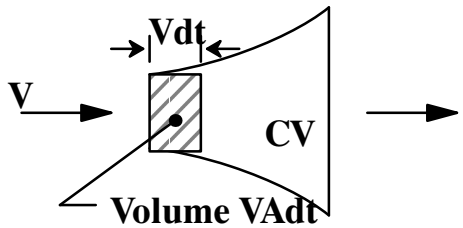
A streamtube is a tubular surface through which the fluid flows. Its surface is composed of streamlines, so that there is no flow across the stream tube surface. It also serves as a convenient CV and CS for our purposes here.

2.1.2 One−Dimensional Flow Assumption (1D)

In many problems, we can assume either that conditions such as velocity, pressure, etc. are constant across the inlet and outlet of a streamtube, or that we are dealing with suitably averaged quantities. In this case, flow variables will be functions only of distance s, along the stream tube or streamline. We refer to this as a 1−D assumption and refer too, to 1−D flows of fluids. All the variables in the conservation equations for steady flows are then functions of only one distance variable s, and the equations are simplified considerably.

2.2 1−D Continuity Equation

We will apply the 1−D assumption to Equation 1.1 as follows:



Referring to the sketch above, in time dt, a volume of fluid VAdt crosses the inlet of area A. If the density of the fluid is ϱ, then the mass dm flows in, where

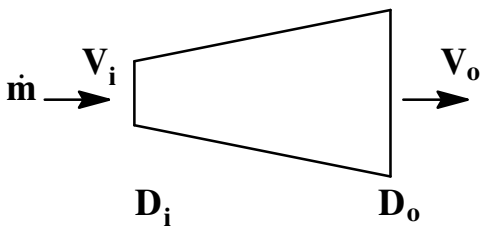
$$dm = \varrho VA dt$$

$$\text{or} \quad \frac{dm}{dt} = \varrho VA \equiv \dot{m} = \text{constant} \qquad \text{2.1 (s,1D)}$$

When the incompressible assumption is valid, this becomes

$$VA = \text{constant} \qquad \text{2.1a (s,1D,inc)}$$

Example:
Water at stp flows through a varying cross–section pipe at a rate of 20 lbm/s. What is the average flow speed at the pipe exit where the diameter is 1 inch?



Given: $\dot{m} = 20$ lbm/s $= 20/32.2$ slugs/s
$D_o = 1$ in $= 1/12$ ft
$\varrho_i = \varrho_{stp}$

Determine: V_o

Assume: steady, incompressible, 1−D flow

Equations & Constants:

Use: equation 2.1 $\dot{m} = \varrho_o V_o A_o$ or $V_o = \dfrac{\dot{m}}{\varrho_o A_o}$ (s,1D)

$$A_o = \frac{\pi D_o^2}{4} \qquad \text{(i)}$$

$$\varrho_o = \varrho_i = 62.4/32.2 \text{ slugs/ft}^3$$

Solution: $V_o = \dfrac{\dot{m}}{\varrho_o A_o} = \dfrac{20/32.2}{(62.4/32.2)\pi(1/12)^2/4} = 58.8$ ft/s

2.3 1−D Momentum Equation

Using the 1−D assumption, Equation 1.2 becomes

$$\vec{F} = \dot{m}_{out}\vec{V}_{out} - \dot{m}_{in}\vec{V}_{in}$$

Using Equation 2.1, this becomes, for the case of a single inlet and single outlet

$$\vec{F} = \dot{m}(\vec{V}_{out} - \vec{V}_{in}) \qquad 2.2 \quad (s,1D)$$

Equation 2.2 is a vector equation which can be written in terms of its 3 Cartesian directions, (x,y,z).

$$\Sigma F_x = \dot{m}(V_{out_x} - V_{in_x}) \qquad 2.3a \quad (s,1D)$$

$$\Sigma F_y = \dot{m}(V_{out_y} - V_{in_y}) \qquad 2.3b \quad (s,1D)$$

$$\Sigma F_z = \dot{m}(V_{out_z} - V_{in_z}) \qquad 2.3c \quad (s,1D)$$

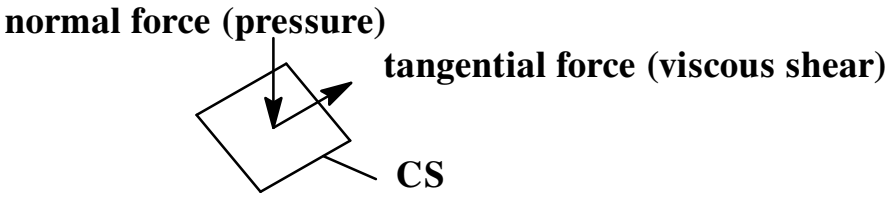
where ΣF_x , ΣF_y , and ΣF_z are the components of the total force acting on the CV

2.4 Aerodynamic Forces

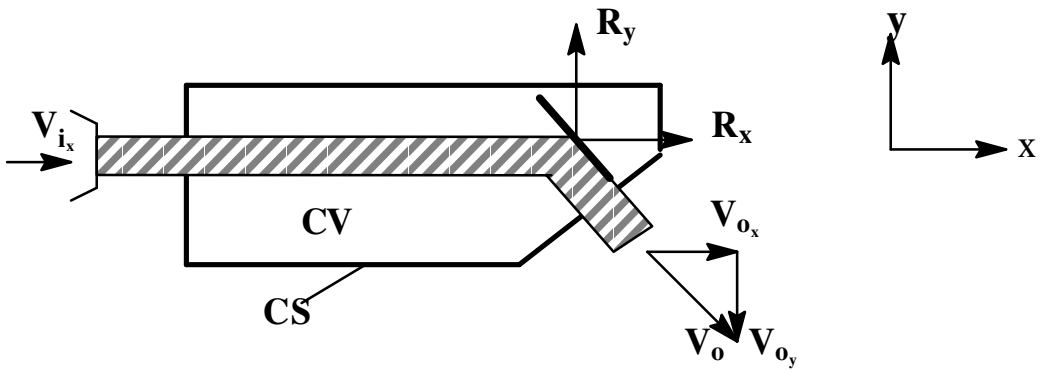
In order to use Equations 2.3, it is necessary to prescribe the total forces acting on the CV. In general such forces can be divided into two types, surface forces and body forces. As their names imply, surface forces act on the CS, whereas body forces act within or on the whole CV. A common example of body forces in fluids are those which result from gravity. It is these forces that give rise to the changes of pressure with altitude in the atmosphere, and are imporant in flows where there are large height changes. They are also important in liquids and are the source of buoyancy forces which keep boats afloat. Although we could include them in our analysis, we will exclude them for simplicity. They are not usually important in aerodynamics. They are the dominant forces however, in studies of the behavior of the atmosphere.

In these notes then, we are interested only in surface forces. In particular we are interested in what we classify as aerodynamic forces, and these are of two types, (i) pressure or normal forces and (ii) viscous or tangential shear forces.

Pressure forces are oriented normal to, and inwards towards the surface on which they act, while viscous forces act tangentially to the surface.



The key to solving problems involving the momentum equation is to choose the appropriate CV. If the surface of a CV is chosen such that it passes through a region where there are negligible velocity gradients, then we can neglect viscous forces there. This does not require that we neglect viscous effects inside the CV, but rather only on its surface. An example is a jet of water hitting an inclined plate as sketched below.

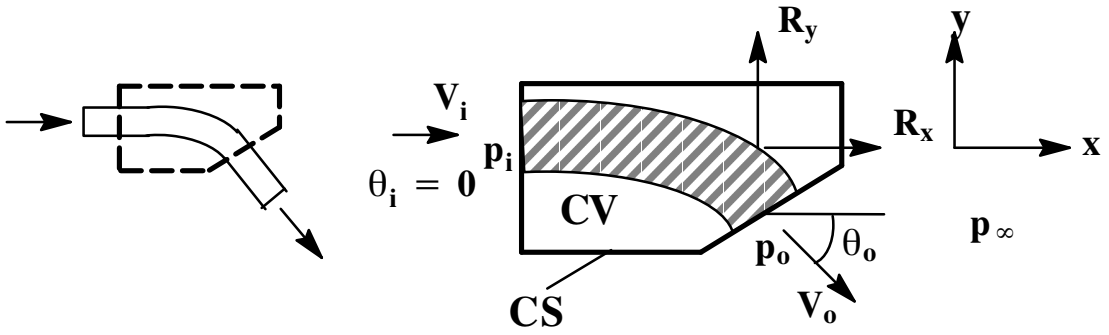


For the CV choice above, there are no appreciable viscous forces on *any* CS.

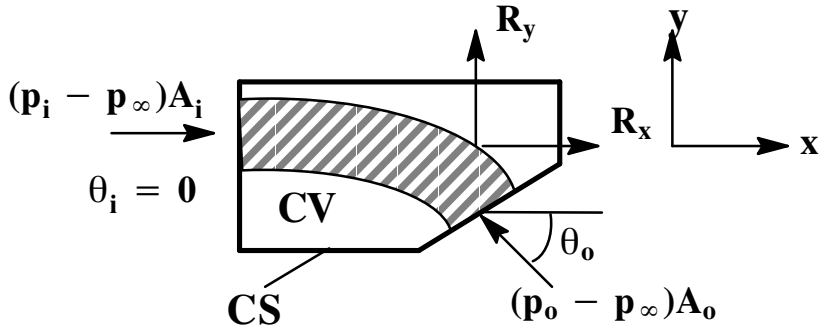
If we now consider the pressure forces, we can see that the pressure is the same on every surface as well. A free low−speed jet has no forces acting on it (when we neglect gravity), and therefore the pressure is constant through it and equal to the ambient pressure. Since the pressure is constant everywhere, there are no pressure forces acting on the surface of the CV. The only forces, then, are the viscous and pressure forces acting *on the fluid* by the plate. If we call this force $\vec{R}$, then equation 2.3a and 2.3b become:

$$F_x = \dot{m}(V_{o_x} - V_{i_x}) = R_x$$
$$F_y = \dot{m}(-V_{o_y} - 0) = R_y$$

Another example is the reaction force on a pipe bend.



In this case, the pressure on the inlet and outlet portion of the surfaces of the CV chosen, are not equal to the ambient pressure and net pressure forces are present on the CV surface. It is only the difference in pressure relative to ambient, $p_i - p_\infty$ and p_o that give pressure forces. These are called gauge pressures. Once again there are only negligible viscous forces on any of the surfaces of the CV.



Now the total force on the CV above can be written as:

$$\sum F_x = (p_i - p_\infty)A_i \cos\theta_i - (p_o - p_\infty)A_o \cos\theta_o + R_x = \dot{m}(V_{o_x} - V_{i_x})$$

$$\sum F_y = (p_i - p_\infty)A_i \sin\theta_i + (p_o - p_\infty)A_o \sin\theta_o + R_y = \dot{m}(-V_{o_y} - 0)$$

This same approach can be applied to the analysis of the thrust of a rocket or jet engine or propellor. These will be studied as examples in class.

2.5 Variable Conditions Across Inlet and Outlet

In the analysis so far developed, we have assumed that V and p are constant across the inlets and outlets. If we cannot make that assumption, then we must integrate across these surfaces. If we do this we must change our equations using the following:

Continuity

$$\dot{m}_o = \int_{A_o} \varrho V_o dA_o \; ; \quad \dot{m}_i = \int_{A_i} \varrho V_i dA_i \qquad \textbf{2.4}$$

Momentum

$$\textbf{Pressure forces: } F_{p_x} = \int_{A_o,A_i} p dA_x \; ; \; \textbf{Momentum fluxes: } \int_{A_o,A_i} \varrho V V_x dA \qquad \textbf{2.5}$$