EE 303, Quiz 8, April 18, 2019, Dr. McCalley
Closed book, closed notes, no calculator, 20 minutes

1. (30pts) A 2500 bus system has 500 generators; all of the generator buses are modeled in a power flow program with constant (known) terminal voltage.
   a. How many type PV buses are there in the power flow model?
   b. How many type PQ buses are there in the power flow model?
   c. What is the minimum number of equations required to solve this problem?
   d. How many bus voltage magnitudes are unknown in this problem?
   e. How many bus voltage angles are unknown in this problem?

Answer:

a. 499
b. 2000
c. 4499
d. 2000
e. 2499

2. (40pts) Consider the following equations: $\underline{f}(\underline{x}) = \begin{bmatrix} f_1(\underline{x}) \\ f_2(\underline{x}) \end{bmatrix} = \begin{bmatrix} x_1^3 - x_2 \\ x_1 - x_2 \end{bmatrix}$. Using an initial guess at the solution of $\underline{x}^{(0)} = \begin{bmatrix} x_1^{(0)} \\ x_2^{(0)} \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$, show one iteration of the Newton-Raphson solution procedure. To get full credit on this problem, you must provide $\underline{J}$, $\underline{J}^{-1}$, $\underline{\Delta x}^{(0)}$, and $\underline{x}^{(1)}$.

$$\underline{f}(\underline{x}^{(0)}) = \begin{bmatrix} f_1(\underline{x}^{(0)}) \\ f_2(\underline{x}^{(0)}) \end{bmatrix} = \begin{bmatrix} 2^3 - 2 \\ 2 - 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$\underline{J}(\underline{x}) = \begin{bmatrix} 3x_1^2 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow\Rightarrow \underline{J}(\underline{x}^{(0)}) = \begin{bmatrix} 12 & -1 \\ 1 & -1 \end{bmatrix} \Rightarrow\Rightarrow \underline{J}^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{-1}{11} \\ \frac{1}{11} & \frac{-12}{11} \end{bmatrix}$$

$$\Delta\underline{x}^{(0)} = -\underline{J}^{-1}\underline{f}(\underline{x}^{(0)}) = -\begin{bmatrix} \frac{1}{11} & \frac{-1}{11} \\ \frac{1}{11} & \frac{-12}{11} \end{bmatrix}\begin{bmatrix} 6 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{-6}{11} \\ \frac{-6}{11} \end{bmatrix} \rightarrow \underline{x}^{(1)} = \underline{x}^{(0)} + \Delta\underline{x}^{(0)} = \begin{bmatrix} 2 \\ 2 \end{bmatrix} + \begin{bmatrix} \frac{-6}{11} \\ \frac{-6}{11} \end{bmatrix} = \begin{bmatrix} \frac{16}{11} \\ \frac{16}{11} \end{bmatrix}$$

3. (30pts) For the following optimization problem, *(a)* express the Lagrangian function you would use to obtain a first solution of the problem; *(b)* If your answer to the problem expressed in *part a* was (x_1, x_2)=(1.2,0.6), what would you do next? (c) for the situation of part b, what is the value of the Lagrange multiplier (μ) corresponding to the inequality constraint?

$$\min \ f(x_1, x_2) = x_1^2 + x_2^2$$
subject to
$$h(x_1, x_2) = 2x_1 + x_2 = 3$$
$$g(x_1, x_2) = x_2 \le 0.7$$

Answer:

a. $F(x_1, x_2, \lambda) = x_1^2 + x_2^2 - \lambda(2x_1 + x_2 - 3)$
b. Declare the solution to the overall problem to be the solution to the problem solved in part *a*.
c. μ=0