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# Treatment of Uncertainty in Long-Term Planning

## 1 Introduction

The problem that the long-term planner is faced with solving is an inherently uncertain one because it addresses the future. In making use of software which implements generation expansion planning (GEP), transmission expansion planning (TEP), or co-optimized expansion planning (CEP), it is necessary to make many assumptions on what that future will be. Examples of attributes characterizing the future about which the planner must make assumptions include:

- Cost of money (discount rate);
- The rate at which technology investment cost will change (maturation rate), including
    - Cost of bulk storage facilities;
    - Cost of transmission;
- Fuel costs forecast;
- Demand forecast (including effect of electrification);
- Plant retirement dates and salvage values;
- Policy changes (e.g., changes in federal production tax credit/investment tax credit, renewable portfolio standards, $CO_2$ reduction requirement)
- Capabilities of renewable (wind and solar) resources;
- Availability of certain technologies (e.g., small modular reactors)

In these notes, we describe different ways to represent uncertainty and different ways to model it within optimization models such as GEP, TEP, and CEP.

## 2 Parameters vs. decision variables

Before addressing the topic of representing uncertainty, it is useful to clarify, for an optimization model, the difference between a parameter and a decision variable, as indicated in Figure 1.

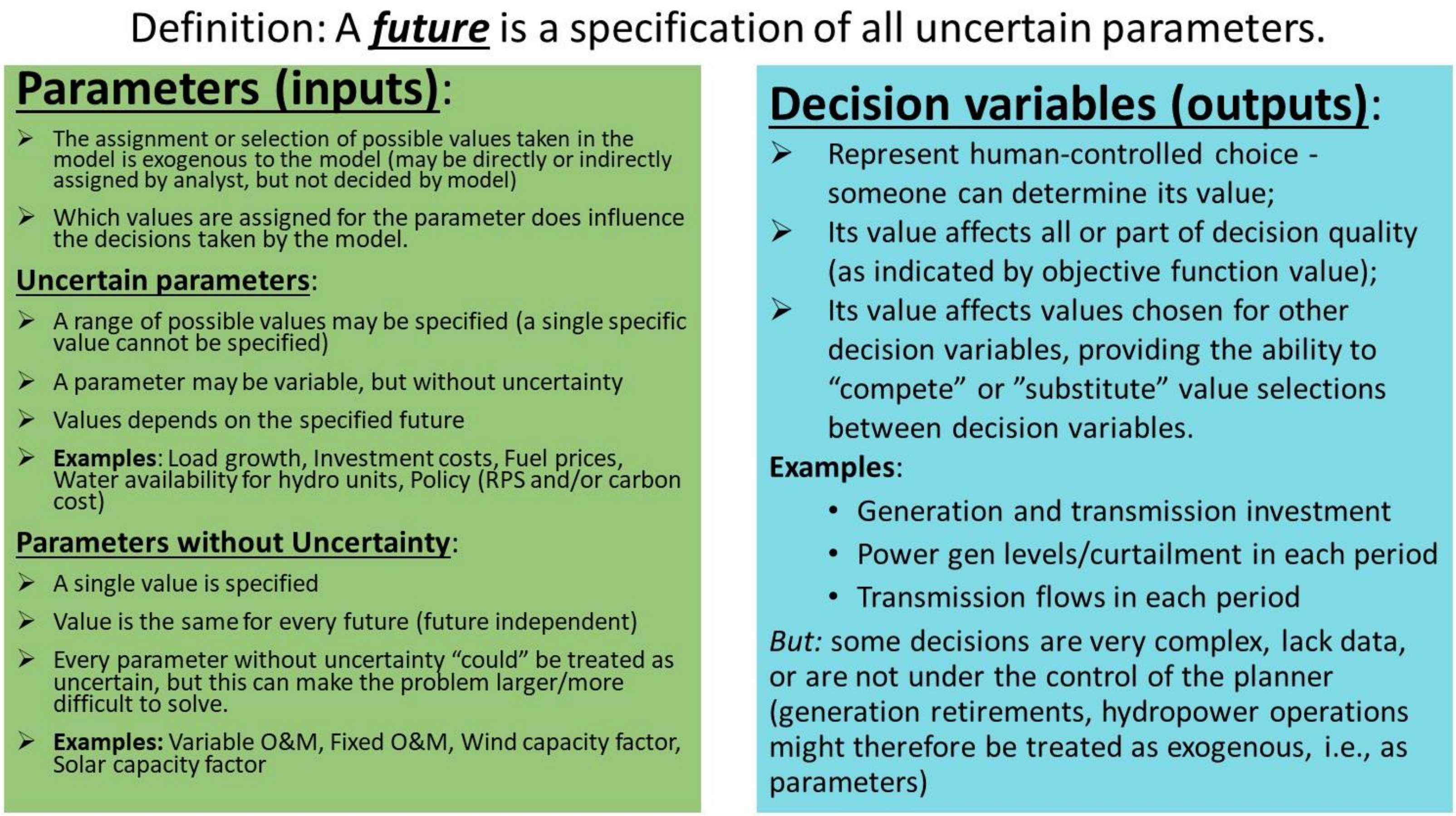



**Figure 1: Parameters and Decision Variables**

Some features of an optimization model may be represented as either a parameter or as a decision variable, and the analyst needs to decide which way such features should be modeled. Distributed energy resources (DER) are like this. DER includes energy efficiency, demand response, rooftop solar (residential, commercial, industrial), and microturbines. Each of these types of DER may be modeled as a parameter, with or without uncertainty, or they may be modeled as a decision variable. DER is often modeled as a parameter when it is considered to be outside the realm of decision and a decision variable when it can be decided.

## 3 Representing uncertainty

One can represent uncertainty by identifying the range within which one may reasonably expect each attribute to lie. For example, we could specify the price of natural gas in one of the following ways:

Time-independent:

- Point value: For each year, it will be \$4.5/MBTU;
- Range: For each year, it lies between \$3/MBTU and \$6/MBTU;
- Distribution: For each year, it is normally distributed with an expected value of \$4.5/MBTU and a standard deviation of \$0.5/MBTU; as shown in Figure 2 below, this means it will fall within the $\mu\pm3\sigma=4.5\pm1.5=(3,6)$ with probability 0.997, i.e., there is only a 0.003 probability of finding it outside the range of (3,6).

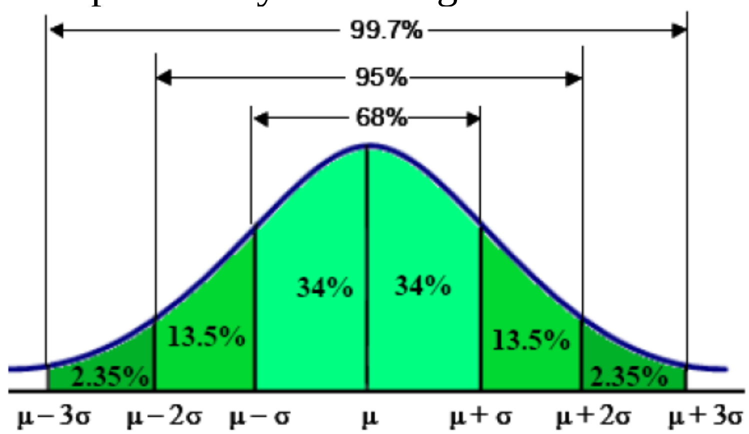



**Figure 2: Confidence intervals for a normally distributed variable**

Time-dependent:

- Point value: The year 1 value will be \$4.5/MBTU and will grow at 2% per year.
- Range: The year 1 value will fall within a range of \$3/MBTU to \$6/MBTU, with the central value of \$4.5/MBTU growing at 2% per year; the lower bound growing at 1% per year and the upper bound growing at 3% per year.
- Distribution: The year 1 expected value will be \$4.5/MBTU with a \$0.5 standard deviation, the expected value will grow 2% per year and the standard deviation will grow 5% per year. A plot of

this uncertainty would appear as in Figure 3. One observes in this figure how (a) the expected price will increase with time, and (b) the uncertainty will also increase with time.

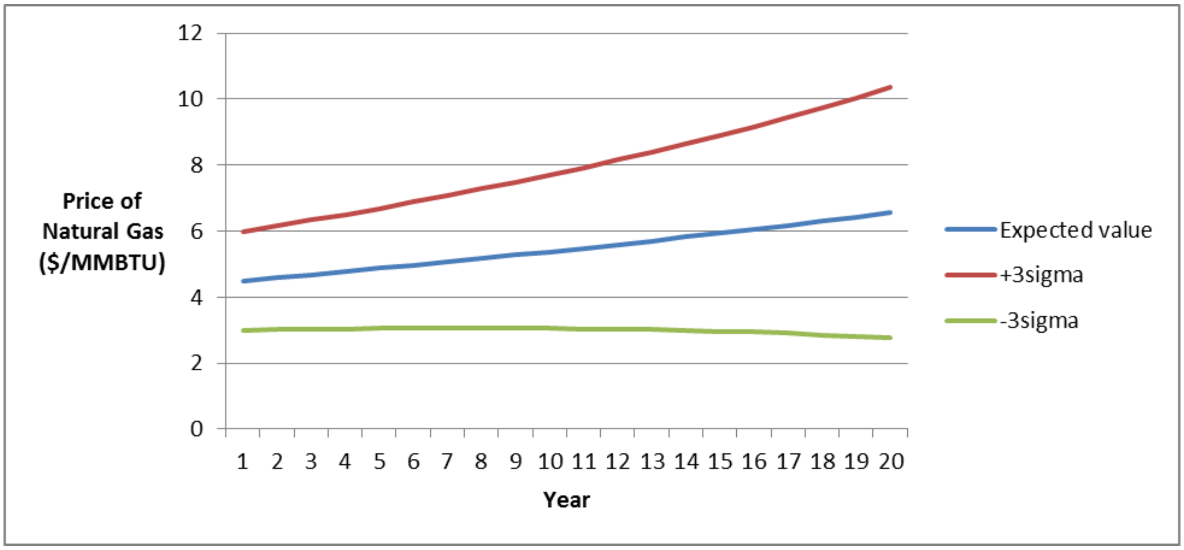



**Figure 3: Specification of uncertainty in natural gas price**

Aside: We may also apply advanced forecasting techniques to provide future estimates of expected value and uncertainty. Some forecasting methods that are commonly used for this purpose include regression, time series forecasting (ARIMA models and exponential smoothing models), or neural networks and other machine learning methods. These are worthy topics of study for uncertainty representation, but we do not have time to address them.

## 4 Two classes of uncertainty

We may group uncertainty into two different classes.

- **Global uncertainties** are those for which different values produce significantly different expansion planning results. Examples of global uncertainties are those related to the implementation of emissions policies, very large changes in demand growth, public rejection of a certain type of resource (nuclear) and its consequential unavailability, or an innovation that results in dramatic change in a technology's investment costs. A set of realizations on global uncertainties are appropriately thought of as a *future* (some literature will use the term *scenario* instead of *future*). In Figure 1 above, we defined a future as a specification of all uncertain parameters; here, we focus that definition to address global uncertainties only. It can be difficult to forecast some types of global uncertainties because (i) they may have occurred rarely or never, so that there is no historical information to be used in making statistical inferences about their future occurrence; or (ii) there is strong reason to believe that the historical information does not characterize the future (weather, due to climate change, may be like this).

- **Local uncertainties** can be parameterized by probability distributions or uncertainty sets based on historical data. Examples of local uncertainties include small variations in near-term load growth, investment costs, and fuel prices.

Figure 4 illustrates conceptualization of a single uncertainty in terms of being represented globally and locally.

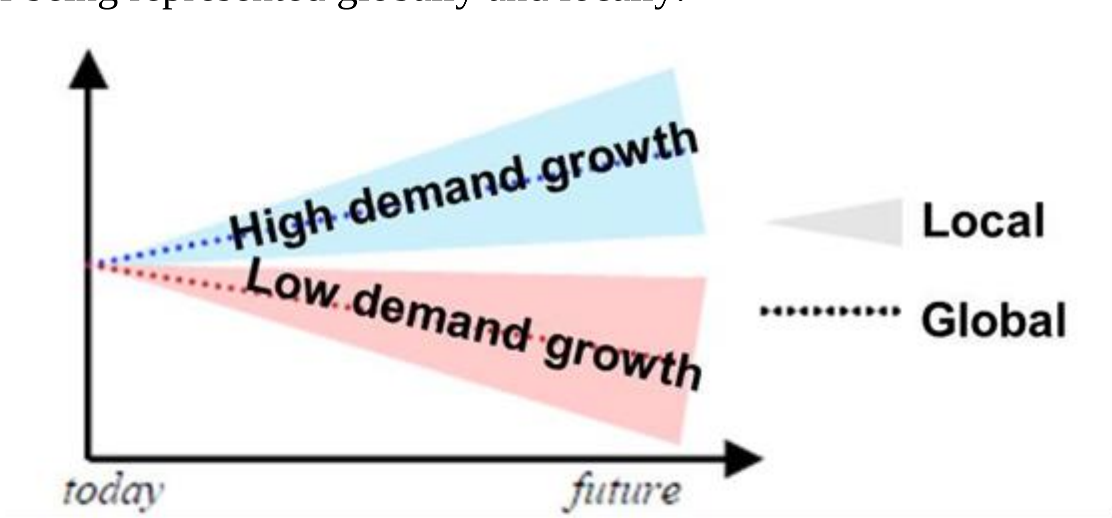



**Figure 4: Conceptualization of a single uncertainty characterized globally and locally**

Figure 5 illustrates conceptualization of multiple uncertainties in terms of being represented globally and locally, for a GEP. Each large red arrow represents a different set of realizations on several global uncertainties, i.e., they are different futures. The grey cones represent local uncertainty within each future. The pie charts terminating each red arrow are generation portfolios corresponding to the GEP solution resulting from consideration of the given uncertainties.

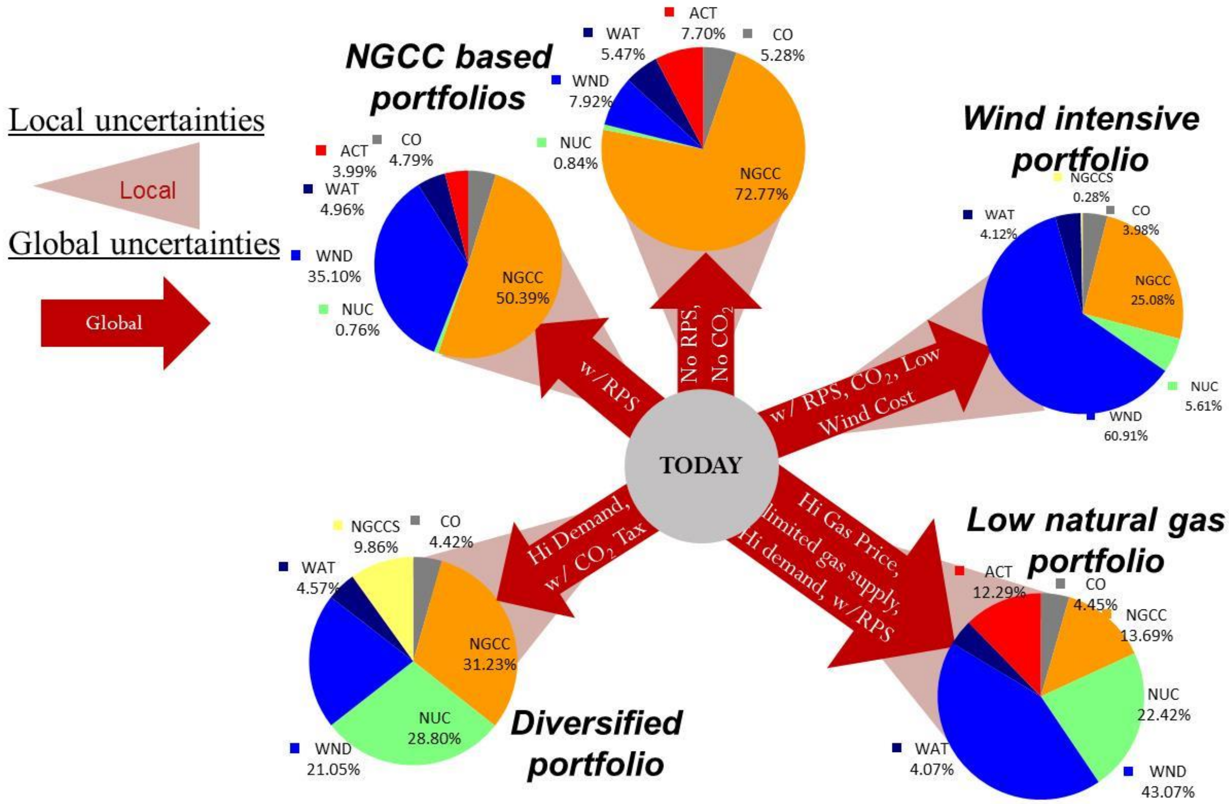



**Figure 5: Conceptualization of multiple uncertainties characterized globally and locally**

## 5 Methods of handling uncertainty within optimization

There are at least five ways of handling uncertainty within expansion planning optimization.

- Scenario (or "future") analysis
- Monte Carlo simulation
- Stochastic programming
- Adaptation: core approach
- Robust optimization

We will describe each of these in the following sections.

## 6 Scenario (or "future") analysis

In the simplest of scenario analyses, each uncertain attribute may take on two or more point values. As we have seen previously, a scenario, or future, is defined as a set of realizations on each uncertain attribute (where we limit uncertain parameters to only the global ones). An example from a 2008 study done by MISO is illustrative[1]. This example was taken from [1]. Table 1 shows an uncertainty matrix which provides six point values (low, med/low, reference, med/high, and high) for each of several uncertainties. The uncertainties are classified into capital costs, load, fuel prices, environmental allowance cost, economic variables, and siting limitations.

**Table 1: Uncertainty matrix**

| Uncertainty | Unit | Low | Mid/Low | Reference | Mid/High | High |
|---|---|---|---|---|---|---|
| **Alternative Capital Costs** | | | | | | |
| Coal | ($/KW) | 1,653 | | 1,835 | | 2,019 |
| CT | ($/KW) | 545 | | 605 | | 665 |
| CC | ($/KW) | 774 | | 859 | | 945 |
| IGCC | ($/KW) | 1,901 | | 2,111 | | 2,323 |
| Nuclear | ($/KW) | 2,245 | | 2,493 | | 2,743 |
| Wind | ($/KW) | 1,720 | | 1,910 | | 2,101 |
| CC w/Sequestration | ($/KW) | 1,003 | | 1,114 | | 1,226 |
| IGCC w/Sequestration | ($/KW) | 2,475 | | 2,748 | | 3,023 |
| Demand Response | ($/KW) | | | | | |
| Storage | ($/KW) | | | | | |
| **Load** | | | | | | |
| Demand Growth Rate | % | PowerBase -25% | | PowerBase | | PowerBase +25% |
| Energy Growth Rate | % | PowerBase -25% | | PowerBase | | PowerBase +25% |
| Energy Efficiency | % | None | | None | | 3% over 10 Years |
| **Fuel Prices** | | | | | | |
| Gas | ($/MMBtu) | Reference -20% | | 2007 w/4% Growth | Ref + 20% | Reference + 50% |
| Oil | ($/MMBtu) | Reference -10% | | 2007 w/4% Growth | | Reference + 10% |
| Coal | ($/MMBtu) | Reference -10% | | 2007 w/2% Growth | | Reference + 10% |
| Uranium | ($/MMBtu) | Reference -10% | | 2007 w/2% Growth | | Reference + 10% |
| **Environmental Allowance Cost** | | | | | | |
| SO2 | ($/ton) | Reference -25% | | PowerBase | | Reference +25% |
| NOx | ($/ton) | Reference -25% | | PowerBase | | Reference +25% |
| CO2 | ($/ton) | 0 | | 0 | 7 | 25 |
| Hg | ($/ton) | Reference -25% | | PowerBase | | Reference +25% |
| **Economic Variables** | | | | | | |
| Wind Credit | $ | 0 | | 19 | | 19 |
| Discount Rate | % | 5 | | 8 | | 10 |
| Inflation Rate | % | 2 | | 3 | | 5 |
| Uneconomic Coal Retirement | | As Scheduled | | As Scheduled | | Forced Retirement |
| **Siting Limitations** | | | | | | |
| Limited Transmission | | No Limitations | | No Limitations | | Limitation |
| Delayed Lead Time on Coal/IGCC | | No Delay | | No Delay | | 5 Year Delay |
| Nuclear Siting Limitation | | Existing & Allowed | | Existing & Allowed | | No Limitation |
| CT & CC Siting Limitation | | No Limitations | | No Limitation | | Limited to Urban Areas |

[1] This was a part of the so-called Joint-Coordinated System Plan (JCSP) studies. Many other analyses were done for the JCSP studies than what are shown here, and certainly, since then, MISO has evolved this procedure in many other studies.

Five different scenarios were created by selecting specific values for the various uncertainties. The five different scenarios were named Reference, DOE 20% Wind Mandate, DOE 30% Wind Mandate, Environmental, and Regulatory Limitation. The specific choices of each uncertain variable for each scenario are listed in the Futures matrix of Table 2 where the entries are L (low), R (reference), M (not sure), and H (high).

**Table 2: Futures (Scenario) Matrix**

| Uncertainty Matrix | Capital Investments | | | | | | | | | | Load | | | Fuel | | | | Effluent $ | | | | Econ Variab. | | | | Siting | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Futures | CC | CT | Coal | IGCC | Wind | Nuclear | CC w/Sequestration | IGCC w/sequestration | Demand Response | Storage | Demand growth Rate | Energy Growth Rate | Energy Efficiency | Gas | Coal | Oil | Uranium | SO2 | NOx | CO2 | Hg | Wind Credit | Inflation | Discount Rate | Coal Uneconomic | Limited Transmission | Delayed Lead Time on Coal/IGCC | Nuclear Siting Limitation | CC & CT Siting Limitation |
| Reference | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R | R |
| DOE 20% Wind Mandate | R | R | R | R | R | R | R | R | R | M | R | R | R | R | R | R | R | R | R | R | R | L | R | R | M | R | R | R | R |
| DOE 30% Wind Mandate | R | R | R | R | R | R | R | R | R | H | R | R | R | R | R | R | R | R | R | R | R | L | R | R | H | R | R | R | R |
| Midwest ISO Additional Futures Being Modeled for MTEP 2009 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| Environmental | H | H | R | R | R | R | H | H | H | H | L | L | H | H | L | H | R | R | R | H | H | R | H | R | H | R | R | H | R |
| Regulatory Limitation | H | H | R | R | R | R | R | R | H | R | R | R | H | M | R | H | R | R | M | M | R | R | R | R | R | H | H | R | H |

A generation expansion plan was created, for each Eastern Interconnection region (see Figure 6), and for each scenario, using a 15% planning reserve margin.

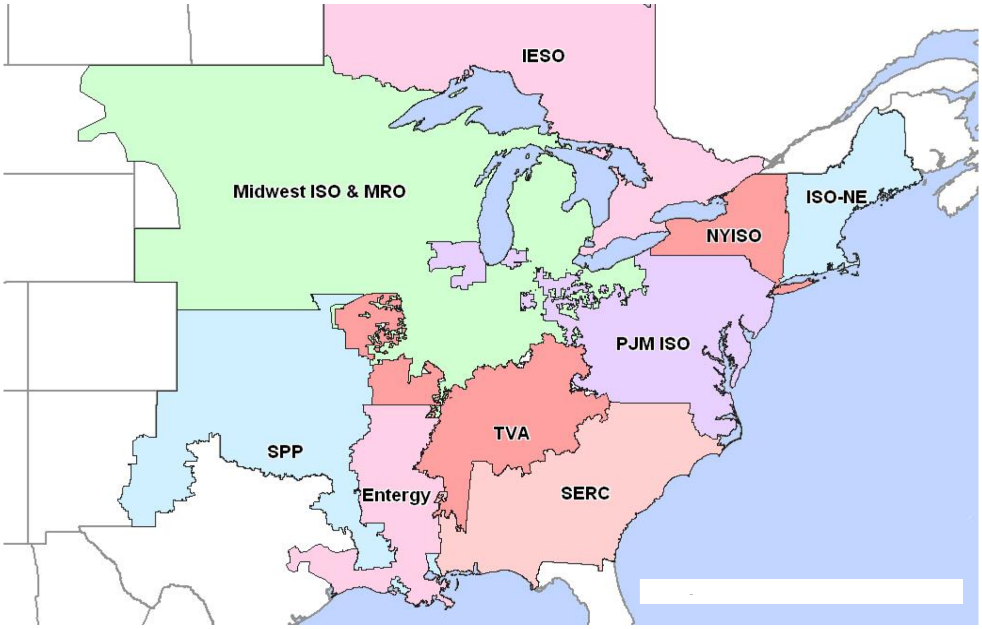



**Figure 6: Eastern Interconnection Regions used in Study**

Two transmission designs were developed, one under the reference scenario and one under the DOE 20% wind mandate scenario. They are illustrated below in Figure 7 and Figure 8.

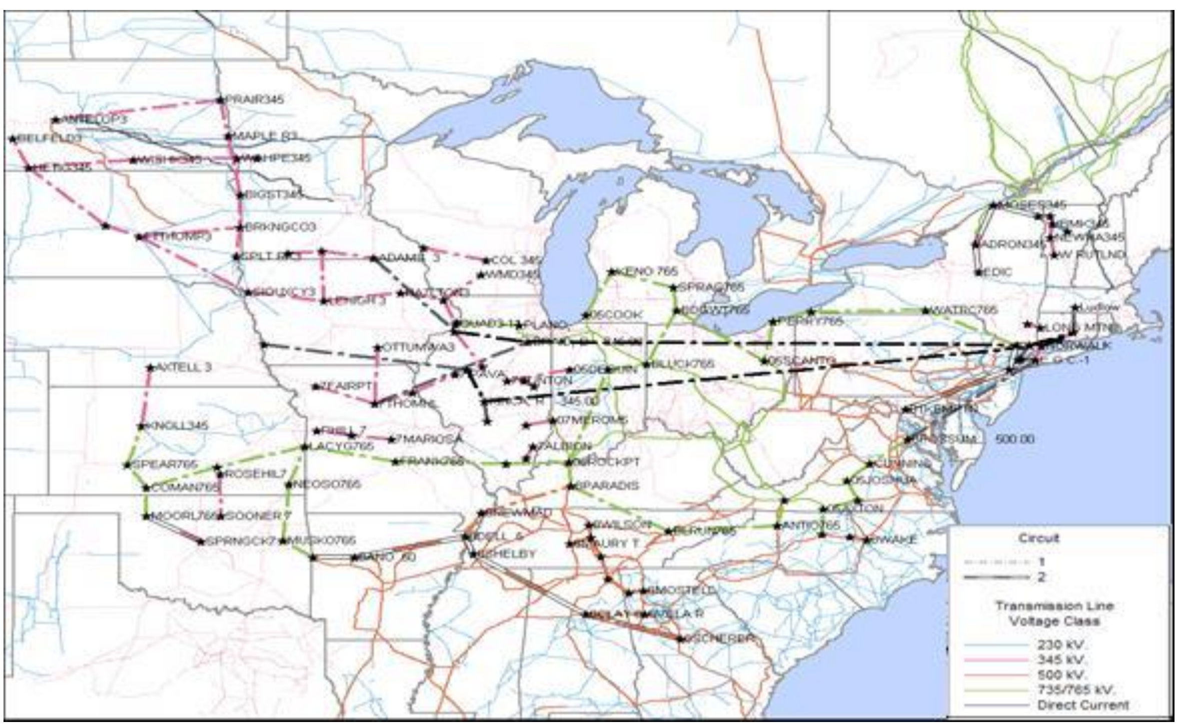



**Figure 7: Transmission design created for Reference Scenario**

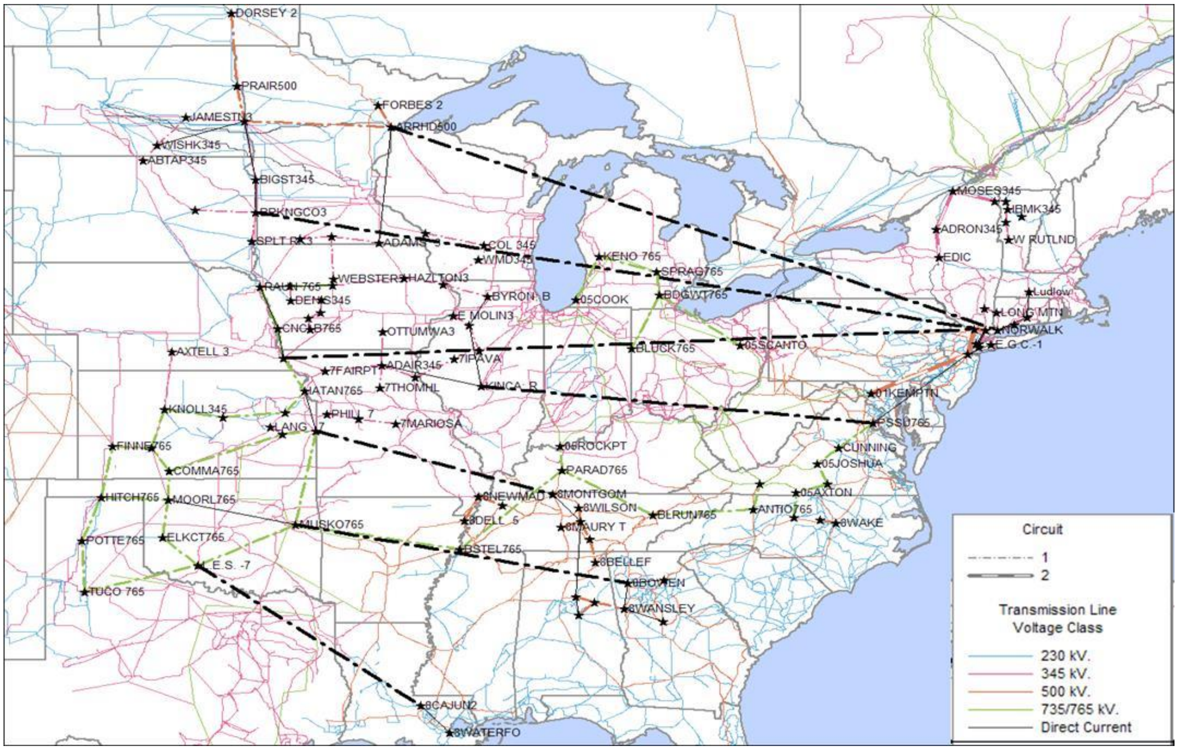



**Figure 8: Transmission design created for 20% DOE scenario**

A robustness testing was performed by evaluating each of the two transmission designs under various scenarios. They were looking for the transmission plan that performs best under the various scenarios.

Four scenarios were used for the robustness testing: Reference, Scenario 2 (20% Wind), Scenario 3 (30% wind), and Scenario 4 (Environmental). The scenario for which the design was developed was not used in the robustness testing.

To evaluate a design under a particular scenario, a set of performance measures were identified, as follows:

- Long-term cost
- Short-term cost
- Benefit/Cost ratio
- Reliability
- Environmental Impacts (carbon emissions)
- Land use criteria

- Local economic impacts
- National security criteria
- Others

Each performance measure was scored on a basis of 1-10 (with the higher score being better) and then a total score was computed as the sum of individual scores. Figure 9 shows the result for the transmission design performed under the reference scenario. Figure 10 shows the result for the transmission design performed under scenario 2. The results indicate that the scenario designed under the reference scenario is more robust to the different futures.

MISO has certainly evolved the approach used in this 2008 study, but the basic approach of identifying futures, each as a particular selection of global uncertain parameters, is still a foundational part of their MTEP (and LRTP) procedure. We return to this approach in Section 9 where we will compare a more recent MISO-MTEP scenario analysis approach to a developed optimization method.

***<u>The underlying objective</u>*** is to find a plan that performs *robustly* in the various possible futures that we think are credible.
***<u>The method of doing so here</u>*** is to (i) design a plan in one future; (ii) test its performance under the other futures; (iii) choose the one that tests the best.

Transmission Design for Reference Scenario

| Attributes | Ranking Under Scenario 2 | Ranking Under Scenario 3 | Ranking Under Scenario 4 |
|---|---|---|---|
| Short –term cost | 10 | 5 | 7 |
| Long-term cost | 9 | 10 | 4 |
| Reserve Margin | 6 | 7 | 8 |
| Investor Impacts | 6 | 7 | 3 |
| Environmental | 5 | 7 | 10 |
| Local economic | 5 | 5 | 10 |
| National Security | 8 | 9 | 10 |
| | 49 | 50 | 52 |

**Figure 9: Scoring for Transmission Design Performed Under Reference Scenario**

Test Portfolio based on ***Scenario 2*** conditions

| Attributes | Ranking Under Scenario 1 | Ranking Under Scenario 3 | Ranking Under Scenario 4 |
|---|---|---|---|
| Short –term cost | 4 | 5 | 3 |
| Long-term cost | 9 | 2 | 4 |
| Reserve Margin | 6 | 7 | 4 |
| Investor Impacts | 2 | 7 | 3 |
| Environmental | 5 | 7 | 5 |
| Local economic | 5 | 5 | 4 |
| National Security | 7 | 4 | 10 |
| | 38 | 38 | 34 |

**Figure 10: Scoring for Transmission Design Performed Under Scenario 2**

## 7 Monte Carlo Simulation

One method of modeling parameter uncertainty is to represent each uncertain parameter $x_1$, $x_2$,… with its numerical distribution. Then repeatedly draw values from each distribution, and for each draw, make the desired computation using those values. If the parameter values are drawn as a function of their probabilities, as indicated by the distribution, then the computed reliability indices will also form a distribution, from which we may compute their statistics, e.g., mean and variance. The process is illustrated in Figure 11, where the loop must be implemented many times before the output converges to a steady-state distribution.

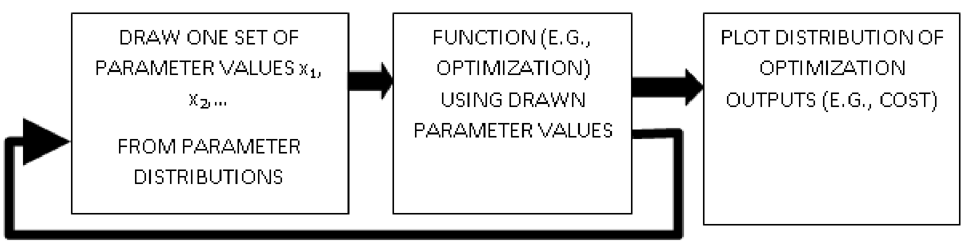



**Figure 11: Monte Carlo Simulation**

The draws (left-hand box in Figure 11) can be made by discretizing the probability density function (PDF) of each uncertain parameter, with each interval of each PDF assigned to an interval on [0,1] in proportion to its probability (area under the PDF curve for the discrete interval). Then a random draw on [0,1], which is then converted to the uncertain parameter value through the assignment, reflects the desired PDF of the uncertain parameter. Figure 12 illustrates the process, where the uncertain parameter is load, assumed to be normally distributed about an expected value.

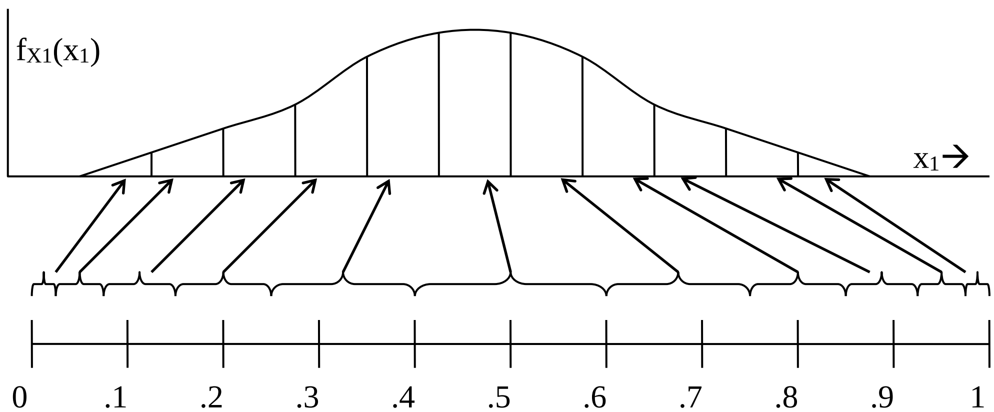



**Figure 12: Drawing Parameter Values According to a PDF**

This process is called Monte Carlo simulation (MCS) and is almost always an available option for making complex computations involving uncertain parameters. An advantage to MCS is that it is conceptually simple to implement.

A disadvantage is that it can be computationally intensive if

- the function (second box in Figure 11) is computationally intensive, because the function must be executed a large number of times to establish enough data to converge to a statistically valid output sample.
- the number of uncertain parameters is large;

It can be especially computationally intensive if both are true, i.e., if the function is computationally intensive and there are a large number of uncertain parameters.

A particularly useful approach is called "Guided MCS." There is a rich literature associated with application of Guided MCS to the development of operating rules, i.e., the rules associated with security-economy decision-making in real-time operations; a representative sample of this literature is [2, 3, 4, 5, 6]. This application is illustrated in Figure 13.

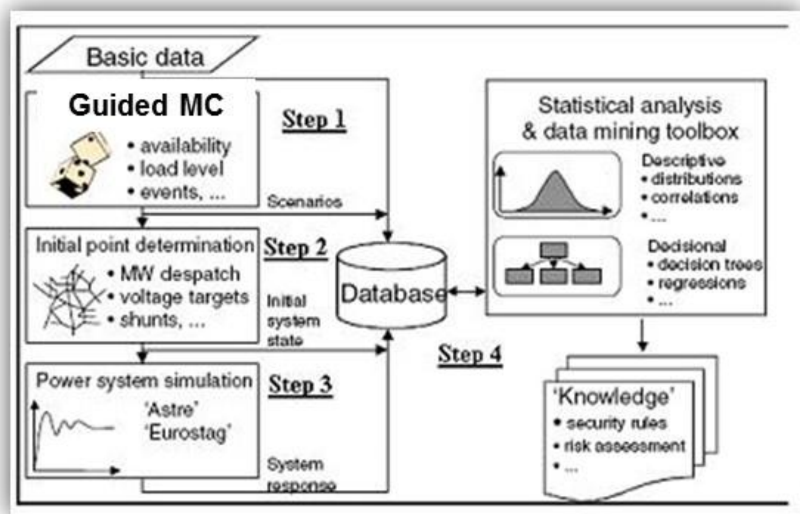



**Figure 13: Guided Monte Carlo Simulation**

This particular application of Guided MCS for developing operating rules is not an expansion planning application. It is presented here because it is a method of treating uncertainty that could be applied to expansion planning if there is information about the investment solution that could be used to weight the uncertainty space.

As observed in Figure 13, there are two main steps to Guided MCS: (A) Database Generation and (B) Statistical analysis. These steps are further broken down into sub-steps as indicated below.

1. Database Generation
    1. Guided MCS
    2. Optimal power flow
    3. Contingency analysis
2. Statistical analysis
    4. Compute reliability indices (LOLE, LOLP, risk, …)

5. Perform statistical analysis on output data to develop the operating rules.

Our interest is the use of step 1 to “guide” the MCS; the implication here is that we will use insight to focus simulations on the part of the decision space of most interest. In the case of generating operating rules, this part of the decision space is the boundary (based on reliability criteria) between acceptable and unacceptable operating conditions. This is illustrated in Figure 14.

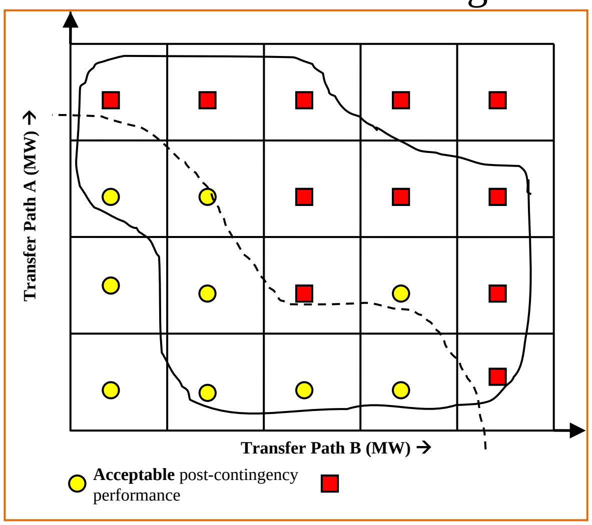
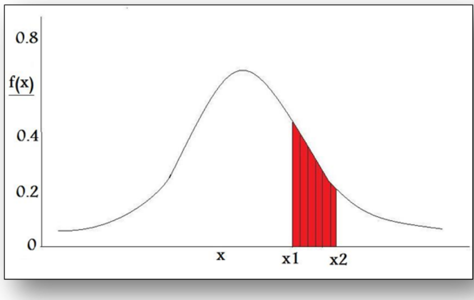
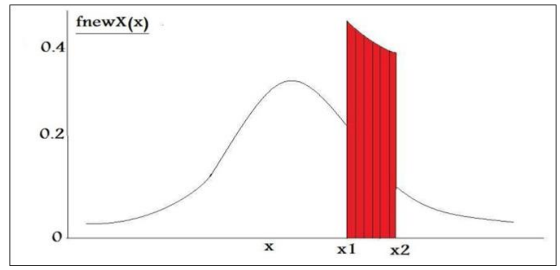



**Figure 14: Illustration of boundary between acceptable and unacceptable conditions**

The “guiding” part of the MCS is also referred to in the literature as importance sampling. The idea in importance sampling is that the selection of operating points is done based on a revised distribution, where the revision is made so as to bias the selection towards the desired conditions. This idea is illustrated in Figure 15 below.




Original distribution $f_X(x)$ and
region S of desired bias




Revised distribution $f_{newX}(x)$
showing bias in region S

$$fnew_{\underline{X}}(\underline{x}) = \begin{cases} p_1 f_{1\,\underline{X}}(\underline{x}) \text{ for } x \in S \\ p_2 f_{2\,\underline{X}}(\underline{x}) \text{ for } x \notin S \end{cases}$$

where $p_1+p_2=1$.
For example, if $p_1=0.75$, then 75% of the points are from S.

**Figure 15: Guided MCS (importance sampling)**

This could be applied to expansion planning by biasing the selection of uncertainty *realizations* (more general term than “conditions”) to focus more heavily on those realizations that motivate investments.

# 8 Stochastic Programming -

These notes are adapted from notes developed by J. Beasley of Brunel University, West London [7].

Stochastic programming can be separated into two distinct classes of problems: those with probabilistic constraints and problems with recourse.

## *8.1 Chance-constrained programming*

Problems with probabilistic constraints are those that are posed with constraints that must be met with a certain probability. An example is provided below.

$$\max \; f(\underline{x}) = 3x_1 + x_2$$
$$\text{s.t.} \quad x_1 + x_2 = 16$$
$$\Pr\left[a_1x_1 + a_2x_2 \le 4\right] \ge \gamma$$

where $a_1$ and $a_2$ are uncertain and described by distributions; $a_1x_1+a_2x_2$ is some condition of interest; and $\gamma$ is a probability level chosen by the decision-maker to be acceptable to the particular situation to which the problem applies.

This problem containing probabilistic constraints has been described as a chance-constrained optimization (CCO) problem, and its solution is referred to as chance-constrained programming; there is a rich literature related to it. Interestingly, up until 2012 there were only a few CCO applications to expansion planning in the literature, including a 2012 paper [8], but one of the best was a 2009 paper by Kit Po Wong's group [9] (Dr. Wong passed away in 2018). One can enter titles of these papers into scholar.google.com to identify related papers published since then.

One solves this problem by choosing values of $x_1$ and $x_2$ such that the objective function is maximized, the deterministic constraint is satisfied, and the probability that the inequality is satisfied is greater than $\gamma$. A conceptual approach for solving this problem is as follows:

1. Identify the $\{x_1, x_2\}$ space that satisfies the equality constraint; call this space $S_1$.
2. Identify the $\{x_1, x_2\}$ space that satisfies the probability constraint; call this space $S_2$.
3. The solution is $\{x_1,x_2\}^*$ contained in $S_1 \cap S_2$ that maximizes $f(\underline{x})$.

Most solution approaches involve transforming the chance constraints into deterministic ones and then applying an appropriate solver accounting for structure and convexity of the problem.

## *8.2 Recourse problems*

Recourse problems are so-called because they enable *recourse* following a decision. What is recourse?

An internet definition indicates it is

> "the act of resorting to a person, course of action, etc., in difficulty or danger."

A less formal equivalent of this is that recourse is an

> "act" that you take, once you have made some decision that gets you in trouble.

There are two "steps" here: a decision and then a recourse action. This very well characterizes recourse-oriented stochastic programs, or recourse problems. Over the past few years, reference to a "stochastic program" without further specification usually implies a recourse problem.

We adapt two examples from Beasley [7].

### 8.2.1 Example 1: Single stage SP recourse problem

We desire to make a decision now (period t=1) about the amount of capacity we need in year 5 (period t=2).

We assume that this capacity is going to cost $2000/kW.

We assume that the growth in peak load (including needed reserves), which drives the need for this capacity, is stochastic. We adopt a simple representation of the demand uncertainty by assuming the increase in peak load will be either

- Low: 500 MW with probability 0.6 or
- High: 700 MW with probability 0.4.

We have to make a decision now (in per t=1) on how much capacity to build because it will take us 5 yrs to build the new capacity. Thus, we must decide before the demand is actually known.

We may represent this situation as a tree-like structure as indicated in Figure 16.

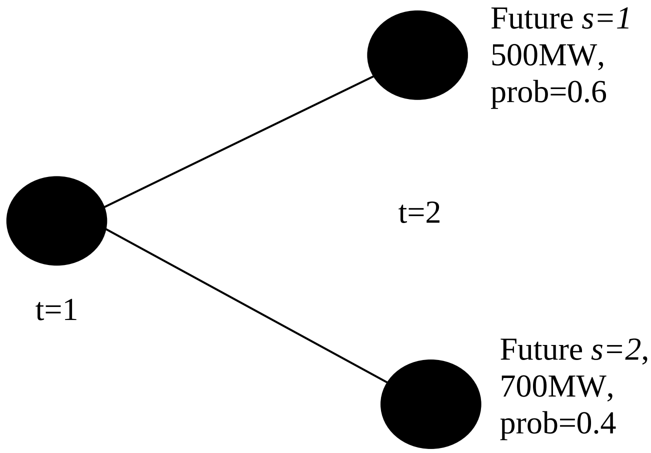



**Figure 16: Illustration of decision problem**

It is clear we will build no less than 500MW; no more than 700MW.

But do we build 500MW? 550MW? 600MW? 650MW? 700MW?

Let's consider that we build 500MW at t=1. This decision will be a good one if the t=2 demand for capacity is indeed 500MW.
However, if we build 500MW at t=1 but the t=2 demand for capacity is 700MW, then we will have to take recourse and add 200MW at time 2 in order to meet that demand. For example, we can purchase (at a cost of $3000/kw) 200MW of capacity from our capacity-rich neighbor, or we can pay some large loads to shut down during peak conditions.

We will assume in this simple model that we can *buy* capacity at t=2 but we cannot *sell* capacity at t=2. This assumption is to keep things simple; we could easily relieve this assumption.

We observe that, in this model:

- We **decide to build** at t=1
- We **observe** the realization of the uncertainties at t=2
- We **employ recourse**, a further decision, depending upon the realization observed.

Let's set up an analytic model to reflect this situation. To do so, we will refer to the two different realizations of the future demand for capacity (i.e., 500 or 700 MW) as "futures" or "scenarios."

Define

- $t,s$ as denoting the time period and the future;
- $x_1$ is the amount of capacity we decide to build at period $t=1$. We might call these the "build" variables.
- $C_s$ is the required capacity corresponding to future $s$ (assume the number of futures is $S$, i.e., $s=1,2,\ldots,S$).
- $y_{2,s}$ is the amount of capacity we will need to purchase at $t=2$ when the value of the demand is realized. We might call these the "recourse" variables.

We can write a constraint to ensure that the capacity requirement is always met:

$$x_1 + y_{2s} \geq C_s \qquad s = 1, 2, \ldots S$$

Observe that the amount of capacity we have in period $t=2$ may exceed the requirement. That is, we are ***not*** requiring

$$x_1 + y_{2s} = C_s \qquad s = 1, 2, \ldots S$$

because the equality sign would require either that we allow capacity sales (enabling $y_{2s}<0$), or our solution would always be $x_1=500MW$ since otherwise, it would be impossible to satisfy the equality if we overbuild (i.e., choosing to build $x_1$ and then learning in period $t=2$ that the required capacity is less than $x_1$).

We desire our objective to minimize total *expected* cost, given by

$$2 \times 10^6 x_1 + \sum_{s=1}^{S} \mathrm{Pr}_s \times 3 \times 10^6 y_{2s}$$

We have already argued that $y_{2,s}<0$ is not allowed. We will also impose the same for $x_1$, i.e., $x_1<0$ is not allowed, meaning we cannot elect to retire capacity in period $t=1$ (again, this is for simplicity and could be lifted if desired).

We can now write down an optimization problem which achieves our objective, as follows:

$$\min \quad 2\times 10^6 x_1 + \sum_{s=1}^{S} \Pr_s \times 3\times 10^6 y_{2s}$$

subject to

$$\begin{aligned} & x_1 + y_{2s} \geq C_s && s = 1,\ldots,S \\ & x_1 \geq 0 && \\ & y_{2s} \geq 0 && s = 1,\ldots,S \end{aligned}$$

What will solving this optimization problem give us?

- A value for $x_1$, which is the amount of capacity we should decide to build now.
- Values for $y_{2s}$, $s=1,\ldots,S$; this provides us with the optimal recourse decisions for all possible futures given that we choose to build $x_1$ now. Only one of these values will be relevant once the actual capacity requirement is known; the other values will be irrelevant.

It is important to observe here that the uncertainty is characterized using a discrete distribution (i.e., a probability mass function) instead of a continuous distribution (i.e., a probability density function). This is typical; if one desires to make use of continuous distributions, the computations become more intensive.

Five comments about terminology:

- Both sets of variables $x_1$ and $y_{2s}$ are *decision variables* in the sense used within the optimization literature.
- The variable $x_1$, previously referred to as “build” variables, is also referred to as a “*here and now*” decision variable.
- The variables $y_{2s}$, previously referred to as “recourse variables, are also referred to as “*wait and see*” decision variables.
- We refer to the problem presented here as a *single-stage problem* because there is only one set of variables $x_1$ corresponding to a decision under uncertainty (the variables $y_{2s}$ correspond to decisions

made only after the uncertainties of the problem are revealed and so do not correspond to decisions made under uncertainty).

- The SP recourse problem may also occur in a *multistage* form, which we address next.

### 8.2.2 Example 2: Two-stage SP recourse problem

Let's now consider that we have a third period $t=3$, in addition to our first two periods $t=1,2$. Here, period $t=1$ is "now," period $t=2$ is "year 5," and period $t=3$ is "year 10." We will retain all information used in Example 1 above, and to it we add information for period $t=3$. The problem is illustrated in Figure 17. Observe that t=2 probabilities are non-conditional, whereas the t=3 probabilities are conditional (they are conditional on being in the previous node).

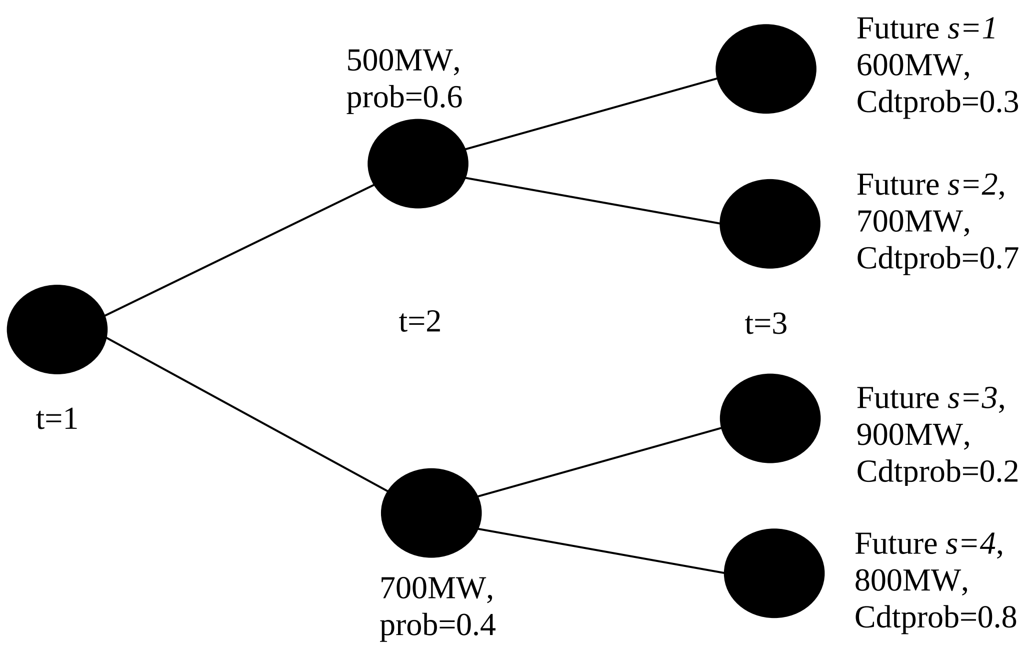



**Figure 17: Illustration of decision problem**

Here, we initially make a decision in period $t=1$ of how much capacity to build in period $t=2$, where we know the capacity requirement will either be 500MW (prob=0.6) or 700MW (prob=0.2). Once the uncertainty in period $t=2$ is revealed, we may make a recourse decision to purchase additional capacity in order to meet the capacity requirement in period $t=2$. All of this seems similar to the situation we had in Example 1.

But now, at period $t=2$, we have another decision to make, which is how much capacity to build in period $t=3$. This is a decision under uncertainty; once made, uncertainty in period $t=3$ is revealed, and we may make a recourse decision to purchase additional capacity.

To summarize then, as we move left to right across the tree of Fig. 11, we encounter the following decision problems:

- In the $t=1$ period, we decide how much capacity to build for the $t=2$ period. This is $x_1$, as in Example 1.
- In the $t=2$ period, the $t=2$ uncertainty is revealed.
- In the $t=2$ period, we make the recourse decision of how much capacity to purchase in order to satisfy capacity requirements of period $t=2$. These are the $y_{2,s}$ variables, as in Example 1. However, these variables may change, depending on the ultimate future we encounter, and there are four such futures. Therefore, we have $y_{2,1}$, $y_{2,2}$, $y_{2,3}$, $y_{2,4}$. ***Note carefully***! By defining these variables across all four futures, we are recognizing that the best recourse decision at the $t=2$ period may differ depending on what happens during the $t=3$ period. But can we use $t$=3 information in our decisions at $t$=2? Can we *anticipate* the future and use that future information?
- In the $t=2$ period, we decide how much capacity to build for the $t=3$ period. This would be $x_2$, but there are four possible futures for $t=2$, $s=1, 2, 3, 4$. Therefore we have $x_{2,1}$, $x_{2,2}$, $x_{2,3}$, $x_{2,4}$. ***Note carefully***! By defining these variables across all 4 futures, we are recognizing that the best decision at the $t=2$ period may differ depending on what happens during the $t=3$ period. But can we use $t$=3 information in our decisions at $t$=2? Can we *anticipate* the future and use that future information?
- In the $t=3$ period, the $t=3$ uncertainty is revealed.
- In the $t=3$ period, we make recourse decision of how much capacity to purchase to satisfy capacity requirements of period $t=3$. These are the $y_{3,s}$ variables; we will have 4 of them, i.e., $y_{3,1}$, $y_{3,2}$, $y_{3,3}$, $y_{3,4}$.

We assume the cost to build in period *t=1* is the same as the cost to build in period *t=2*. We also assume the cost to buy capacity in period *t=2* is the same as the cost to buy capacity in period *t=3*.

We first consider period *t=2*, requiring that what we build in period *t*=1 plus capacity we buy via recourse during period *t*=2 must equal or exceed the required capacity in period *t*=2, i.e.,

$$x_1 + y_{2s} \geq C_s \qquad s = 1, 2, \ldots S$$

These constraints will be:

$$x_1 + y_{2s} \geq 500 \qquad s = 1, 2$$

$$x_1 + y_{2s} \geq 700 \qquad s = 3, 4$$

At the *t=2* period, we may have excess capacity given by

$$x_1 + y_{2s} - 500 \qquad s = 1, 2$$

$$x_1 + y_{2s} - 700 \qquad s = 3, 4$$

And then at the *t=2* period, we will make a decision to build additional capacity, and then at the *t=3* period, we will learn the capacity requirement and subsequently take a recourse decision to purchase additional capacity. Thus, we will require that:

*Excess Capacity+Capacity built+Capacity Purchased>=CapRequired*

Writing in terms of our defined nomenclature, we have

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 600 \qquad s = 1$$

$$x_1 + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 700 \qquad s = 2$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 900 \qquad s = 3$$

$$x_1 + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 800 \qquad s = 4$$

> These equations (and original Beasley reference) imply the *t*=3 capacity values represent a *growth* from *t*=2 capacity values. To interpret all capacity values as peak values, then we should remove the left-hand-side constants (500, 700) from these equations.

We might think we are done with constraints; however, we need to reconsider our build/recourse variables at the *t=2* period; these are:

build variables: $x_{2,1}, x_{2,2}, x_{2,3}, x_{2,4}$

recourse variables: $y_{2,1}, y_{2,2}, y_{2,3}, y_{2,4}$

Here, the first subscript is the time; the second subscript is the future. There are 3 ways to think about these variables, as follows:

1. *When you can know*: When we are at period $t=2$, how will we know what is going to happen at period $t=3$? Answer: we will not know! Therefore, since we cannot know the future:
   - we can only distinguish between variables if their past is different;
   - we cannot distinguish between variables that have a different future but a common past!
2. *Implication of same information-histories*: Futures that share the same info-history until a particular time should also make the same decisions up to that particular time;
3. *How many decisions you can make*: A decision maker at $t$=2 can only make a single decision, s\he cannot make two separate decisions at $t$=2 depending on which $t$=3 future occurs.

This means that period $t$=2 variables originating from the 500MW node must be equal, i.e.,

$$x_{2,1}=x_{2,2}$$
$$y_{2,1}=y_{2,2}$$

and $t$=2 variables originating from the 700MW node must be equal, i.e.,

$$x_{2,3}=x_{2,4}$$
$$y_{2,3}=y_{2,4}$$

These are called the *non-anticipativity constraints* (NACs), implying we cannot anticipate the future. This further implies that futures with a common history must have the same set of decisions. These constraints result from the fact that the decision maker does not know, when s/he is in the initial time period, which scenario will occur, and so in that time period, investments are constrained to be the same across all $s$. This means the decision-maker cannot optimize pre-scenario decisions based on information about scenarios that have not yet occurred. Reference [10], a tutorial, provides the following additional perspective concerning NACs:

> "Depending on the manner in which the problem is formulated, it may be necessary to include specific conditions *to ensure that the decision sequence honors the information structure associated with the scenario tree*. These conditions are known as the nonanticipativity constraints, and impose the condition that scenarios that share the same history (of information) until a particular decision epoch should also make the

Why not define a single variable for each pair to start with? One answer is to clearly retain the expression of the nonanticipativity concept in the problem formulation, to remind us all of its necessity. Another answer is that stochastic programming problems are very "L-shaped," and as a result amendable to solution by decomposition methods where the nonanticipa-tivity constraints are relaxed in the subproblem.

Note in the tutorial quote, the qualifying statement "*to ensure the decision sequence honors the information structure associated with the scenario tree*" (italics added). It seems this author sees the motivation for NACs as a need to remain true to an "info-structure." What is this info-structure? It appears to be that **info comes at a certain time**. And to satisfy "the decision sequence honors" this info -structure, then decisions cannot use info that comes after the time at which the decision is taken.

Epoch – "the beginning of a distinctive period in the history of someone or something." (def: Oxford Languages). In this case, it means "node" in the decision tree.

As far as the optimization problem is concerned, NACs are simply additional mathematical constraints. The problem is solvable without them, i.e., if you solve the problem without them, you will get a solution in terms of what & when to build (build variables) and what & when to buy (recourse variables). However, such a solution will give, for each particular time, build & buy instructions for each future emanating from that particular time. In reality, only one such set of instructions may be followed. Thus, such a solution is not ***implementable***. Thus, per the quote, implementability & nonanticipativity are equivalent.

same decisions. In reality, the nonanticipativity constraints ensure that the solutions obtained are implementable, i.e., that actions that must be taken at a specific point in time depend only on information that is available at that time. For that reason, the terms nonanticipativity and implementability are sometimes used interchangeably. These nonanticipativity constraints, which are derived from the scenario tree, are a distinguishing characteristic of stochastic programs—solution methods are typically designed to exploit their structure."

We now formulate our objective function. We have just one cost incurred with certainty, namely that associated with $x_1$. All other costs are probabilistic. Let's identify the probability of each future and the cost of each future, using total probabilities for each future. We also repeat our tree of Figure 17 below, for convenience.

| Future | Total prob of each future | Cost |
|---|---|---|
| 1 | 0.6×0.3=0.18 | $2\times10^6 x_{21}+3\times10^6 y_{21}+3\times10^6 y_{31}$ |
| 2 | 0.6×0.7=0.42 | $2\times10^6 x_{22}+3\times10^6 y_{22}+3\times10^6 y_{32}$ |
| 3 | 0.4×0.2=0.08 | $2\times10^6 x_{23}+3\times10^6 y_{23}+3\times10^6 y_{33}$ |
| 4 | 0.4×0.8=0.32 | $2\times10^6 x_{24}+3\times10^6 y_{24}+3\times10^6 y_{34}$ |

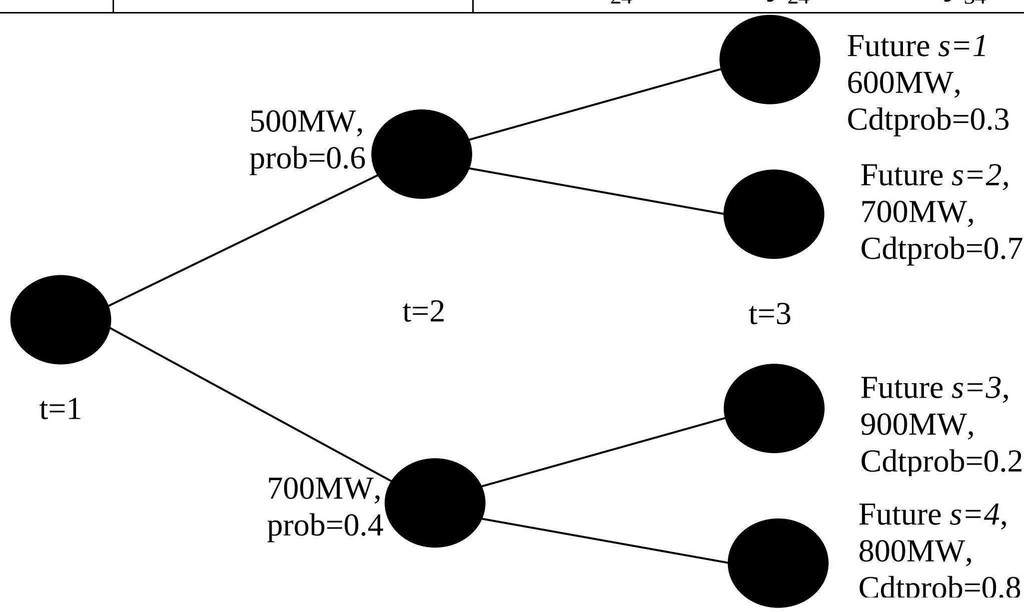



**Figure 17: Illustration of decision problem**

We can now write down our optimization problem. The objective is the cost of each future weighted by its probability, and we want to minimize it. The constraints are the need to satisfy the capacity requirements at the t=2 and t=3 periods, together with the non-anticipativity constraints. Thus,

$$\begin{aligned}\min\quad & 2\times10^6 x_1\\ & +0.18\left[2\times10^6 x_{21}+3\times10^6 y_{21}+3\times10^6 y_{31}\right]\\ & +0.42\left[2\times10^6 x_{22}+3\times10^6 y_{22}+3\times10^6 y_{32}\right]\\ & +0.08\left[2\times10^6 x_{23}+3\times10^6 y_{23}+3\times10^6 y_{33}\right]\\ & +0.32\left[2\times10^6 x_{24}+3\times10^6 y_{24}+3\times10^6 y_{34}\right]\end{aligned}$$

Subject to

$$x_1+y_{2s}\geq 500 \qquad s=1,2$$

$$x_1+y_{2s}\geq 700 \qquad s=3,4$$

$$x_1+y_{2s}-500+x_{2s}+\mathrm{y}_{3s}\geq 600 \qquad s=1$$

$$x_1+y_{2s}-500+x_{2s}+\mathrm{y}_{3s}\geq 700 \qquad s=2$$

$$x_1+y_{2s}-700+x_{2s}+\mathrm{y}_{3s}\geq 900 \qquad s=3$$

$$x_1+y_{2s}-700+x_{2s}+\mathrm{y}_{3s}\geq 800 \qquad s=4$$

These equations (and original Beasley reference) imply the *t*=3 capacity values represent a <u>*growth*</u> from *t*=2 capacity values. To interpret all capacity values as peak values, then we should remove the left-hand-side constants (500, 700) from these equations.

$$x_{2,1}=x_{2,2}$$

$$y_{2,1}=y_{2,2}$$

$$x_{2,3}=x_{2,4}$$

$$y_{2,3}=y_{2,4}$$

and all variables $\geq 0$

One good question is this: why not also define separate variables for the *t*=1 node? Indeed, we could do so if we applied NACs for those variables as well. Below is the (equivalent) formulation that results. Whether we want to do this depends on whether doing so offers computational benefits.

min

$$0.18\left[2\times10^6 x_{11} + 2\times10^6 x_{21} + 3\times10^6 y_{21} + 3\times10^6 y_{31}\right]$$
$$+0.42\left[2\times10^6 x_{12} + 2\times10^6 x_{22} + 3\times10^6 y_{22} + 3\times10^6 y_{32}\right]$$
$$+0.08\left[2\times10^6 x_{13} + 2\times10^6 x_{23} + 3\times10^6 y_{23} + 3\times10^6 y_{33}\right]$$
$$+0.32\left[2\times10^6 x_{14} + 2\times10^6 x_{24} + 3\times10^6 y_{24} + 3\times10^6 y_{34}\right]$$

Subject to

$$x_{1s} + y_{2s} \geq 500 \qquad s = 1,2$$
$$x_{1s} + y_{2s} \geq 700 \qquad s = 3,4$$
$$x_{1s} + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 600 \qquad s = 1$$
$$x_{1s} + y_{2s} - 500 + x_{2s} + \mathrm{y}_{3s} \geq 700 \qquad s = 2$$
$$x_{1s} + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 900 \qquad s = 3$$
$$x_{1s} + y_{2s} - 700 + x_{2s} + \mathrm{y}_{3s} \geq 800 \qquad s = 4$$

These equations (and original Beasley reference) imply the $t$=3 capacity values represent a *growth* from $t$=2 capacity values. To interpret all capacity values as peak values, then we should remove the left-hand-side constants (500, 700) from these equations.

$$x_{11} = x_{12} = x_{13} = x_{14}$$
$$x_{2,1} = x_{2,2}$$
$$y_{2,1} = y_{2,2}$$
$$x_{2,3} = x_{2,4}$$
$$y_{2,3} = y_{2,4}$$

and all variables ≥0

Stochastic programming of this sort has been applied to electric power system investment planning. There are many papers on this topic; some good work was done by the group led by Ben Hobbs of Johns Hopkins University [11, 12, 13, 14]. We introduce one such formulation in the next section.

# 9 Traditional stochastic programming formulation

This formulation is adapted from [15][2]. We define time period $t$=1,...$T$ and $s$ is a scenario or future in a set of scenarios $S$.

The following are decision vectors:

$\Delta C_1$ : Incremental cost investments in period 1 (MW) for "core"

$O_1$ : Operational costs in period 1 ($ present worth)

$\Delta A_{t,s}$ : Incremental scenario-specific capacity investment in period $t$, scenario $s$ (MW)

$A_{t,s}$ : Cumulative capacity investment in period $t$, scenario s (MW)

$O_{t,s}$ : Operational costs in period $t$, scenario $s$ ($ present worth)

The following are parameters:

$I_t$ : Investment cost in period $t$ ($/MW) for "core"

$I_{t,s}$ : Investment cost in period $t$ ($/MW) for scenario $s$

$P_s$ : Probability of scenario $s$

Then we write the traditional stochastic programming problem as:

$$\min I_1 \Delta C_1 + O_1 + \sum_{t\geq 2}\sum_{s} P_s \left( I_{t,s}\Delta A_{t,s} + O_{t,s} \right)$$

subject to:

1) Non-anticipativity constraints (build decisions same for all futures s):

$$\left.\begin{aligned} \Delta A_{2,s} &= \Delta A_{2,s-1} \quad \forall s \geq 2 \\ \Delta A_{3,s} &= \Delta A_{2,s-1} \quad \forall s \geq 2 \\ \Delta A_{4,s} &= \Delta A_{2,s-1} \quad \forall s \geq 2 \end{aligned}\right\} \leftarrow \begin{array}{l}\text{At each time } t = 2,3,4, \text{ build decisions are} \\ \text{same for all futures } s.\end{array} \qquad \text{(TSP)}$$

2) Accumulation (memory) constraint:

$$\Delta A_{1,s} = \Delta C_1 \qquad \forall s \qquad \leftarrow \text{Enables } \Delta \text{C}_1 \text{ to appear in memory constraint}$$

$$A_{t,s} = A_{t-1,s} + \Delta A_{t,s} \quad \forall t, s$$

3) Operational constraints for each future $s$ (not shown)

Problem (TSP) was simplified by eliminating technology and locational indices from the nomenclature and representing both transmission and generation within the same capacity vector. This problem is illustrated in Figure 18, where the three solid axes represent time (horizontal), accumulated generation capacity (vertical), and accumulated transmission investment.

[2] In the reference, a delay $d$ between decision and operation was included; here, to avoid unnecessary complications to an already complicated topic, we have dropped the modeling of delay.

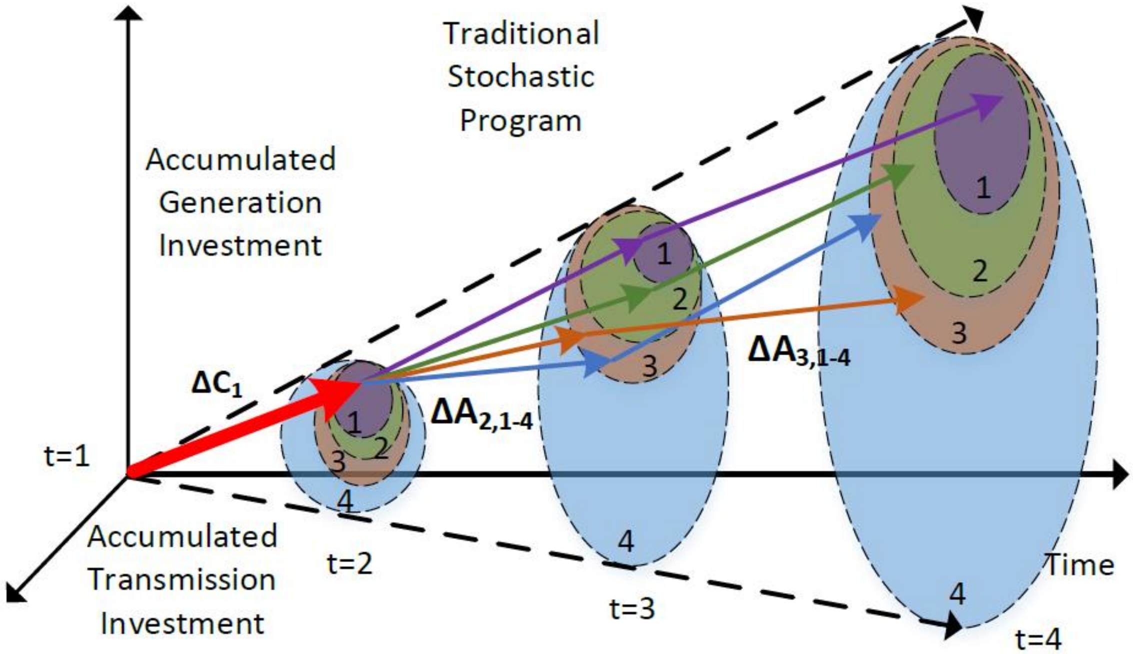



**Figure 18: Illustrating traditional stochastic program (TSP)**

In Figure 18, one observes the first dark red line, which represents the core investment $\Delta C_1$ in the first time period from which extend four additional smaller arrows each of which represent the recourse investments $\Delta A_{2,1}$, $\Delta A_{2,2}$, $\Delta A_{2,3}$, $\Delta A_{2,4}$, and $\Delta A_{3,1}$, $\Delta A_{3,2}$, $\Delta A_{3,3}$, $\Delta A_{3,4}$, made in times $t$=2 and $t$=3, respectively, for the four scenarios. Although this formulation has four time periods, it has only two decision stages, one at $t$=1 and another at $t$=2. The capacity additions corresponding to the time $t$=3 period, $\Delta A_{3,1}$, $\Delta A_{3,2}$, $\Delta A_{3,3}$, $\Delta A_{3,4}$, do not represent an additional decision stage, because the decision on those $t$=3 period additions are made at the $t$=2 period.

We will keep this illustration of TSP in mind, as we refer back to it when describing the adaptation approach Section 11.

# 10 Adaptive Coordinated Expansion Planning (ACEP)

In this section, we explain ACEP, starting from the concept illustrated in Figure 19.

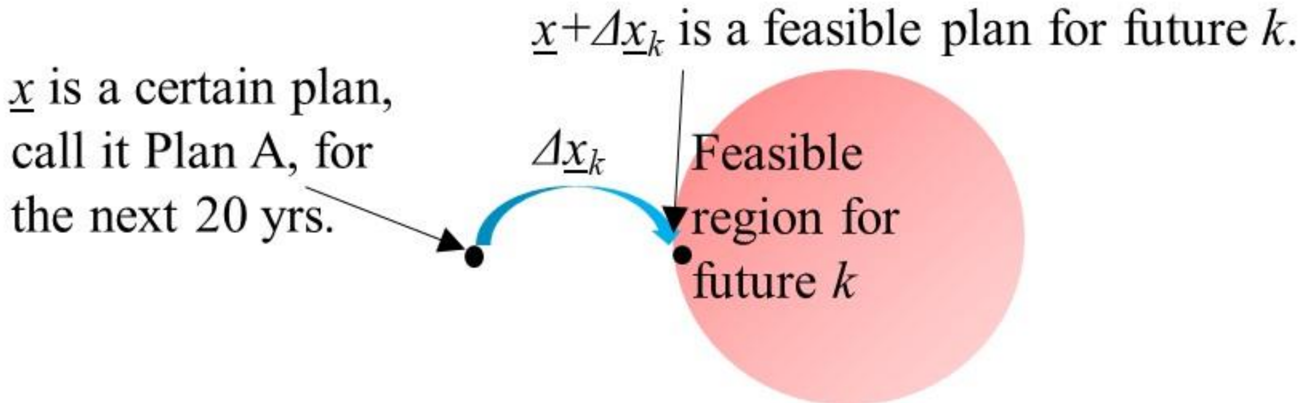



**Figure 19: Illustration of adaptation for a single future *k***

There are five things in Figure 19 that need explanation:

- Plan A: This is any particular plan for the system and time frame of interest; it may have been designed under a certain, specific future; it may have been designed using TSP or some other design paradigm. Exactly how Plan A was designed is not important to us, with one caveat… Plan A was not designed under future *k*.
- Future *k*: Future *k* represents to Plan A an alternative future, one for which Plan A was not explicitly designed.
- Core investments $\underline{x}$: This is the vector of investments (capacities of each generation and transmission investment) identified for Plan A.
- Adaptation investments $\Delta\underline{x}$: This is a vector of changes to Plan A investments necessary to make the system under Plan A feasible in future *k*.
- Future *k* investments $\underline{x}+\Delta\underline{x}$: This is the vector of investments necessary to make the system under Plan A feasible in future *k*.

The ***adaptation cost*** of Plan A to future k is the minimum cost to move Plan A to a feasible design in future *k*. It measures the additional cost to our Plan A cost if scenario *k* happens.

Now assume that we have several futures *k*. Figure 20(a) shows the situation where we have a Plan A (and investments $\underline{x}$) and four possible futures and four adaptations $\Delta\underline{x}_1$, $\Delta\underline{x}_2$, $\Delta\underline{x}_3$, and $\Delta\underline{x}_4$.

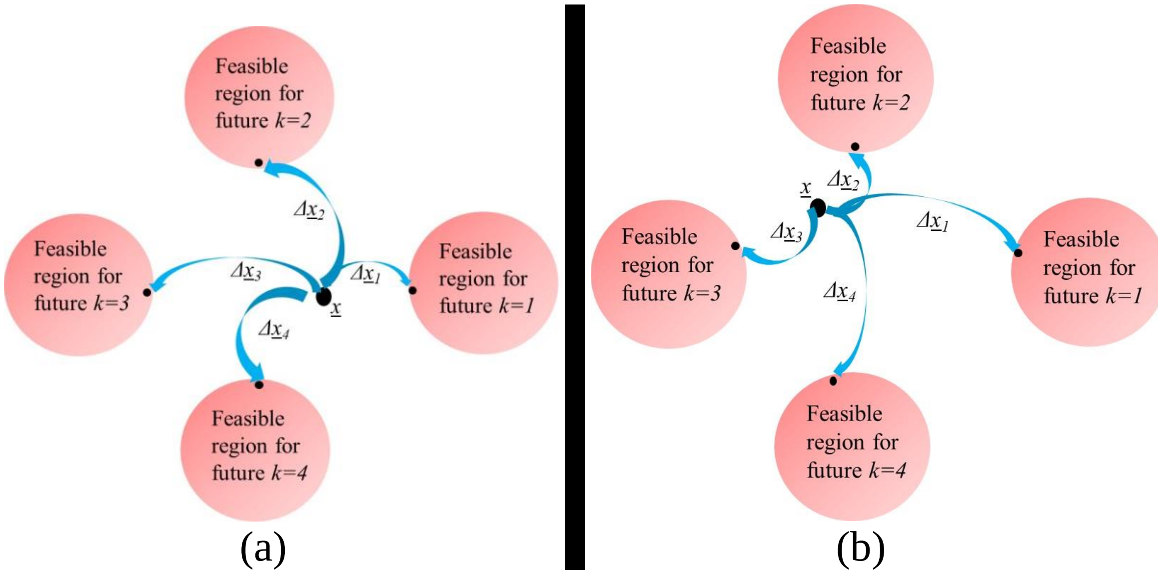



**Figure 20: Illustration of 2 adaptation strategies for four futures *k*=1,2,3,4**

It has been implicit in our discussion so far that Plan A is known, and so, in Figure 20(a), the Plan A investments $\underline{x}$ are known, and thus, if each future's feasible regions are known, the $\Delta\underline{x}_k$ are the decision vectors we use to place the four points in the respective feasible regions of each future *k*. In such an approach, the objective would be to place the four points in their respective feasible regions so as to minimize the cost of doing so. Since the futures are uncertain, if we can associate a probability $P_k$ with each one, then a better approach would be to minimize the expected cost of doing so. That is, our problem becomes:

**FORMULATION A1:**

**Minimize:**

$$\Sigma_k P_k \times \text{AdaptationCost}(\Delta\underline{x}_k) + [\Sigma_k P_k \times \text{OperationalCost}(\underline{x}+\Delta\underline{x}_k)]$$

**Subject to:**

**Operational constraints for future k=1,…N:** $\underline{g}_k(\underline{x}+\Delta\underline{x}_k)\leq\underline{b}_k$

Some observations on Formulation A1:

- The objective consists of two terms, an investment term and an operational term, and both are expectations.

- Investment term: For each $k$, only the cost of adaptation is included (the Plan A is fixed, and so $\underline{x}$ cannot be varied and therefore the cost of $\underline{x}$ should not influence the solution).
- Operational term: For each $k$, the operational cost is a function of $\underline{x}+\Delta\underline{x}_k$ (not the *cost* of $\underline{x}+\Delta\underline{x}_k$) because the infrastructure that will be in place for each future $k$ is the Plan A infrastructure ($\underline{x}$) plus the future $k$ adaptation ($\Delta\underline{x}_k$).
- Expectation: The presence of the probabilities $P_k$ on adaptation cost and on operational cost imposes that we are computing the expected value of these costs.

- The constraints will include operational constraints.
  - Operational constraints: These include total generation must equal total load, power balance at each bus, unit dispatch within the unit's $P_{min}$ and $P_{max}$, branch flow dependence on angles at terminal buses, branch flows within branch flow limits, reserve requirements must be satisfied, and carbon reduction must reach a certain level. These constraints, for each future $k$, will depend on the infrastructure in place for that future $k$. Therefore, these constraints are a function of $\underline{x}+\Delta\underline{x}_k$ (not the *cost* of $\underline{x}+\Delta\underline{x}_k$).
  - Nonoperational constraints: There may also be some non-operational constraints; these would be constraints on investments, e.g., limits on generation investments per year, and planning reserve margin. Although non-operational, they are a function of $\underline{x}+\Delta\underline{x}_k$ and so the above $g_k$ expression is OK.

Let's now extend this thinking by assuming that we may vary our Plan A, i.e., that $\underline{x}$ is also a decision vector. This makes some sense because $\underline{x}$ represents what we actually build, i.e., it is "the plan." Since we certainly want to be able to decide what we actually build, it is satisfying to represent $\underline{x}$ as a decision vector (in contrast, the adaptations $\Delta\underline{x}_k$ represent what we need to build only if future $k$ happens).

Then we observe an important question raised by Figure 20(a): **where to place $\underline{x}$**? Figure 20(b) shows an alternative placement of $\underline{x}$, i.e., we have modified Plan A (and therefore moved $\underline{x}$). In moving $\underline{x}$, observe that this modification also causes change to the four adaptations $\Delta\underline{x}_1$, $\Delta\underline{x}_2$, $\Delta\underline{x}_3$, and $\Delta\underline{x}_4$.

These observations raise a question: what should be our objective in placing $\underline{x}$? In thinking about that question for a bit, we conclude that there should be two components. One component should be the expected value of the adaptation costs and operational costs, exactly as expressed in Formulation A1 above. The other component should be the cost of what we indeed build. To distinguish between adaptations and what we indeed build, we identify what we indeed build as the "core," as represented by the vector $\underline{x}$. And the cost of that core is the "core costs." Thus, we want to minimize the core costs plus the expected adaptation cost plus the expected operational cost. This leads to Formulation A2.

**FORMULATION A2:**
**Minimize:**

$$\mathbf{CoreCosts(\underline{x}) + [\Sigma_k\ P_k \times AdaptationCost(\Delta\underline{x}_k)] + [\Sigma_k\ P_k \times OperationalCost(\underline{x}+\Delta\underline{x}_k)]}$$

**Subject to:**

**Operational constraints for future k=1,…N:** $\mathbf{\underline{g}_k(\underline{x}+\Delta\underline{x}_k)\leq\underline{b}_k}$

In considering Formulation A2, we observe that there will be tradeoffs between the core costs and the adaptation costs. That is, the more we actually build, the less we will need to adapt. For example, we could spend a very large amount of money to build G&T infrastructure that would be feasible in every future *k*. Then the adaptation cost would be zero. Or, we could spend no money at all and build nothing; in this case, the adaptation cost would be extremely large. And there are obviously graduations between these two extremes that we could implement.

Indeed, Formulation A2 is a type of multiobjective optimization problem - the first two terms of our objective are conflicting, i.e., they are functions of the same variables[3], and when one increases the other decreases. One way to handle a multiobjective optimization problem is to institute a multiplier that allows the modeler-analyst to control the tradeoffs between the conflicting objectives, in this case, between the core and the adaptations. We do this using a multiplier $\beta$ on the adaptation cost, as in Formulation A3. We call $\beta$ the *robustness parameter*.

**FORMULATION A3:**
**Minimize:**

$$\mathbf{CoreCosts}(\underline{\mathbf{x}}) + \beta\times[\Sigma_k\ P_k\times\mathbf{AdaptationCost}(\Delta\underline{\mathbf{x}}_k)] +[\Sigma_k\ P_k\times\mathbf{OperationalCost}(\underline{\mathbf{x}}+\Delta\underline{\mathbf{x}}_k)]$$

**Subject to:**

**Operational constraints for future k=1,…N:** $\underline{\mathbf{g}_k}(\underline{\mathbf{x}}+\Delta\underline{\mathbf{x}}_k)\underline{\leq}\underline{\mathbf{b}}_k$

Observe the influence of $\beta$: it acts as a "dial" on adaptation cost, i.e., we can use $\beta$ to make adaptation

- very cheap ($\beta$ small), in which case ACEP makes very little core investment and very large adaptation – in the extreme ($\beta$=0), it makes no core investment because adaptation is free;
- very expensive ($\beta$ large), in which case ACEP makes very large core investment and very little adaptation – in the extreme ($\beta$=∞), it makes no adaptation because it is infinitely expensive to do so, and all investment goes into the core.

### *10.1 An early example*

This approach was applied to a GEP problem at the national level [16]. Figure 21 shows the geographical scale of the problem.

[3] At first glance, it may not appear that the first two terms are functions of the same variables; however, $\Delta\underline{x}$ is implicitly a function of $\underline{x}$ in that, as $\underline{x}$ increases, the adaptations $\Delta\underline{x}$ decrease.

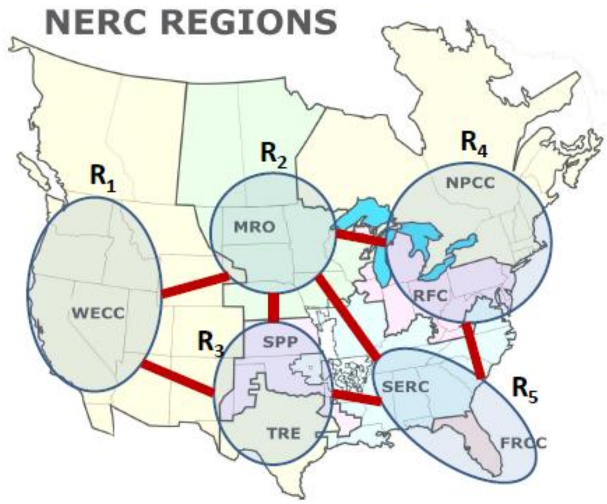



**Figure 21: Geography of the problem addressed**

Sixty-four futures were developed in terms of 6 uncertain parameters, each of which could take on one of two values ($2^6$=64):

- Natural gas price
- Natural gas production limits
- Demand growth
- Existence of a national renewable portfolio standard
- Existence of a $CO_2$ cap
- Wind plant investment cost

An aggregation approach was used to identify 10 futures that best represented the 64. These 10 futures are listed in Figure 22.

| Cluster | Gas price GP | Gas production limits GPL | Demand D | RPS | $CO_2^{cap}$ | Wind investment cost WC |
|---|---|---|---|---|---|---|
| Benchmark | L | No | L | No | No | H |
| 1 | L | No | L | No | No | L |
| 2 | L | No | L | Yes | No | H |
| 3 | L | No | L | Yes | Yes | L |
| 4 | L | No | H | No | Yes | H |
| 5 | L | Yes | L | Yes | Yes | H |
| 6 | H | No | L | Yes | Yes | L |
| 7 | H | No | H | No | Yes | L |
| 8 | H | Yes | L | No | No | L |
| 9 | H | Yes | L | Yes | Yes | H |
| 10 | H | Yes | H | Yes | No | H |

**Figure 22: Selected future (scenarios)**

The optimization problem was then solved for different values of $\beta$, and the results are plotted in Figure 23.

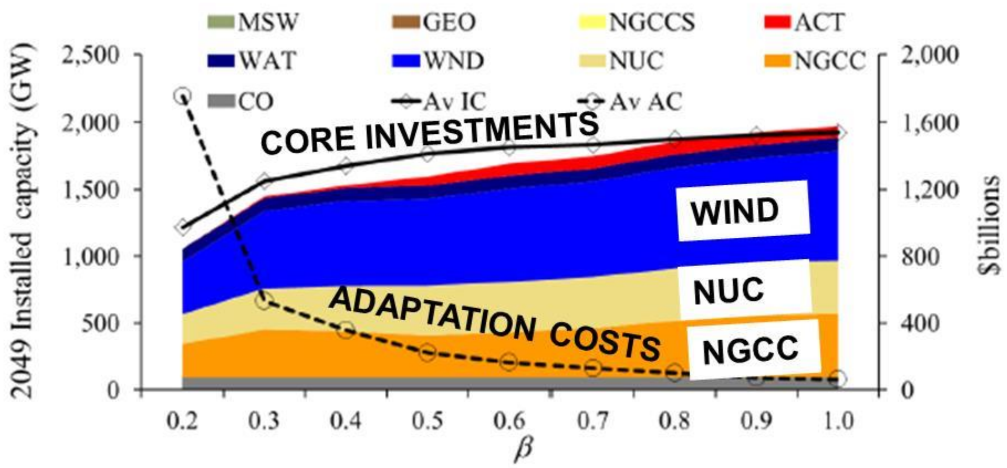



**Figure 23: Adaptation solutions for different values of *β***

A single value of $\beta$ was selected, and a complete solution was produced over a 40-year horizon. The total installed capacity of the solution is shown in Figure 24.

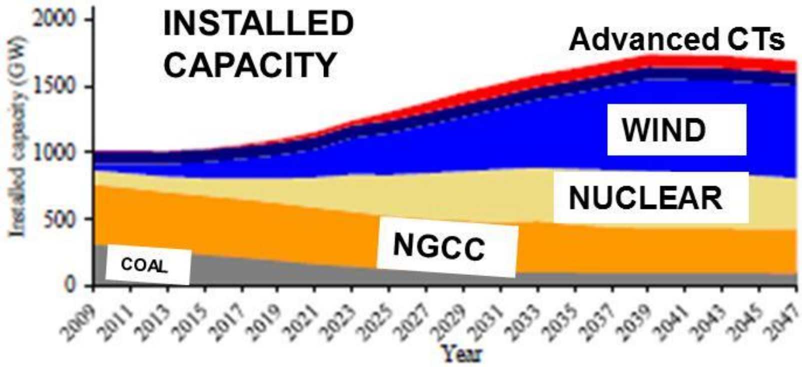



**Figure 24: Total installed capacity over 40 years**

The solution shown is considered to be adaptable, or "flexible," to the futures used to develop it. We observe that, with respect to the scenarios studied, adaptability means:

- Increase Advanced CTs
- Increase WIND
- Increase NUCLEAR
- Maintain NGCC
- Retire COAL

### *10.2 A more recent example*

More recent work comes from [17], where the ACEP approach was characterized in terms of its benefits to the MISO transmission expansion planning (MTEP) process used by MISO in 2020 (this discussion extends from that given in Section 6 of these notes, which presented the "scenario analysis" performed by MISO in their 2008 MTEP process).

This work compared the future (scenario) evaluation process, using the deterministic expansion planning tool CEP, to the ACEP optimization method, as indicated in Figure 25.

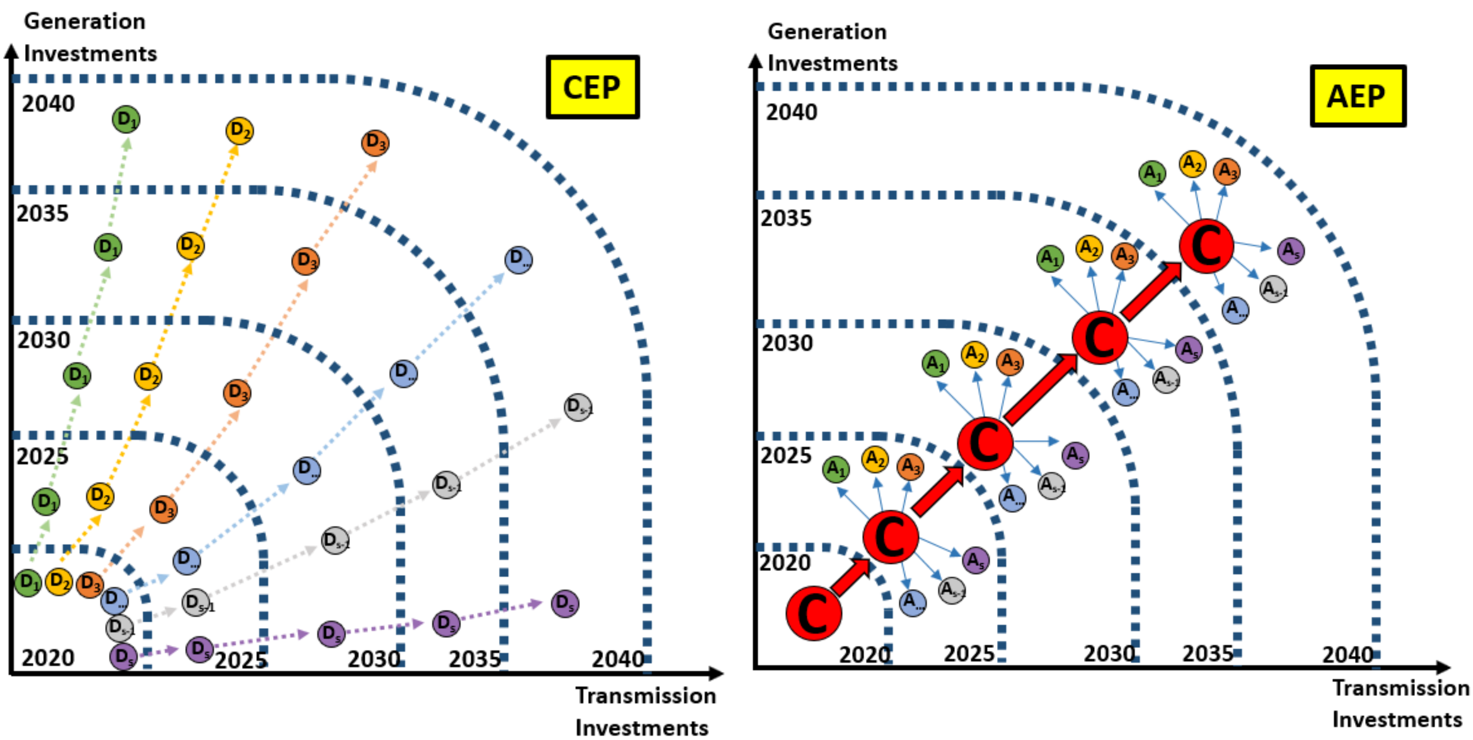



**Figure 25: Comparison of future (scenario) evaluation using CEP (left) to the ACEP optimization method (right)**

The problem solved is the same as the problem provided as (ACEP) and also on p. 35, expressed here as Figure 26.

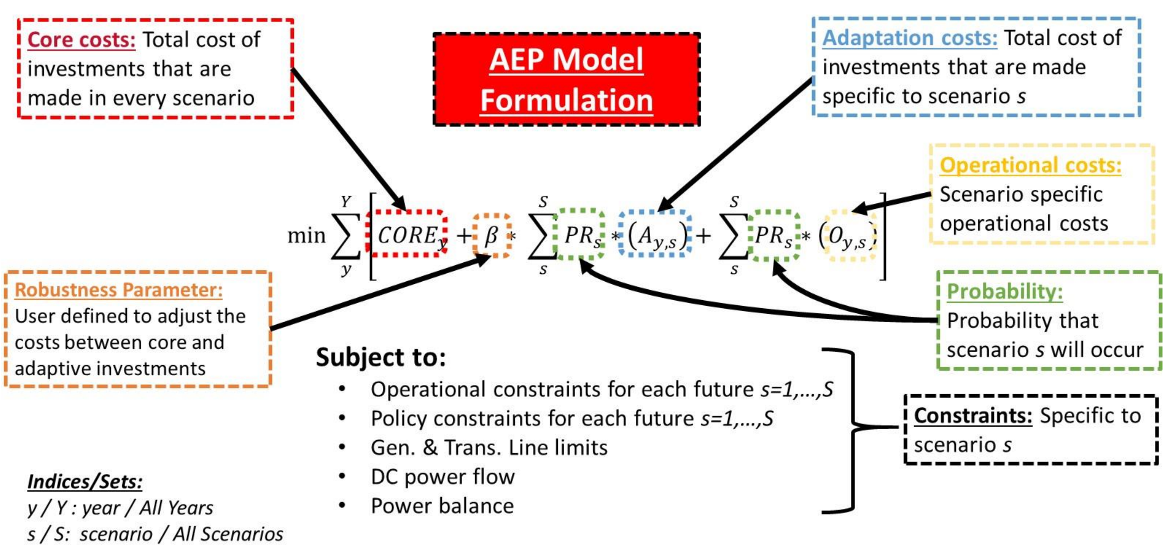



**Figure 26: Optimization problem**

Uncertainties were modeled as shown in Figure 27.

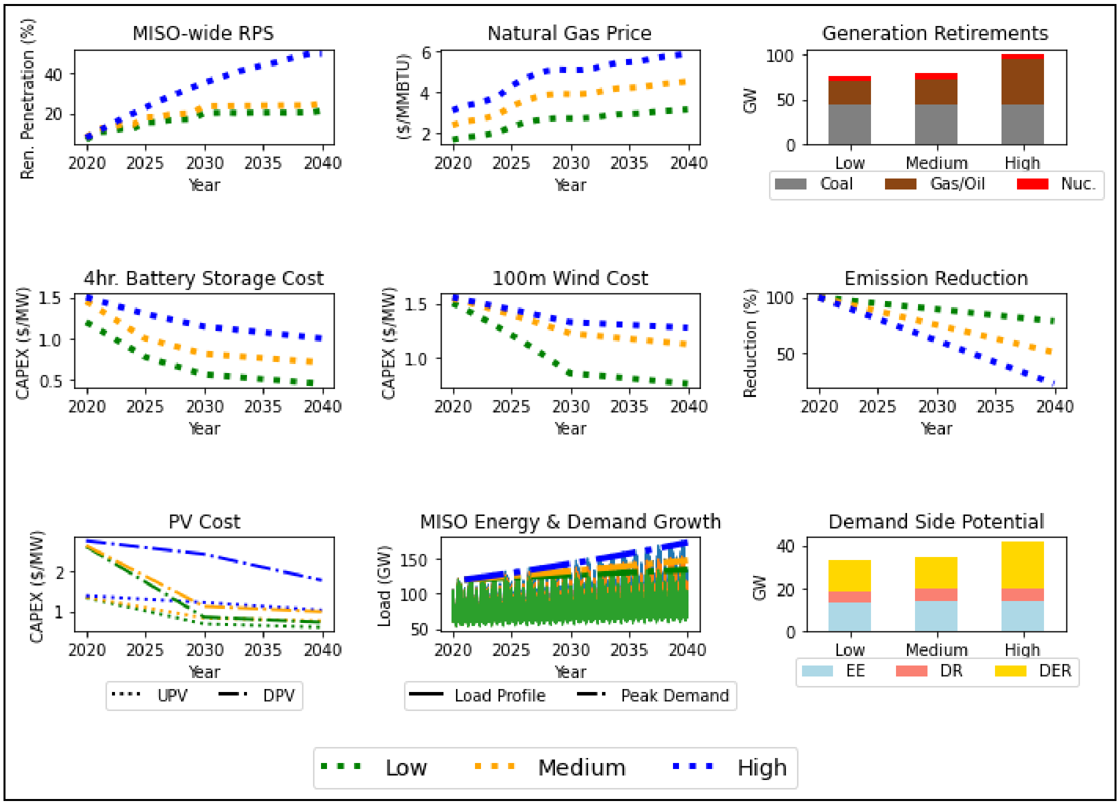



**Figure 27: Uncertainty modeling [17]**

There are nine uncertainties each with three possible values; this gives a number of possible scenarios of $3^9$=19,683, far too many to model within the ACEP optimization formulation. Therefore the scenario space was reduced to a set of 7 scenarios, expressed by

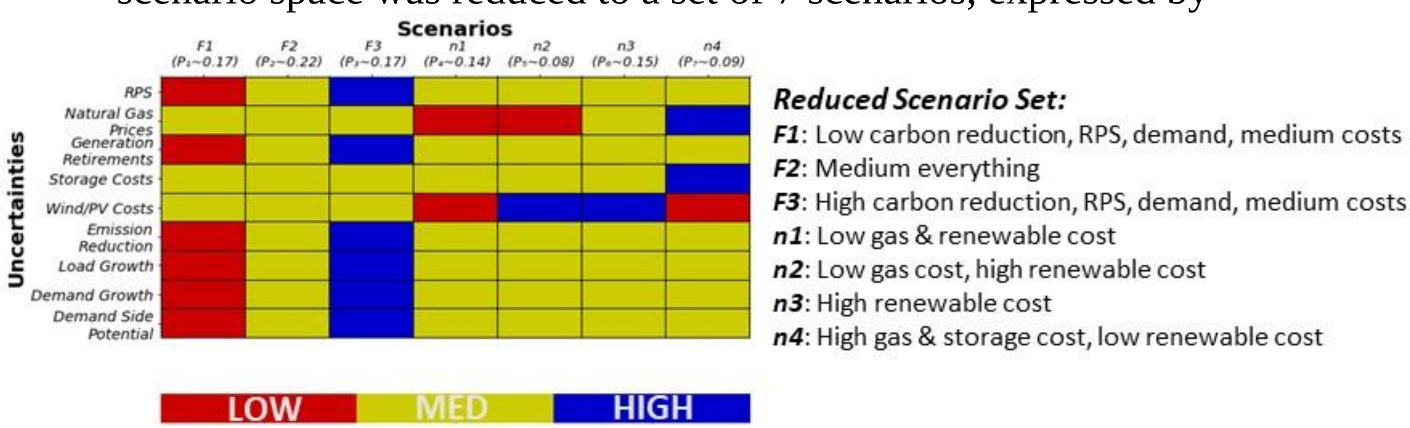



**Figure 28: Scenarios [17]**

A 74,000 bus model of the Eastern Interconnection was reduced using a combined Kron/heuristics method, shown in Figure 29.

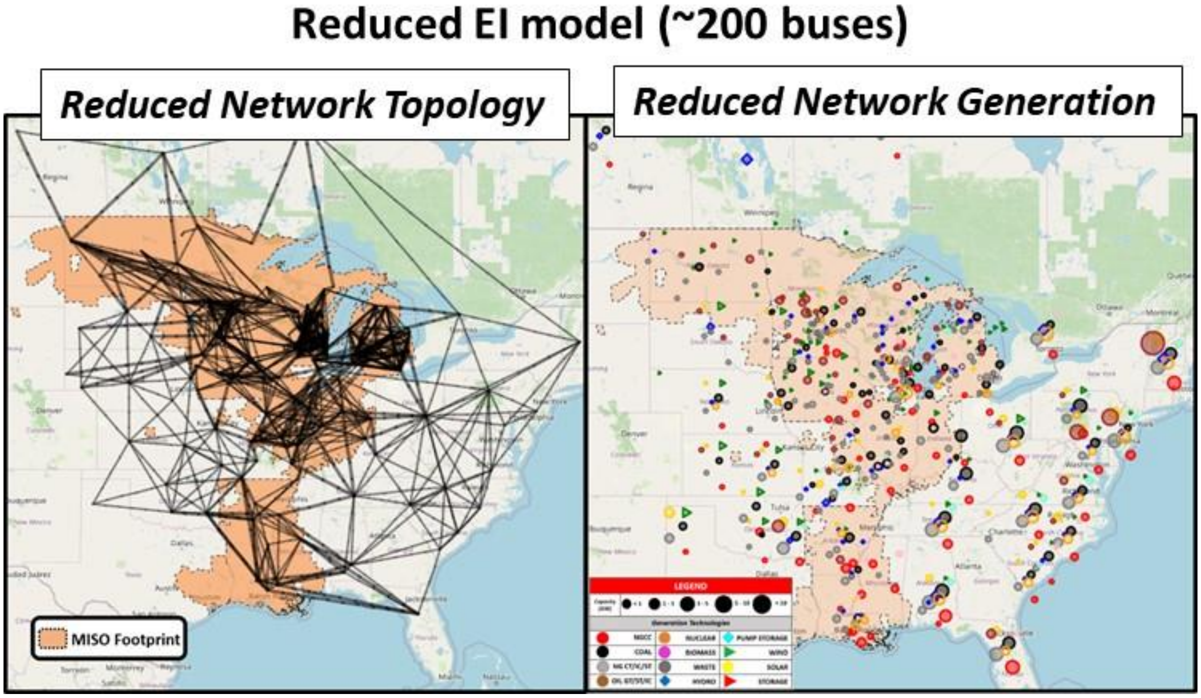



**Figure 29: Reduced model of the Eastern Interconnection**

Study assumptions included the following:

- 20-year planning horizon; with investment years 2020, 2026, 2032, 2038;
- 30-year end effects to calculate future operational costs of later investments;
- Investments only in an expanded MISO footprint, but operational costs in the entire EI.

Figure 30 shows the ***core investments***, as bars, in terms of capacity of generation (for each technology) and capacity of transmission, for increasing robustness parameter $\beta$. Figure 30 also shows the cost of the core investments, as the blue curve.

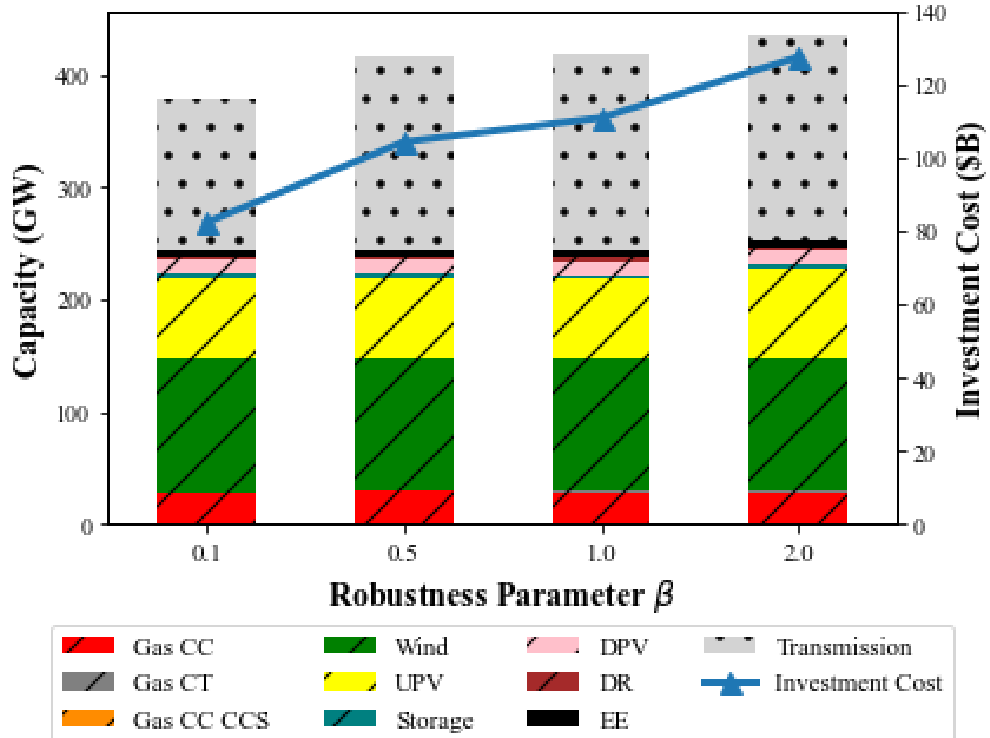


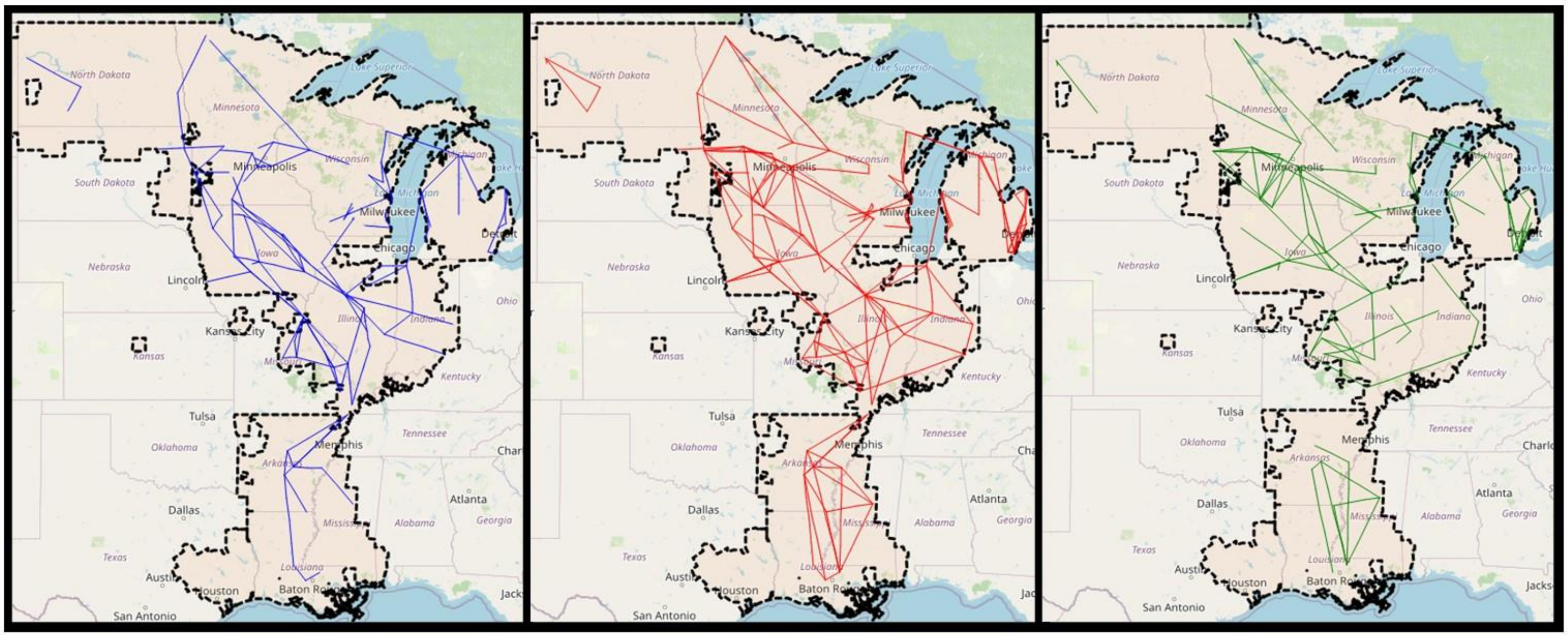

**Figure 30: ACEP results**

The following observations are made:

- As $\beta$ increases, the core becomes more robust (and expensive), and adaptations are diminished;
- Significant increase in core transmission investments → ACEP model favors transmission in comparison to the CEP.
- **<u>Transmission provides robustness to the future uncertainties!!</u>**

Figure 31 shows on the left the "least-regrets" solution, which is the set of investments (transmission, in this case) that appear in all CEP scenarios. Figure 31 also shows in the middle the ACEP core for a value of $\beta$=1.0. Finally, Figure 31 shows, on the right, the transmission investments appearing in the ACEP core that do not appear in the least regrets solution. These investments may be due to the relative high value of $\beta$ (=1.0) that was chosen for the ACEP solution. However, there is also some possibility that ACEP selects investments that are not in the least regrets solution because, in the words of [18], "For example, a particular transmission investment might perform well in many scenarios because it gives the system some flexibility, e.g., to develop any of several renewable energy zones. But that investment might never be the very best choice in any single scenario of renewable development. However, when considered stochastically, that line would provide a hedge against uncertainty and could be optimal overall. For this reason, scenario planning and heuristics are unable to quantify the full value of alternatives that increase the adaptability of transmission plans."

# AEP Results –Potential Benefits




***CEP:*** Transmission investments common to all scenarios from the CEP simulations (*displaying Investments > 10MW)*

***AEP:*** Transmission investments in the core for $\boldsymbol{\beta} = \mathbf{1.0}$ (*displaying Investments > 10MW)*

→ ***Unique*** investments identified by the AEP and not by the deterministic solution (*displaying Investments > 10MW)*

**Figure 31: Comparison of "least regrets" solution (left) to ACEP core (middle) and investments in ACEP core that do not appear in the least regrets solution**

## 11 Comparison between TSP and ACEP

Adaptation, like TSP, is an approach to design an investment strategy under uncertainty. The formulation (TSP) above may be converted to the adaptive cooptimized expansion planning (ACEP) formulation. This process simply identifies changes necessary to TSP in order to express ACEP; it by no means indicates equivalence between the two formulations (they are not equivalent). There are three steps necessary for this conversion; these three steps are summarized in [15] but are not given here.

There are three main differences between TSP and ACEP:

1. *Difference in core*:
   - In TSP, the core investment (that which is common to all investments, also known as "here and now") is in period $t$=1.
   - In ACEP, the core "here and now" investment is a trajectory through time.
2. *Scenario representation; memory*:
   - In TSP, the scenarios and associated recourse investments begin at period $t$=2 and extend through the remaining time periods; in each scenario, there is inter-temporal memory from one time period to another, i.e., what gets built in time period $t$ is in addition to what was built in time period $t$-1.
   - In ACEP, the futures (scenarios) and associated recourse investments extend from the core trajectory at each time period; there is no inter-temporal memory to the investments made for each future (scenario), i.e., at each time period, a new set of recourse investments are identified for each future (scenario) which are independent of any recourse investments identified for that future (scenario) in earlier time periods.

*ASIDE*: It is important to consider at this point the difference between memory and non-anticipative behavior. Whereas memory is intertemporal and specific to a single future (scenario), non-anticipative behavior is intertemporal but concerned with several futures (scenarios).

A system can have memory but be either anticipative or non-anticipative, i.e., each time step in each scenario can accumulate from the previous time step, but

- the system may be anticipative across scenarios if scenarios with common histories *are not constrained* to have the same decisions;
- the system can be non-anticipative across scenarios if those scenarios having common histories *are constrained* to have the same decisions.

However, a system without memory cannot be anticipative since there is no mechanism (i.e., there is no accumulation equation) for the investment decisions at time $t'$ to be influenced by investment decisions at time $t > t'$.

I am unsure of this reasoning.

3. *Non-anticipativity constraints*:
   - In TSP, as we have seen (p. 25), NACs are imposed to ensure that the decision maker does not know, when s/he is in the initial time period, which scenario will occur, and so in that time period, investments are constrained to be the same across all $s$.
   - In ACEP, we drop the NACs for the scenarios. However, we do not need them in order to impose non-anticipatory behavior, because we also drop the memory in scenarios from one time period to the other, i.e., there is no accumulation equation for the scenarios; this implies that all scenario investments (called the adaptations) are computed anew for each time step.

It needs additional thinking.

4. *Purpose*: Because of the differences in the core, where the TSP core is at the $t$=1 time period and the ACEP core is a trajectory through time, TSP is sometimes said to be good for identifying what to build "today," whereas ACEP is sometimes said to be good for identifying what to build through time, that is, whereas TSP is good for decision-making, ACEP is good for planning.

In attempt to illustrate these ideas, we compare in Figure 32 the illustration of TSP from Figure 18 (on the left) with an illustration of ACEP (on the right).

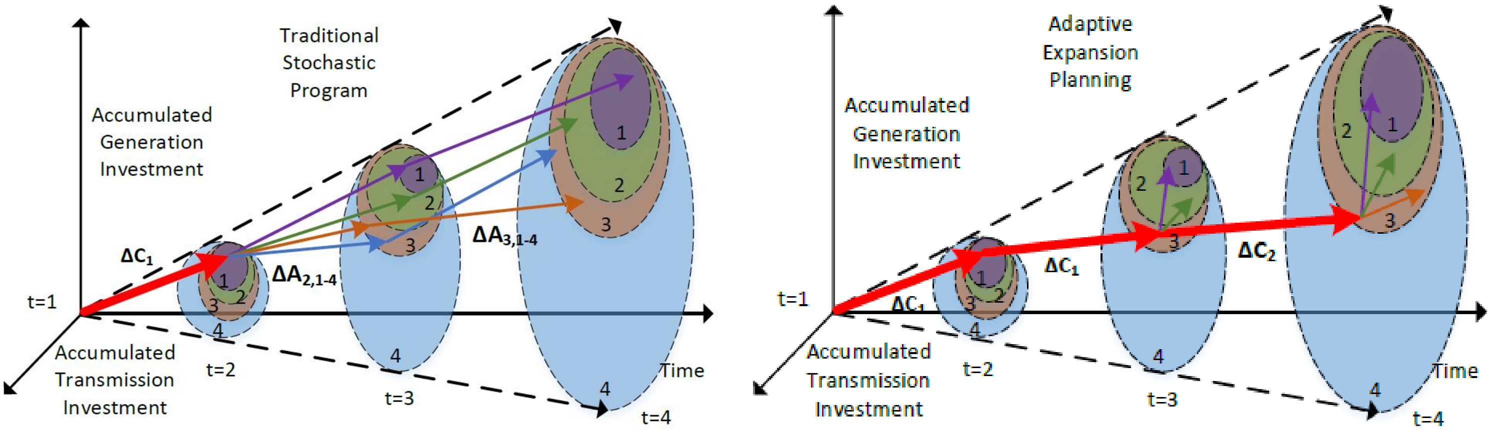



**Figure 32: Comparison of TSP (on left) with ACEP (on right) [15]**

The problem is expressed in [15] as

$$\min \sum_t I_{t,c} \Delta C_t + \sum_t \sum_s P_s \left( I_{t,s} \beta \Delta A'_{t,s} + O_{t,s} \right)$$

subject to:

1) Core accumulation (memory) constraint:

$$C_t = C_{t-1} + \Delta C_t \ \ \forall t$$

(ACEP

2) Total scenario investment evaluation:

$$A_{t,s} = C_t + \Delta A'_{t,s} \ \ \forall t, s$$

3) Operational constraints for each scenario $s$ (not shown)

## 12 Robust optimization

The basic concept underlying robust optimization is that

- some parameters in our optimization problem are uncertain
- and though we don't know distributions on those uncertain parameters,
- we do know ranges outside of which those parameters will not take on values.

In other words, for each time period, t=1, 2, …, we know a region within the parameter space where the values of those parameters may be found. Figure 33 below illustrates for two parameters U1 and U2, and for two time periods t=1, 2.

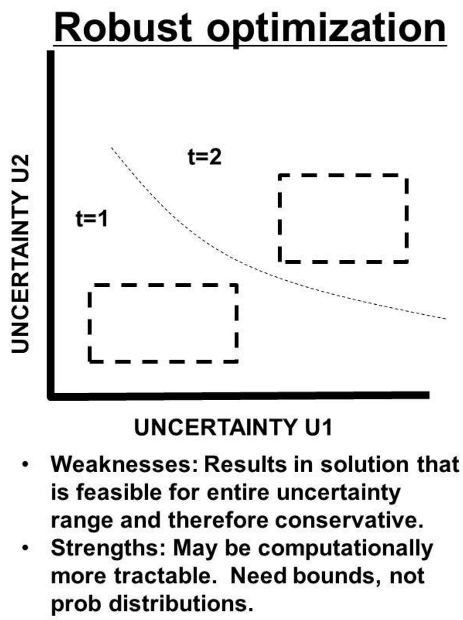



**Figure 33: Robust optimization**

The weakness (conservatism) and strength (computational tractability) of robust optimization is expressed in the figure above. An extension of robust optimization that addresses the conservatism issue, called adjustable robust optimization, expresses decision-variables as affine functions of the uncertain parameters [19].

## 13 Folding horizon simulation (FHS)

One major problem with long-term planning optimization is that it is very difficult to proof-test the investment solution obtained from the optimizer, since it does little good in the decision-making moment of today to wait 15-20 years to see how the plan performed. In other words, there is no real lab-bench or other environment to examine a plan's performance within a controlled environment.

- It would be good to have a way to *validate* plans produced by the optimizer. Validate means to "check or prove the validity or accuracy of something." That is, is the plan valid? Is it accurate? This is not possible to do, although there has been some effort towards evaluating tools by applying them using the system & relevant data for conditions 20 yrs ago (2004) to develop a 2004-'24 plan.
- It is possible to *evaluate* the plan. There are several features that could be evaluated, including:

1. *Reliability*
    - Resource adequacy (LOLE)
    - Congestion cost
2. *Resilience* (to assess severity/duration of defined extreme events):
    - Energy not served (a distribution-related metric);
    - Increase in the total cost of electric energy (a transmission-related metric)
3. *Cost*
    - Investment
    - Operational
    - Total
4. *Economic development impact*
    - revenues from additional property tax
    - revenues from land lease payments
    - savings on industrial, commercial, and residential cost of energy
5. *Environmental impact*
    - Impact on air (criteria pollutants) – those for which acceptable levels of exposure can be determined & for which an ambient air quality standard has been set: Ozone, CO, $SO_2$, $NO_x$, particulate
    - Mercury (Hg)
    - Greenhouse gases ($CO_2$, methane, ozone, $NO_x$, chlorofluorocarbons)
    - Impact on water

In evaluating the above, it is necessary to choose conditions, i.e., futures (scenarios) under which we perform the evaluation. However, it is clear that there are many futures, and we do not know which one will actually take place. Therefore, we are interested in the *robustness* of the plan's evaluation metrics to all possible futures. Reference to Figure 9 and Figure 10 earlier in these notes indicate that this is exactly what MISO was doing in the work of theirs that is displayed there.

However, which futures should be used to evaluate the robustness of the plan's evaluation metrics? One approach, when using ACEP, would be to evaluate this robustness using all of the futures that were used in the ACEP optimization. Yet, we would expect an ACEP-developed plan to perform well under these futures, since it was designed under these futures.

A better approach might be to evaluate the plan using futures that were not used in the ACEP optimization (and there are many such futures!). To this end, we have proposed the folding horizon simulation (FHS).

FHS provides a computational means of simulating the time period associated with the decision horizon. In this approach:

1. FHS uses as input the optimized plan throughout time together with uncertainty realizations to generate scenarios that were not used in the optimization; we refer to these scenarios as "out-of-sample."
2. It simulates one year at a time using production simulation.
3. Reliability is assessed within the production simulation, and if it is unacceptable, a re-investment step is taken and the production simulation performed again over that year.
4. Once reliability is satisfied, the FHS moves to the next year and repeats until it has assessed all years of the decision-horizon.
5. Metrics produced that provide an evaluation can be averaged over all out-of-sample scenarios and include the reliability values each year together with the cost of reinvestments.

A key feature of all of this is how these tools relate to the RTO planning processes – see figure at the end.

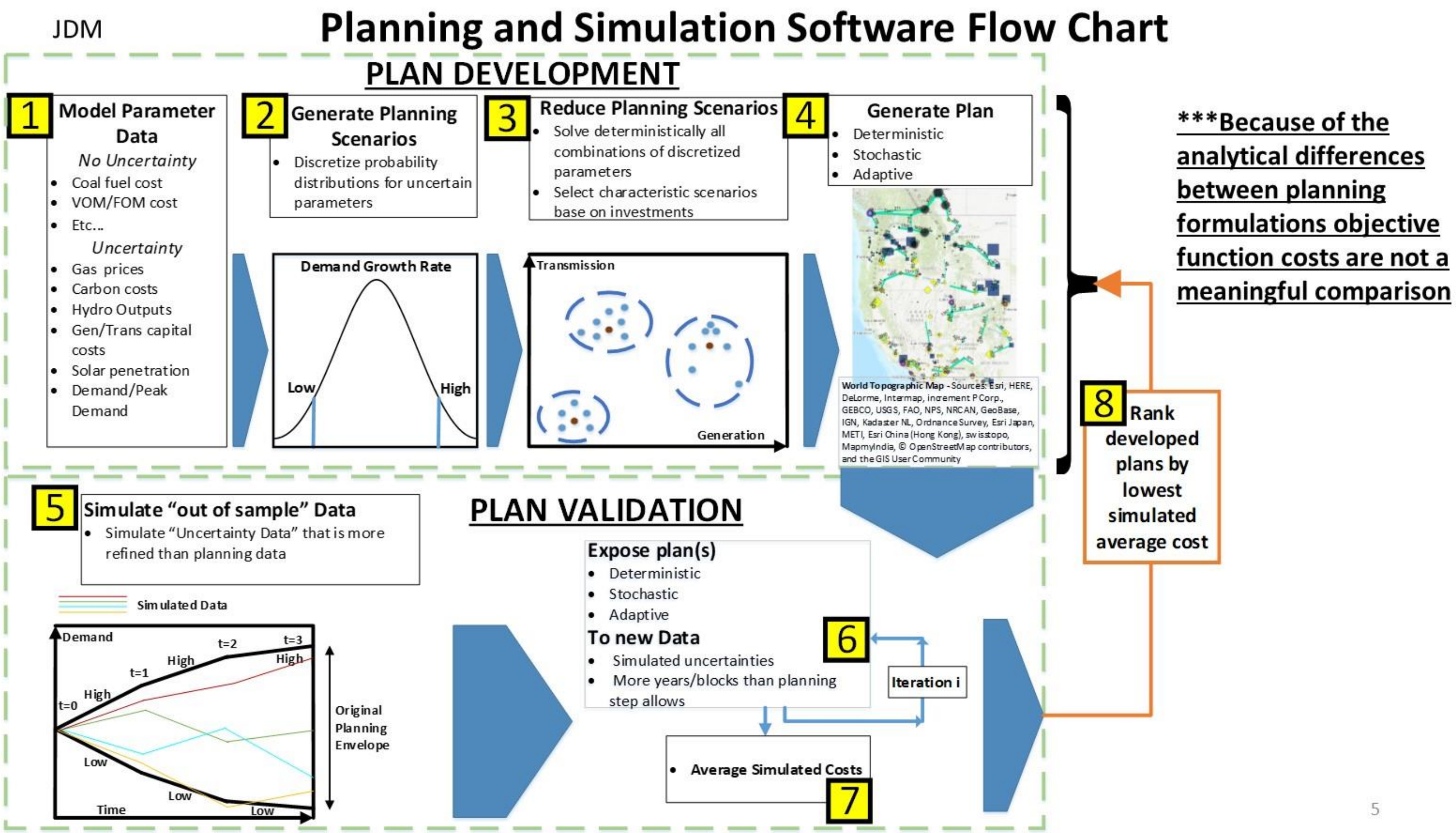
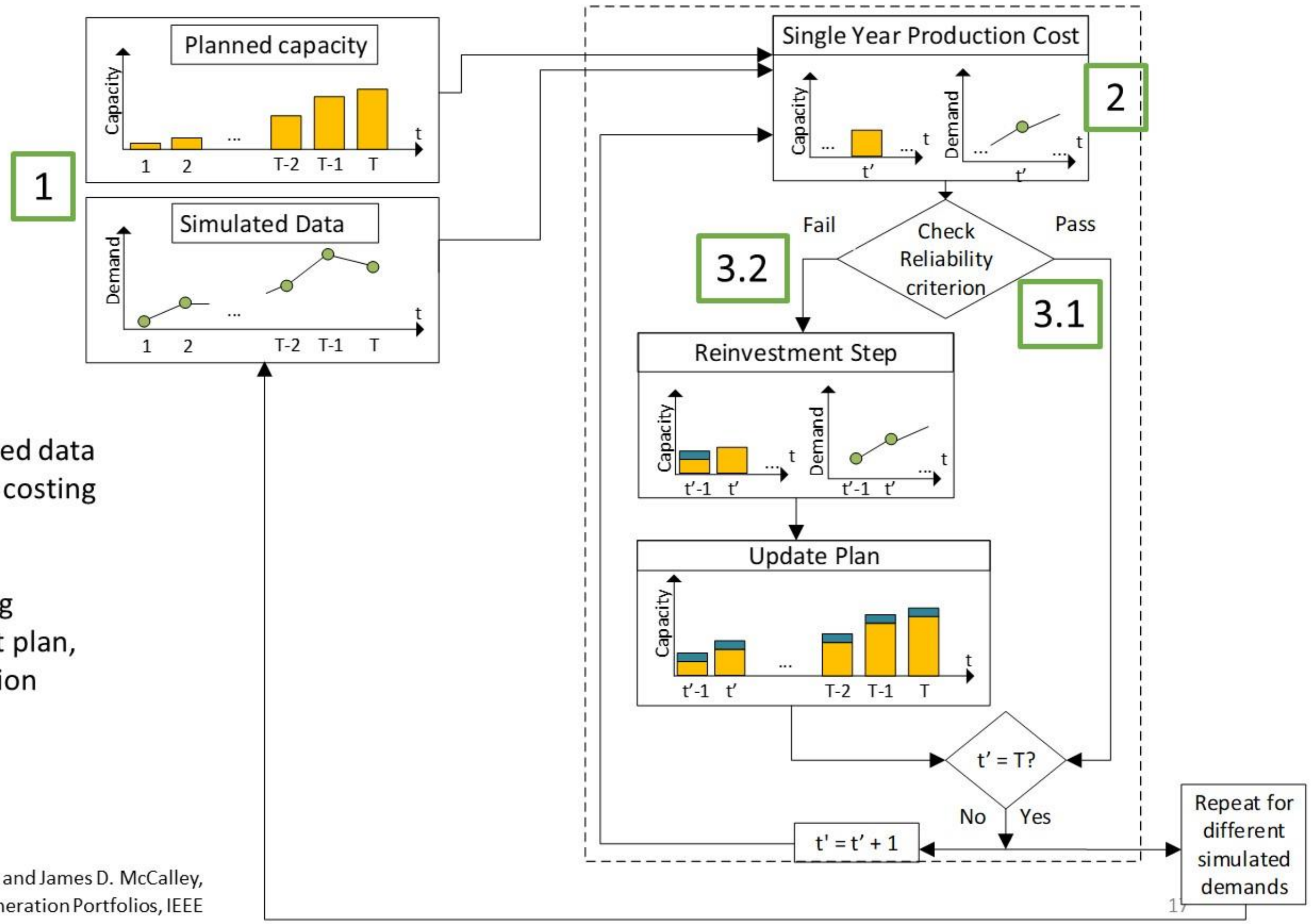
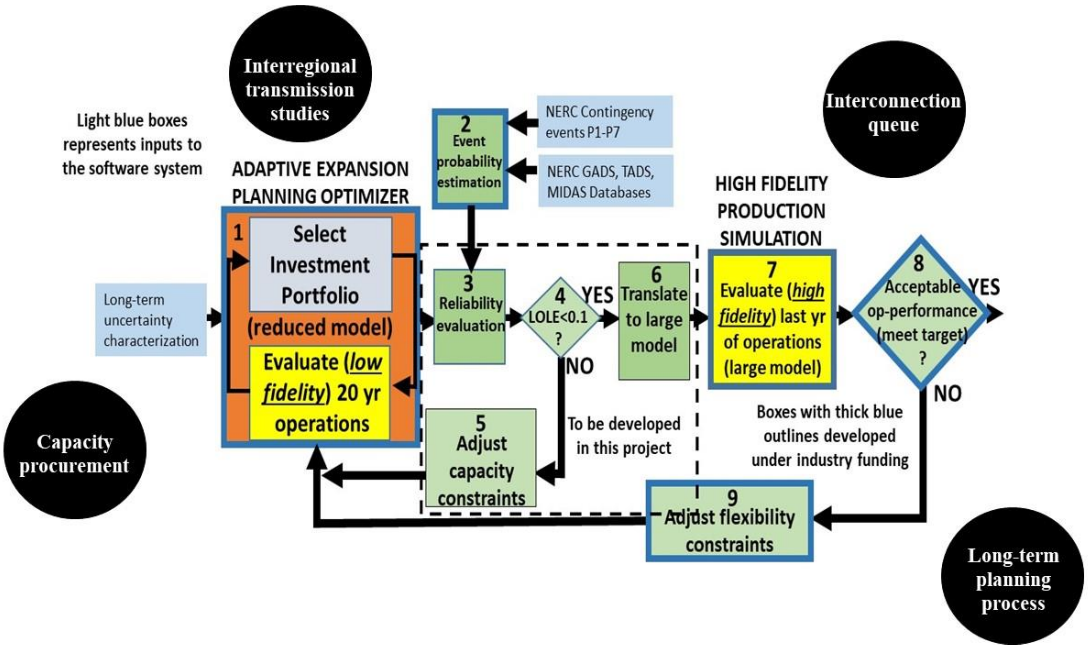



**Figure 34: Plan development and plan validation**




1. Input data and simulated data
2. Single Year Production costing
3. Test Reliability
    1. If Ok – Return to production costing
    2. If violation, adapt plan, return to production costing

Figure adapted from "Diego Mejia-Giraldo and James D. McCalley, Maximizing Future Flexibility in Electric Generation Portfolios, IEEE transactions on power Systems, 2014"


Interregional transmission studies
Light blue boxes represents inputs to the software system
ADAPTIVE EXPANSION PLANNING OPTIMIZER
1
Select Investment Portfolio
(reduced model)
Evaluate (low fidelity) 20 yr operations
Long-term uncertainty characterization
Capacity procurement
2
Event probability estimation
NERC Contingency events P1-P7
NERC GADS, TADS, MIDAS Databases
3
Reliability evaluation
4
LOLE<0.1 ?
YES
NO
6
Translate to large model
5
Adjust capacity constraints
To be developed in this project
HIGH FIDELITY PRODUCTION SIMULATION
7
Evaluate (high fidelity) last yr of operations (large model)
8
Acceptable op-performance (meet target) ?
YES
NO
Interconnection queue
Boxes with thick blue outlines developed under industry funding
9
Adjust flexibility constraints
Long-term planning process

## 14 Compare and contrast

It would be good to compare and contrast the various ways of handling uncertainty.

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