Test systems and mathematical models for transmission network expansion planning

R. Romero, A. Monticelli, A. Garcia and S. Haffner

Abstract: The data of four networks that can be used in carrying out comparative studies with methods for transmission network expansion planning are given. These networks are of various types and different levels of complexity. The main mathematical formulations used in transmission expansion studies—transportation models, hybrid models, DC power flow models, and disjunctive models are also summarised and compared. The main algorithm families are reviewed—both analytical, combinatorial and heuristic approaches. Optimal solutions are not yet known for some of the four networks when more accurate models (e.g. the DC model) are used to represent the power flow equations—the state of the art with regard to this is also summarised. This should serve as a challenge to authors searching for new, more efficient methods.

1 Introduction

This paper presents the data of four different systems which can be used for testing alternative algorithms for transmission network expansion planning. The main motivation for giving these data in a systematical and organised way is to allow meaningful comparative studies—the one thing that is certainly lacking in this important research area. In most publications, practitioners have used the well known Garver's six-bus network to illustrate the proposed methods, along with some other networks, for which as a rule the relevant data is not entirely available. Comparative studies using known data are practically non-existent. To a lesser degree, the same is true for the different models used to represent the transmission networks. Comparative studies dealing with the alternative representations for different networks are badly needed to properly evaluate the performances of proposed algorithms. This paper gives the data of four systems which differ very widely in computational complexity. It also summarises in a systematic way the alternative models that are normally used for representing a transmissions network in transmission planning studies. A summary of the main methodologies available is also presented.

2 Mathematical modelling

Four main types of model have been used in the literature for representing the transmission network in transmission expansion planning studies: the transportation model, the hybrid model, the disjunctive model, and the DC power flow model. Full AC models are considered only at later stages of the planning process when the most attractive topologies have been determined.

2.1 DC model

When the power grid is represented by the DC power flow model, the mathematical model for the one-stage transmission expansion planning problem can be formulated as follows:

Minimise

$$v = \sum_{(i,j)} c_{ij} n_{ij} + \alpha \sum_k r_k \tag{1}$$

Subject to

$$Sf + g + r = d \tag{2}$$

$$f_{ij} - \gamma_{ij}\left(n_{ij}^0 + n_{ij}\right)\left(\theta_i - \theta_j\right) = 0 \tag{3}$$

$$|f_{ij}| \leq \left(n_{ij}^0 + n_{ij}\right)\bar{f}_{ij} \tag{4}$$

$$0 \leq g \leq \bar{g} \tag{5}$$

$$0 \leq r \leq d \tag{6}$$

$$0 \leq n_{ij} \leq \bar{n}_{ij} \tag{7}$$

$$n_{ij} \text{ integer, } f_{ij} \text{ and } \theta_j \text{ unbounded} \tag{8}$$

$$(i,j) \in \Omega,\ k \in \Gamma$$

where c_{ij}, γ_{ij}, n_{ij}, n_{ij}^0, f_{ij} and $\bar{f}_{ij}$ represent, respectively the cost of a circuit that can be added to right-of-way i–j, the susceptance of that circuit, the number of circuits added in right-of-way i–j, the number of circuits in the base case, the power flow, and the corresponding maximum power flow. v is the total investment, S is the branch-node incidence matrix, f is a vector with elements f_{ij} (power flows), g is a vector with elements g_k (generation in bus k) whose maximum value is $\bar{g}$, $\bar{n}_{ij}$ is the maximum number of circuits that can be added in right-of-way i–j, Ω is the set of all right-of-ways, Γ is the set of indices for load buses and r is the vector of artificial generations with elements r_k (they are used in certain formulations and to represent loss of load,

IEE Proceedings online no. 20020026

DOI: 10.1049/ip-gtd:20020026

Paper first received 14th March 2001

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and normally appear in the formulation multiplied by a cost α measured in \$/MW).

The constraint in eqn. 2 represents the conservation of power in each node if we think in terms of an equivalent DC network, this constraint models Kirchhoff's current law (KCL). The constraint in eqn. 3 is an expression of Ohm's law for the equivalent DC network. Notice that the existence of a potential function θ associated with the network nodes is assumed, and so Kirchhoff's voltage law (KVL) is implicitly taken into account (the conservation of energy in the equivalent DC network)—these are nonlinear constraints. The constraint in eqn. 4 represents power flow limits in transmission lines and transformers. The constraints in eqns. 5 and 6 refer to generation (and pseudo-generation) limits.

The transmission expansion problem as formulated above is an integer nonlinear problem (INLP). It is a difficult combinatorial problem which can lead to combinatorial explosion on the number of alternatives that have to be searched.

2.2 Transportation model

This model is obtained by relaxing the nonlinear constraint eqn. 3 of the DC model described above. In this case the network is represented by a transportation model, and the resulting expansion problem becomes an integer linear problem (ILP). This problem is normally easier to solve than the DC model although it maintains the combinatorial characteristic of the original problem. An optimal plan obtained with the transportation model is not necessarily feasible for the DC model, since part of the constraints have been ignored; depending on the case, additional circuits are needed in order to satisfy the constraint in eqn. 3, which implies higher investment cost.

2.3 Hybrid model

The hybrid model combines characteristics of the DC model and the transportation model. There are various ways of formulating hybrid models, although the most common is that which preserves the linear features of the transportation model. In this model it is assumed that the constraint in eqn. 2, KCL, is satisfied for all nodes of the network, whereas the constraint in eqn. 3, which represents Ohm's law (and indirectly, KVL), is satisfied only by the existing circuits (and not necessarily by the added circuits).

The hybrid model is obtained by replacing the constraints in eqns. 2 and 3 of the DC model by the following constraints:

$$S_o f + S f' + g + r = d \tag{9}$$

$$f_{ij} - \gamma_{ij} n_{ij}^0 (\theta_i - \theta_j) = 0, \quad \forall (i,j) \in \Omega_0 \tag{10}$$

$$|f_{ij}| \le n_{ij}^0 \bar{f}_{ij}, \quad \forall (i,j) \in \Omega_0 \tag{11}$$

$$|f'_{ij}| \le n_{ij} \bar{f}_{ij}, \quad \forall (i,j) \in \Omega \tag{12}$$

where S_o is the branch-node incidence matrix for the existing circuits (initial configuration), f is the vector of flows in the existing circuits (with elements f_{ij}), and f' is the vector of flows in the added circuits (with elements f'_{ij}).

2.4 Disjunctive model

A linear disjunctive model has been used in [1–3]. It can be shown that under certain conditions the optimal solution for the disjunctive model is the same as the one for the DC model. This model can be formulated as follows.

Minimise

$$v = \sum_{(i,j)} c_{ij} y_{ij}^p + \alpha \sum_k r_k \tag{13}$$

Subject to

$$S_0 f^0 + S_1 f^1 + g + r = d \tag{14}$$

$$f_{ij}^0 - \gamma_{ij} n_{ij}^0 (\theta_i - \theta_j) = 0, \quad \forall (i,j) \in \Omega_0 \tag{15}$$

$$|f_{ij}^p - \gamma_{ij}(\theta_i - \theta_j)| \le M(1 - y_{ij}^p), \quad \forall (i,j) \in \Omega \tag{16}$$

$$|f_{ij}^0| \le \bar{f}_{ij} n_{ij}^0 \tag{17}$$

$$|f_{ij}^k| \le \bar{f}_{ij} y_{ij}^p \tag{18}$$

$$0 \le g \le \bar{g} \tag{19}$$

$$0 \le r \le d \tag{20}$$

$$y_{ij}^p \in \{0, 1\}, \quad (i,j) \in \Omega, \quad p = 1, 2, \ldots, p \tag{21}$$

$$f_{ij}^0, f_{ij}^p \text{ and } \theta_j \text{ unbounded}$$

where p is the number of circuits that can be added to a right-of-way (these are binary variables of the type y_{ij}^k), f^0 is the vector of flows in the circuits of the initial configuration (with elements f_{ij}^0), S_I is the node-branch incidence matrix of the candidate circuits (which are considered as binary variables) f^I is the vector of flows in the candidate circuits (with elements f_{ij}^p), n_{ij}^0 are the circuits of the initial configuration, and M is a number of appropriate size.

The appeal of this model is that the resulting formulation can be approached by binary optimisation techniques. On the other hand, it has two main disadvantages: the increase in the number of problem variables due to the use of binary variables, and the need to determine the value of M. An additional feature of this method is that it can be extended to AC models: this, however, is not of great value in practice, since most of the long term studies are performed with DC models only.

3 Data sets

Is this Section the data sets for transmission expansion planning of four systems are presented. These systems show a wide range of complexities and are of great value for testing new algorithms. The reactance data are in p.u. considering a 100MW base.

3.1 6-bus system

This system has six buses and 15 right-of-ways for the addition of new circuits. The demand is of 760MW and the relevant data are given in Tables 1 and 2. This system was originally used in [4], and since then has become the most popular test system in transmission expansion planning. The initial topology is shown in Fig. 1.

3.2 46-bus system

This system is a medium sized system that represents the southern part of the Brazilian interconnected network. It has 46 buses and 79 right-of-ways for the addition of new circuits (all relevant data can be found in [5]). The total demand for this system is 6800MW. There is no limit for circuit additions in each right-of-ways.

Table 1: Generation and load data for 6-bus system

Bus no.	Generation, MW		Load, MW	Bus no.	Generation, MW		Load, MW
	Maximum	Level			Maximum	Level	
1	150	50	80	4	0	0	160
2	0	0	240	5	0	0	240
3	360	165	40	6	600	545	0

3.3 78-bus system

This system corresponds to a reduced version of the Brazilian southeastern network; the reduced model has 78 buses, 142 right-of-ways for the addition of new circuits, and a total demand of 37999MW. All relevant data about this case can be found at http://www.dsee.fee.unicamp.br/planning.pdf. There is no limit for circuit additions in each right-of-way. In order to carry out the expansion planning with generation redispatch the maximum generation levels should be specified. We suggest determining these values using the following relationship $\bar{g}_k = \lfloor 1.15 g_k \rfloor$, that is, the maximum generation level is the biggest integer value contained in the product of the current generation by 1.15 (15% increase).

3.4 87-bus system

This system is a reduced version of the Brazilian north-northeastern network: the reduced model has 87 buses, 183 right-of-ways for the addition of new circuits, and a total demand of 20316MW for plan P1 and 29748MW for plan P2. All relevant data about this case can be found in Tables 3 and 4. There is no limit for the number of circuit additions in each right-of-way.

This system shows a high degree of complexity due to the large number of islanded buses in the initial network. In order to run the cases considering generation redispatch, it is necessary to consider generation limits: it is suggested to consider the following: $\bar{g}_k \lfloor 1.3 g_k \rfloor$, that is, the maximum generation level is equal to the largest integer contained in the product of the current generation by 1.3 (i.e. a 30% margin).

4 Illustrative example

To illustrate the differences among the four mathematical modelling approaches discussed in this paper, as well as the quality of the optimal topologies for each of these models, a detailed example is presented herein based on Garver's 6-bus system [4] (the optimal solution for this network when the DC model is used can be found in Table 5). Only eight rights-of-way have been used for new circuit additions: six for circuit reinforcements (1-2, 1-4, 1-5, 2-3, 2-4, and 3-5) and two for new circuits (2-6 and 4-6, which are the circuits connecting the initially isolated bus 6 to the existing part of the network) as shown in Fig. 1. In buses 1 and 3 we only represent the equivalent load (bus 1) or generation (bus 3). The *pu* basis is 100 MVA

The DC model for this system is given by the following set of equations

Minimise

$$v = 40n_{12} + 60n_{14} + 20n_{15} + 20n_{23} + 40n_{24} + 30n_{26} + 20n_{35} + 30n_{46} + \alpha(r_1 + r_2 + r_4 + r_5) \tag{22}$$

Subject to

$$-f_{12} - f_{14} - f_{15} = 0.30 \tag{23}$$

$$f_{12} - f_{23} - f_{24} - f_{26} + r_2 = 2.40 \tag{24}$$

$$f_{23} - f_{35} + g_3 = 0.00 \tag{25}$$

$$f_{14} + f_{24} - f_{46} + r_4 = 1.60 \tag{26}$$

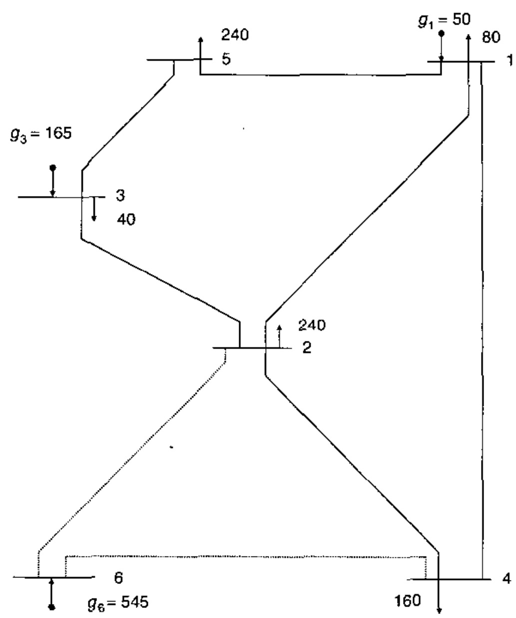

Fig. 1 *Initial configuration of Garver's network*

Table 2: Branch data for 6-bus system

From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$	From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$
1–2	1	0.40	100	40	2–6	0	0.30	100	30
1–3	0	0.38	100	38	3–4	0	0.59	82	59
1–4	1	0.60	80	60	3–5	1	0.20	100	20
1–5	1	0.20	100	20	3–6	0	0.48	100	48
1–6	0	0.68	70	68	4–5	0	0.63	75	63
2–3	1	0.20	100	20	4–6	0	0.30	100	30
2–4	1	0.40	100	40	5–6	0	0.61	78	61
2–5	0	0.31	100	31					

Table 3: Generation and load data for 87-bus system

Bus no.	Plan P1		Plan P2		Bus no.	Plan P1		Plan P2	
	Generation, MW	Load, MW	Generation, MW	Load, MW		Generation, MW	Load, MW	Generation, MW	Load, MW
1	0	1857	0	2747	30	0	189	0	273
2	4048	0	4550	0	31	0	110	0	225
4	517	0	6422	0	34	0	28	0	107
7	0	31	0	31	35	1635	0	1531	0
8	403	0	82	0	36	0	225	0	325
9	465	0	465	0	37	169	0	114	0
10	538	0	538	0	39	0	186	0	269
11	2200	0	2260	0	40	0	1201	0	1738
12	2257	0	4312	0	41	0	520	0	752
13	4510	0	5900	0	42	0	341	0	494
14	542	0	542	0	44	0	4022	0	5819
19	0	86	0	125	46	0	205	0	297
20	0	125	0	181	48	0	347	0	432
21	0	722	0	1044	49	0	777	0	1124
22	0	291	0	446	50	0	5189	0	7628
23	0	58	0	84	51	0	290	0	420
24	0	159	0	230	52	0	707	0	1024
25	0	1502	0	2273	67	1242	0	1242	0
26	0	47	0	68	68	888	0	888	0
27	0	378	0	546	69	902	0	902	0
28	0	189	0	273	85	0	487	0	705
29	0	47	0	68					

Table 4: Branch data for 87-bus System

From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$	From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$
01–02	2	0.0374	1000	44056	12–15	1	0.0256	1200	31594
02–04	0	0.0406	1000	48880	12–17	1	0.0246	1200	30388
02–60	0	0.0435	1000	52230	12–35	2	0.0117	600	8926
02–87	1	0.0259	1000	31192	12–84	0	0.0058	1200	21232
03–71	0	0.0078	3200	92253	13–14	0	0.0075	1200	10690
03–81	0	0.0049	3200	60153	13–15	0	0.0215	1200	26770
03–83	0	0.0043	3200	53253	13–17	0	0.0232	1200	28780
03–87	0	0.0058	1200	21232	13–45	1	0.0290	1200	35480
04–05	1	0.0435	1000	52230	13–59	1	0.0232	1200	28780
04–06	0	0.0487	1000	58260	14–17	0	0.0232	1200	28780
04–32	0	0.0233	300	7510	14–45	0	0.0232	1200	28780
04–60	0	0.0215	1000	26770	14–59	0	0.0157	1200	20070
04–68	0	0.0070	1000	10020	15–16	2	0.0197	1200	24760
04–69	0	0.0162	1000	20740	15–45	0	0.0103	1200	13906
04–81	0	0.0058	1200	21232	15–46	1	0.0117	600	8926
04–87	1	0.0218	1000	26502	15–53	0	0.0423	1000	50890
05–06	1	0.0241	1000	29852	16–44	4	0.0177	600	8926
05–38	2	0.0117	600	8926	16–45	0	0.0220	1200	27440
05–56	0	0.0235	1000	29182	16–61	0	0.0128	1000	16720
05–58	0	0.0220	1000	27440	16–77	0	0.0058	1200	21232
05–60	0	0.0261	1000	32130	17–18	2	0.0170	1200	21678
05–68	0	0.0406	1000	48880	17–59	0	0.0170	1200	21678

Table 4: *(continued)*

From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US;dollar;	From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$
05–70	0	0.0464	1000	55580	18–50	4	0.0117	600	8926
05–80	0	0.0058	1200	21232	18–59	1	0.0331	1200	40170
06–07	1	0.0288	1000	35212	18–74	0	0.0058	1200	21232
06–37	1	0.0233	300	7510	19–20	1	0.0934	170	5885
06–67	0	0.0464	1000	55580	19–22	1	0.1877	170	11165
06–68	0	0.0476	1000	56920	20–21	1	0.0715	300	6960
06–70	0	0.0371	1000	44860	20–21	1	0.1032	170	6435
06–75	0	0.0058	1200	21232	20–38	2	0.1382	300	12840
07–08	1	0.0234	1000	29048	20–56	0	0.0117	600	8926
07–53	0	0.0452	1000	54240	20–66	0	0.2064	170	12210
07–62	0	0.0255	1000	31460	21–57	0	0.0117	600	8926
08–09	1	0.0186	1000	23420	22–23	1	0.1514	170	9130
08–12	0	0.0394	1000	47540	22–37	2	0.2015	170	11935
08–17	0	0.0447	1000	53570	22–58	0	0.0233	300	7510
08–53	1	0.0365	1200	44190	23–24	1	0.1651	170	9900
08–62	0	0.0429	1000	51560	24–25	1	0.2153	170	12705
08–73	0	0.0058	1200	21232	24–43	0	0.0233	300	7510
09–10	1	0.0046	1000	7340	25–26	2	0.1073	300	29636
10–11	1	0.0133	1000	17390	25–26	3	0.1691	170	10120
11–12	1	0.0041	1200	6670	25–55	0	0.0117	600	8926
11–15	1	0.0297	1200	36284	26–27	2	0.1404	300	25500
11–17	1	0.0286	1200	35078	26–27	3	0.2212	170	12760
11–53	1	0.0254	1000	31326	26–29	1	0.1081	170	6710
12–13	1	0.0046	1200	7340	26–54	0	0.0117	600	8926
27–28	3	0.0826	170	5335	60–66	0	0.0233	300	7510
27–35	2	0.1367	300	25000	60–87	0	0.0377	1000	45530
27–53	1	0.0117	600	8926	61–64	0	0.0186	1000	23420
28–35	3	0.1671	170	9900	61–85	0	0.0233	300	7510
29–30	1	0.0688	170	4510	61–86	0	0.0139	1000	18060
30–31	1	0.0639	170	4235	62–67	0	0.0464	1000	55580
30–63	0	0.0233	300	7510	62–68	0	0.0557	1000	66300
31–34	1	0.1406	170	8525	62–72	0	0.0058	1200	21232
32–33	0	0.1966	170	11660	63–64	0	0.0290	1000	35480
33–67	0	0.0233	300	7510	65–66	0	0.3146	170	18260
34–39	2	0.1160	170	7510	65–87	0	0.0233	300	7510
34–39	2	0.2968	80	6335	67–68	0	0.0290	1000	35480
34–41	2	0.0993	170	6215	67–69	0	0.0209	1000	26100
35–46	4	0.2172	170	12705	67–71	0	0.0058	1200	21232
35–47	2	0.1327	170	8085	68–69	0	0.0139	1000	18060
35–51	3	0.1602	170	9625	68–83	0	0.0058	1200	21232
36–39	2	0.1189	170	7315	68–87	0	0.0186	1000	23240
36–46	2	0.0639	170	4235	69–87	0	0.0139	1000	18060
39–42	1	0.0973	170	6105	70–82	0	0.0058	1200	21232
39–86	0	0.0233	300	7510	71–72	0	0.0108	3200	125253
40–45	1	0.0117	600	8926	71–75	0	0.0108	3200	125253
40–46	3	0.0875	170	5500	71–83	0	0.0067	3200	80253
41–64	0	0.0233	300	7510	72–73	0	0.0100	3200	116253
42–44	2	0.0698	170	4565	72–83	0	0.0130	3200	149253
42–85	2	0.0501	170	3465	73–74	0	0.0130	3200	149253
43–55	0	0.0254	1000	31326	73–75	0	0.0130	3200	149253
43–58	0	0.0313	1000	38160	73–84	0	0.0092	3200	107253
44–46	3	0.1671	170	10010	74–84	0	0.0108	3200	125253
47–48	2	0.1966	170	11660	75–76	0	0.0162	3200	185253
48–49	1	0.0757	170	4895	75–81	0	0.0113	3200	131253

Table 4: (*continued*)

From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US;dollar;	From–To	n_{ij}^0	Reactance p.u.	$\bar{f}_{ij}$, MW	Cost, 10^3 US$
48–50	2	0.0256	170	2090	75–82	0	0.0086	3200	101253
48–51	2	0.2163	170	12760	75–83	0	0.0111	3200	128253
49–50	1	0.0835	170	5335	76–77	0	0.0130	3200	149253
51–52	2	0.0560	170	3795	76–82	0	0.0086	3200	101253
52–59	1	0.0117	600	8926	76–84	0	0.0059	3200	70953
53–54	0	0.0270	1000	32120	77–79	0	0.0151	3200	173253
53–70	0	0.0371	1000	44860	77–84	0	0.0115	3200	132753
53–76	0	0.0058	1200	21232	78–79	0	0.0119	3200	137253
53–86	0	0.0389	1000	46870	78–80	0	0.0051	3200	62253
54–55	0	0.0206	1000	25028	79–82	0	0.0084	3200	98253
54–58	0	0.0510	1000	60940	80–81	0	0.0101	3200	117753
54–63	0	0.0203	1000	25430	80–82	0	0.0108	3200	125253
54–70	0	0.0360	1000	43520	80–83	0	0.0094	3200	110253
54–79	0	0.0058	1200	21232	81–83	0	0.0016	3200	23253
56–57	0	0.0122	1000	16050	82–84	0	0.0135	3200	155253
58–78	0	0.0058	1200	21232					

Table 5: Optimal solutions obtained for Garver network

No.	Added circuits				Investment cost
	n_{15}	n_{26}	n_{35}	n_{46}	v
1	0	4	1	2	200
2	0	3	1	3	200
3	0	5	1	1	200
4	1	4	0	2	200
5	1	3	0	3	200

$$f_{15} + f_{35} + r_5 = 2.40 \quad (27)$$

$$f_{26} + f_{46} + g_6 = 0 \quad (28)$$

$$f_{12} - \frac{5}{2}(1 + n_{12})(\theta_1 - \theta_2) = 0 \quad (29)$$

$$f_{14} - \frac{5}{3}(1 + n_{14})(\theta_1 - \theta_4) = 0 \quad (30)$$

$$f_{15} - 5(1 + n_{15})(\theta_1 - \theta_5) = 0 \quad (31)$$

$$f_{23} - 5(1 + n_{23})(\theta_2 - \theta_3) = 0 \quad (32)$$

$$f_{24} - \frac{5}{2}(1 + n_{24})(\theta_2 - \theta_4) = 0 \quad (33)$$

$$f_{26} - \frac{10}{3}n_{26}(\theta_2 - \theta_6) = 0 \quad (34)$$

$$f_{35} - 5(1 + n_{35})(\theta_3 - \theta_5) = 0 \quad (35)$$

$$f_{46} - \frac{10}{3}n_{46}(\theta_4 - \theta_6) = 0 \quad (36)$$

$$|f_{12}| \le (1 + n_{12}) \quad (37)$$

$$|f_{14}| \le 0.8(1 + n_{14}) \quad (38)$$

$$|f_{15}| \le (1 + n_{15}) \quad (39)$$

$$|f_{23}| \le (1 + n_{23}) \quad (40)$$

$$|f_{24}| \le (1 + n_{24}) \quad (41)$$

$$|f_{26}| \le n_{26} \quad (42)$$

$$|f_{35}| \le (1 + n_{35}) \quad (43)$$

$$|f_{46}| \le n_{46} \quad (44)$$

$$0 \le g_3 \le 1.25$$

$$0 \le g_6 \le 5.45$$

$$0 \le r_1 \le 0.30$$

$$0 \le r_2 \le 2.40$$

$$0 \le r_4 \le 1.60$$

$$0 \le r_5 \le 2.40$$

$$n_{12}, n_{14}, n_{15}, n_{23}, n_{24}, n_{26}, n_{35}, n_{46} \text{ integer}$$

$$\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6 \text{ unbounded}$$

$$f_{12}, f_{14}, f_{15}, f_{23}, f_{24}, f_{26}, f_{35}, f_{46} \text{ unbounded}$$

The transportation model can be obtained from the DC model given above by eliminating the constraints in eqns. 29–36. The DC model can have its size reduced by eliminating the power flow variables f_{ij}; this can be done using the equality constraints in eqns. 29–36. In this case, the constraints in eqns. 23–44 are replaced by the following constraints:

$$-\left[\frac{55}{6}+\frac{5}{2}n_{2}+\frac{5}{3}n_{14}+5n_{15}\right]\theta_1$$
$$+\frac{5}{2}(1+n_{12})\theta_2+\frac{5}{3}(1+n_{14})\theta_4+5(1+n_{15})\theta_5+r_1=0.30$$

$$\frac{1}{2}(1+n_{12})\theta_1-\left[2+\frac{1}{2}n_{12}+n_{23}+\frac{1}{2}n_{24}+\frac{2}{3}n_{26}\right]\theta_2$$
$$+(1+n_{23})\theta_3+\frac{1}{2}(1+n_{24})\theta_4+\frac{2}{3}n_{26}\theta_6+0.2r_2=0.48$$

$$(1+n_{23})\theta_2-[2+n_{23}+n_{35}]\theta_3+(1+n_{35})\theta_5$$
$$+0.2g_3=0.0$$

$$\frac{5}{3}(1+n_{14})\theta_1$$
$$+\frac{5}{2}(1+n_{24})\theta_2-\left[\frac{25}{6}+\frac{5}{3}n_{14}+\frac{5}{2}n_{24}+\frac{10}{3}n_{46}\right]\theta_4$$
$$+\frac{10}{3}n_{46}\theta_6+r_4=1.60$$

$$(1+n_{15})\theta_1+(1+n_{35})\theta_3-[2+n_{15}+n_{35})]\theta_5$$
$$+0.2r_5=0.48$$

$$n_{26}\theta_2+n_{46}\theta_4-[n_{26}+n_{46}]\theta_6+0.3g_6=0.0$$

$$|\theta_1-\theta_2|\le 0.40$$
$$|\theta_1-\theta_4|\le 0.48$$
$$|\theta_1-\theta_5|\le 0.20$$
$$|\theta_2-\theta_3|\le 0.20$$
$$|\theta_2-\theta_4|\le 0.40$$
$$n_{26}|\theta_2-\theta_6|\le 0.30n_{26}$$
$$|\theta_3-\theta_5|\le 0.20$$
$$n_{46}|\theta_4-\theta_6|\le 0.30n_{46}$$

For the hybrid model, the constraints in eqns. 9–12 are rewritten as follows:

$$-f_{12}-f_{14}-f_{15}-f'_{12}-f'_{14}-f'_{15}+r_1=0.30 \quad (45)$$
$$f_{12}-f_{23}-f_{24}+f'_{12}-f'_{23}-f'_{24}-f'_{26}+r_2=2.40 \quad (46)$$
$$f_{23}-f_{35}+f'_{23}-f'_{35}+g_3=0.0 \quad (47)$$
$$f_{14}+f_{24}+f'_{14}+f'_{24}-f'_{46}+r_4=1.60 \quad (48)$$
$$f_{15}+f_{35}+f'_{15}+f'_{35}+r_5=2.40 \quad (49)$$
$$f'_{26}+f'_{46}+g_6=0 \quad (50)$$
$$f_{12}-\frac{5}{2}(\theta_1-\theta_2)=0 \quad (51)$$
$$f_{14}-\frac{5}{3}(\theta_1-\theta_4)=0 \quad (52)$$
$$f_{15}-5(\theta_1-\theta_5)=0 \quad (53)$$
$$f_{23}-5(\theta_2-\theta_3)=0 \quad (54)$$
$$f_{24}-\frac{5}{2}(\theta_2-\theta_4)=0 \quad (55)$$
$$f_{35}-5(\theta_3-\theta_5)=0 \quad (56)$$
$$|f_{12}|\le 1.0 \quad (57)$$
$$|f_{14}|\le 0.8 \quad (58)$$
$$|f_{15}|\le 1.0 \quad (59)$$
$$|f_{23}|\le 1.0 \quad (60)$$
$$|f_{24}|\le 1.0 \quad (61)$$
$$|f_{35}|\le 1.0 \quad (62)$$
$$|f'_{12}|\le n_{12} \quad (63)$$
$$|f'_{14}|\le 0.8\,n_{14} \quad (64)$$
$$|f'_{15}|\le n_{15} \quad (65)$$
$$|f'_{23}|\le n_{23} \quad (66)$$
$$|f'_{24}|\le n_{24} \quad (67)$$
$$|f'_{26}|\le n_{26} \quad (68)$$
$$|f'_{35}|\le n_{35} \quad (69)$$
$$|f'_{46}|\le n_{46} \quad (70)$$

where the objective function and other trivial constraints are the same as in the DC model. As happens with the DC model the power flow variables can be eliminated from the model, resulting the following reduced system:

$$f'_{12}-f'_{14}-f'_{15}-\frac{55}{6}\theta_1+\frac{5}{2}\theta_2+\frac{5}{3}\theta_4+5\theta_5+r_1=0.30$$
$$f'_{12}-f'_{23}-f'_{24}-f'_{26}+\frac{5}{2}\theta_1-10\theta_2+5\theta_3+\frac{5}{2}\theta_4+r_2=2.40$$
$$f'_{23}-f'_{35}+5\theta_2-10\theta_3+5\theta_5+g_3=0.00$$
$$f'_{14}+f'_{24}-f'_{46}+\frac{5}{3}\theta_1+\frac{5}{2}\theta_2-\frac{25}{6}\theta_4+r_4=1.60$$
$$f'_{15}+f'_{35}+5\theta_1+5\theta_3-10\theta_5+r_5=2.40$$
$$f'_{26}+f'_{46}+g_6=0.0$$
$$|\theta_1-\theta_2|\le 0.40$$
$$|\theta_1-\theta_4|\le 0.48$$
$$|\theta_1-\theta_5|\le 0.20$$
$$|\theta_2-\theta_3|\le 0.20$$
$$|\theta_2-\theta_4|\le 0.40$$
$$|\theta_3-\theta_5|\le 0.20$$
$$|f'_{12}|\le n_{12}$$
$$|f'_{14}|\le 0.8\,n_{14}$$
$$|f'_{15}|\le n_{15}$$
$$|f'_{23}|\le n_{23}$$
$$|f'_{24}|\le n_{24}$$
$$|f'_{26}|\le n_{26}$$
$$|f'_{35}|\le n_{35}$$
$$|f'_{46}|\le n_{46}$$

For the disjunctive model, let us assume that the maximum number of circuit additions is of two circuits, except for right-of-way 2–6, where the limit is four circuits. Under these assumptions, the disjunctive model can be formulated as follows.

Minimise

$$
\begin{aligned}
v =\,& 40(y_{12}^1 + y_{12}^2) + 60(y_{14}^1 + y_{14}^2) + 20(y_{15}^1 + y_{15}^2)\\
&+ 20(y_{23}^1 + y_{23}^2) + 40(y_{24}^1 + y_{24}^2)\\
&+ 30(y_{26}^1 + y_{26}^2 + y_{26}^3 + y_{26}^4)\\
&+ 20(y_{35}^1 + y_{35}^2)\\
&+ 30(y_{46}^1 + y_{46}^2) + \alpha(r_1 + r_2 + r_4 + r_5)
\end{aligned}
$$

Subject to

$$
\begin{aligned}
&- f_{12}^0 - f_{12}^1 - f_{12}^2 - f_{14}^0 - f_{14}^1 - f_{14}^2 - f_{15}^0\\
&- f_{15}^1 - f_{15}^2 + r_1 = 0.30
\end{aligned}
$$

$$
\begin{aligned}
&f_{12}^0 + f_{12}^1 + f_{12}^2 - f_{23}^0 - f_{23}^1 - f_{23}^2 - f_{24}^0 - f_{24}^1 - f_{24}^2\\
&- f_{26}^1 - f_{26}^2 - f_{26}^3 - f_{26}^4 + r_2 = 2.40
\end{aligned}
$$

$$f_{23}^0 + f_{23}^1 + f_{23}^2 - f_{35}^0 - f_{35}^1 - f_{35}^2 + g_3 = 0.00$$

$$f_{14}^0 + f_{14}^1 + f_{14}^2 + f_{24}^0 + f_{24}^1 + f_{24}^2 - f_{46}^1 - f_{46}^2 + r_4 = 1.60$$

$$f_{15}^0 + f_{15}^1 + f_{15}^2 + f_{35}^0 + f_{35}^1 + f_{35}^2 + r_5 = 2.40$$

$$f_{26}^1 + f_{26}^2 + f_{26}^3 + f_{26}^4 + f_{46}^1 + f_{46}^2 + g_6 = 0$$

$$f_{12}^0 - \frac{5}{2}(\theta_1 - \theta_2) = 0$$

$$f_{14}^0 = \frac{5}{3}(\theta_1 - \theta_4) = 0$$

$$f_{15}^0 - 5(\theta_1 - \theta_5) = 0$$

$$f_{23}^0 - 5(\theta_2 - \theta_3) = 0$$

$$f_{24}^0 - \frac{5}{2}(\theta_2 - \theta_4) = 0$$

$$f_{35}^0 - 5(\theta_3 - \theta_5) = 0$$

$$|f_{12}^p - \frac{5}{2}(\theta_1 - \theta_2)| \le M(1 - y_{12}^p), \quad p = 1, 2$$

$$|f_{14}^p - \frac{5}{3}(\theta_1 - \theta_4)| \le M(1 - y_{14}^p), \quad p = 1, 2$$

$$|f_{15}^p - 5(\theta_1 - \theta_5)| \le M(1 - y_{15}^p), \quad p = 1, 2$$

$$|f_{23}^p - 5(\theta_2 - \theta_3)| \le M(1 - y_{23}^p), \quad p = 1, 2$$

$$|f_{24}^p - \frac{5}{2}(\theta_2 - \theta_4)| \le M(1 - y_{24}^p), \quad p = 1, 2$$

$$|f_{26}^p - \frac{10}{3}(\theta_2 - \theta_6)| \le M(1 - y_{26}^p), \quad p = 1, 2, 3, 4$$

$$|f_{35}^p - 5(\theta_3 - \theta_5)| \le M(1 - y_{35}^p), \quad p = 1, 2$$

$$|f_{46}^p - \frac{10}{3}(\theta_4 - \theta_6)| \le M(1 - y_{46}^p), \quad p = 1, 2$$

$$|f_{12}^0| \le 1.0$$

$$|f_{14}^0| \le 0.8$$

$$|f_{15}^0| \le 1.0$$

$$|f_{23}^0| \le 1.0$$

$$|f_{24}^0| \le 1.0$$

$$|f_{35}^0| \le 1.0$$

$$|f_{12}^p| \le y_{12}^p, \quad p = 1, 2$$

$$|f_{14}^p| \le 0.8 y_{14}^p, \quad p = 1, 2$$

$$|f_{15}^p| \le y_{15}^p, \quad p = 1, 2$$

$$|f_{23}^p| \le y_{23}^p, \quad p = 1, 2$$

$$|f_{24}^p| \le y_{24}^p, \quad p = 1, 2$$

$$|f_{26}^p| \le y_{26}^p, \quad p = 1, 2, 3, 4$$

$$|f_{35}^p| \le y_{35}^p, \quad p = 1, 2$$

$$|f_{46}^p| \le y_{46}^p, \quad p = 1, 2$$

$$0 \le g_3 \le 1.25$$

$$0 \le g_6 \le 5.45$$

$$0 \le r_1 \le 0.30$$

$$0 \le r_2 \le 2.40$$

$$0 \le r_4 \le 1.60$$

$$0 \le r_5 \le 2.40$$

$$y_{ij}^p \in \{0, 1\};\ f_{ij}^p,\ f_{ij}^p \text{ and } \theta_j \text{ unbounded}$$

The transportation model has the five alternative optimal solutions shown in Table 5. The first three solutions are also optimal solutions for the hybrid model. Only, the first solution in the Table is the optimal solution for the DC model.

5 Solution techniques

A variety of algorithms for solving the one-stage transmission expansion planning problem have been suggested in the literature. The proposed method can be classified in three large groups: (i) heuristic algorithms, (ii) classical mathematical optimisation algorithms, and (iii) algorithms based on metaheuristics. Heuristic algorithms are simple to implement and require relatively small computational effort: as a rule, they are able to find good quality solutions for small systems, although for larger networks the solutions can be very poor. Examples of heuristic algorithms can be found in [4, 6–12].

There are very few proposals in the literature of classical optimization algorithms applied to transmission expansion planning. A popular choice in the area is the Benders decomposition approach. This technique has been used with different types of network model. Although this technique works for small and medium sized systems, the computational effort for larger systems can be prohibitive. Numerical stability problems, as well as local optimal solutions, have also been reported [3, 13]. The branch-and-bound algorithm has been used in connection with the transportation model [14]. The combined use of optimisation and heuristics has also been tried [15, 16].

More recently metaheuristic algorithms—simulated annealing, genetic algorithms, tabu search GRASP, etc. —have been applied to the transmission expansion planning problem [13, 17–19]. These algorithms are usually robust and yield near-optimal solutions for large complex networks. As a rule, these methods require high computational effort. This limitation, however, is not necessarily critical in planning applications. New, more efficient algorithms are still needed to solve the problems classified as very complex (VC) in the next Section.

Even more complex problems such as the dynamic (through time) expansion, integrated generation/transmission planning, and planning in competitive environments have received very little attention in the literature, perhaps due to the fact that the apparently easier part of the problem, the one-stage expansion, still remains unsolved for more complex networks.

6 Proposed tests and known solutions

In this Section, the tests that can be performed with the four test systems presented are summarised. The optimal solutions (if they are known) or the best solutions known for each system and each model (DC, hybrid, transportation etc.) are also indicated. Tables 6–9 give the costs (in 1000US$) of the best known solutions for the various combinations of models and test systems. The following notation has been used in these tables: NOR means without redispatch, WR means with redispatch, NRNN meass without redispatch and without initial network (green-field expansion). WRNN means with redispatch but without initial network (green-field), VS means very simple, S means simple. N means normal. C means complex and VC means very complex. All solutions presented herein have been obtained with no loss of load, i.e. $w = \Sigma_k r_k = 0$.

7 Conclusions

The paper gives the data for the one-stage transmission expansion planning of four systems with different levels of complexity. These systems are intended to be used in tests of algorithms designed to find optimal expansion plans. In addition, the most popular models used in transmission expansion studies are summarised and compared: the DC

Table 6: Results for 6-bus system

Test type	Model					
	Transport		Hybrid		DC	
	v	Complexity	v	Complexity	v	Complexity
NOR	200	VS	200	VS	200	VS
WR	110	VS	110	VS	110	NS
NRNN	291	S	291	S	291	S
WRNN	190	S	190	S	190	S

Table 7: Results for 46-bus system

Test type	Model					
	Transport		Hybrid		DC	
	v	Complexity	v	Complexity	v	Complexity
NOR	127272	N	141350	N	154420	C
WR	53334	S	63136	N	72780	N
NRNN	473208	C	—	C	—	C
WRNN	402748	C	402748	C	402748	C

Table 8: Results for the 78-bus system

Test type	Model					
	Transport		Hybrid		DC	
	v	Complexity	v	Complexity	v	Complexity
NOR	284142	N	—	N	424800	C
WR	—	S	—	N	—	C
NRNN	—	VC	—	VC	—	VC
WRNN	—	VC	—	VC	—	VC

Table 9: Results for 87-bus system

Test type		Model					
		Transport		Hybrid		DC	
		v	Complexity	v	Complexity	v	Complexity
Plane NOR	P1:	1194240	C	—	C	1356272	VC
Plane WR	P1:	614900	C	—	C	737147	VC
Plane NRNN	P1:	—	VC	—	VC	—	VC
Plane WRNN	P1:	—	VC	—	VC	—	VC
Plane NOR	P2:	—	C	—	C	—	VC
Plane WR	P2:	—	C	—	C	2474750	VC
Plane NRNN	P2:	—	VC	—	VC	—	VC
Plane WRNN	P2:	—	VC	—	VC	—	VC

model, transportation model, hybrid model and disjunctive model. Finally, the best known solutions for each system and each alternative model are presented—the importance of these data is that for certain combinations of system/model the optimal solutions are not yet, known and so the data should serve as a benchmark for further developments in the area.

8 Acknowledgement

This project was partly supported by the Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), and the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).

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