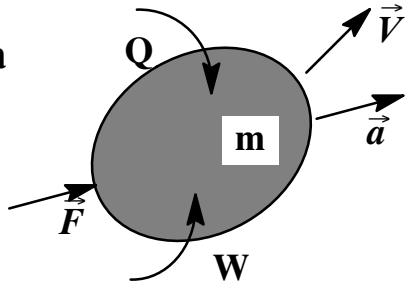
1. THE EQUATIONS OF MOTION

1.1 The Conservation Principles

The basic laws of motion are formulated with reference to a fixed quantity, or identity of matter.

Consider a free−body of mass m, moving at a velocity $\vec{V}$ and acted on by a force $\vec{F}$. This force results in an acceleration $\vec{a} = \frac{d\vec{V}}{dt}$

Suppose too, that an amount of work W, is done on the body, and that a quantity of heat Q, flows into the body.



The following three basic Laws, or Conservation Principles are then applied to determine the ensuing behavior of the body:

Name	Word Statement	Mathematical Statement
Mass Conservation	Mass cannot be created or destroyed	$m = \text{constant}$
Momentum Conservation (Newton's 2nd Law of Motion)	Force applied to a body increases its momentum at a rate proportional to the force	$\vec{F} = k\frac{d(m\vec{V})}{dt}$
Energy Conservation (First Law of Thermodynamics)	The internal (thermal) energy of a body is increased by the flow of energy (heat) across its surface and by the work done on it	$\Delta U = Q + \frac{W}{J}$

By using consistent sets of fundamental units, such as the SI or the English slug−foot−second system as we will throughout these notes, the constant of proportionality k, can be taken as unity. Joule's constant (J) is also not required if Q and W are in the same units.

Using mass conservation (m = constant), Newton's 2nd Law may be written in its more common form:

$$\vec{F} = m\frac{d\vec{V}}{dt} = m\vec{a}$$

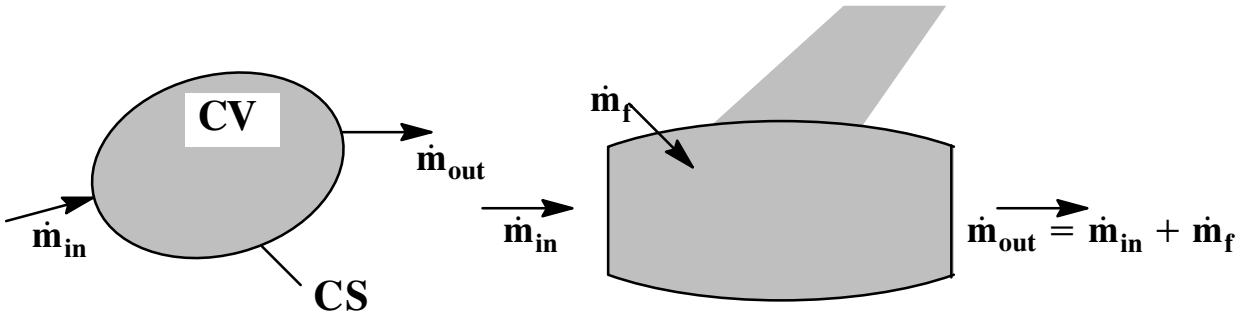
1.2 Application to Fluids

A solid material generally retains its integrity and does not break up into smaller pieces . If it does, the equations above still hold but it is more difficult to analyze such cases, because we must follow the motion of all of the pieces. This problem always exists for a fluid. As it moves, a single parcel of fluid not only distorts into very irregular shapes under the action of external forces, but it also breaks up into numerous, smaller parcels of fluid. We therefore find it mathematically more tractable, not to consider a fixed mass of fluid in our analyses, but rather consider a fixed volume in space, and analyze the flow of the fluid through it. Obviously, now the mass of fluid in the volume changes in time and we must re−word the conservation principles above.

It is still possible to analyze a fluid flow by considering a fixed mass of fluid, and following it, but it is generally more difficult for the reasons given above. The fixed mass approach is called the "Control Mass" approach, abbreviated as CM here. When we consider a fixed volume, we refer to the analysis as "Control Volume" approach, abbreviated CV here.

1.3 Steady Flow (s)

In Engineering, when we apply results of analysis to real problems we have to make certain assumptions in order to solve the problems. In each of the equations we develop here, we will indicate assumptions involved by using a code to indicate them directly following the equation. The first of these we will present is the steady flow assumption, and for it we use the code "s" as indicated in parentheses in the heading of this section.



1.3.1 Mass Conservation

Referring to the sketch above, we see that the mass within a CV is increased if there is a surplus of fluid flowing into it across its boundary, called the Control Surface, CS. If conditions exist such that no changes occur in time, within the CV or at any point on the CV, i.e., the influx of mass just balances the efflux, then we call the flow a "steady flow." Such conditions are very closely approximated in a large number of important Engineering problems. An example would be a jet engine operating on an aircraft flying straight and level with no accelerations. The rate at which mass flows from the exhaust equals the rate at which mass flows into the inlet plus the rate at which fuel mass is supplied to the engine.

We will assume steady flow for the remainder of these notes.

1.3.2 Momentum Conservation

For steady flow, momentum within the control volume will be constantin time too. The momentum that exists in the fluid within the CV will however be determined by the action of force $\vec{F}$. The momentum will be just the difference between the rate of momentum flow out of the CV and the rate of momentum flow into the CV, both flowing across the CS.

1.3.3 Energy Conservation

In the same way, energy of the fluid in the CV will not change in time for steady flow, but its value will be determined by the heat added and work done. Now, however, we must consider not only the internal (thermal) energy of the fluid, but also its kinetic energy and its potential energy. The energy for steady flow, however, will again be just the difference between the efflux of

energy and the influx of energy and will be balanced by the heat added and work done at the boundary of the CV.

Re−wording the Conservation Principles for steady flow through a control volume then, we can say:

Conserved Quantity	Word Statement
Mass	**All fluid flowing into the system flows out at the same rate.**
Momentum	**A force *on* a control volume *increases* the net rate of momentum flow out of the system relative to that flowing in. The rate of increase is proportional the force applied.**
Energy	**Heat added *to*, and work done *on*, a contol volume *increase* the net rate of energy flow out of the system, relative to that flowing in. The increase is proportional to the sum of the rate at which heat is added and work is done at the control surface.**

Note that the conservation laws for a CV are concerned with the flow *rate* of the conserved quantities. In other words we are dealing with the *rate* at which mass, momentum and energy are convected past a surface (the CS) fixed in space.

1.4 Mathematical Form of Conservation Equations

1.4.1 Mass Conservation or Continuity Equation

If a CV system has multiple inlets and outlets, then the mass conservation principle may be written

$$\Sigma(\dot{m})_{in} = \Sigma(\dot{m})_{out} \qquad 1.1 \quad (s)$$

where $\dot{m}$ is the mass flow rate, and the summations are carried out over all in(lets) and out(lets)

and $\dot{m} = \left\{\frac{dm}{dt}\right\}_{CS}$

1.4.2 Momentum Equation

Similarly,

$$\vec{F} = \Sigma\left\{\frac{d(m\vec{V})}{dt}\right\}_{out} - \Sigma\left\{\frac{d(m\vec{V})}{dt}\right\}_{in}$$

But $$\left\{\frac{d(m\vec{V})}{dt}\right\}_{CS} = m\frac{d\vec{V}}{dt} + \vec{V}\frac{dm}{dt}$$

$$= 0 + \vec{V}\dot{m}$$

since from the steady state assumption $\frac{d\vec{V}}{dt} = 0$ on the CS

$$\therefore \vec{F} = \Sigma(\dot{m}\vec{V})_{out} - \Sigma(\dot{m}\vec{V})_{in} \qquad 1.2 \quad (s)$$

Remember, $\vec{F}$ is the *total* force acting on the fluid in the CV.

1.4.3 Energy Equation

$$\dot{Q} + \dot{W} = \Sigma(\dot{E})_{out} - \Sigma(\dot{E})_{in} \qquad 1.3 \quad (s)$$

where $\dot{E} = \dot{U} + \dot{m}V^2/2$

= internal energy flow rate + kinetic energy flow rate

Note that the energy and continuity equations are scalar equations, whereas the momentum equation is a vector equation.

1.5 Incompressible Fluid, Constant Density Assumption (inc)

The density of a fluid under conditions of small or moderate temperature change, will itself change only because of compression due to pressure change. Liquids do not compress easily and for engineering purposes it is acceptable to assume that they are incompressible over a wide range of conditions. It is also usual to assume that they remain at constant density if temperature changes are small.

Gases are considerably more easily compressed than are liquids but we find that we can still assume they are incompressible as long as pressure changes are limited. They can also be assumed to be of constant density if temperature changes are small. Since pressure changes in a gas result from velocity changes, the constant density, or incompressible assumption can only be made for low speed gas flows. In air, a good rule of thumb is to limit the assumption to $M < 0.4$, where the Mach number M is the ratio of flow speed to speed of sound in air.