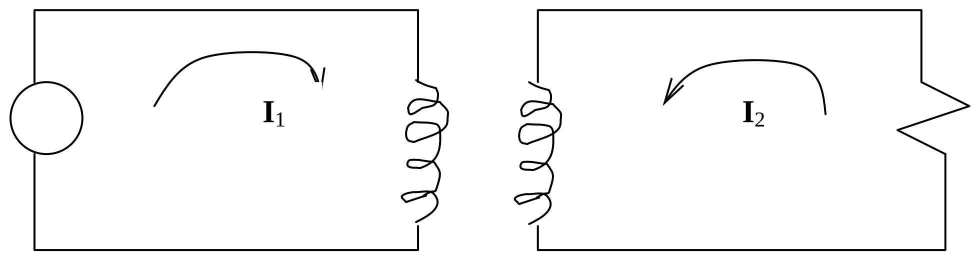


Left loop: $5\cos(4t+45°) = 8i_1(t) + 4\frac{di_1(t)}{dt} + 2\frac{di_2(t)}{dt}$

Right loop: $3\frac{di_2(t)}{dt} + 2\frac{di_1(t)}{dt} + 12i_2(t) = 0$

Now let's change to phasor NOTATION:

$5\angle 45° = 8\bar{I}_1 + j\omega 4\bar{I}_1 + j\omega 2\bar{I}_2$

$j\omega 3\bar{I}_2 + j\omega 2\bar{I}_1 + 12\bar{I}_2 = 0$

And with $\omega = 4$ (from the source voltage), we have

$5\angle 45° = 8\bar{I}_1 + j16\bar{I}_1 + j8\bar{I}_2 \Rightarrow 5\angle 45° = (8+j16)\bar{I}_1 + j8\bar{I}_2$ (1)

$j12\bar{I}_2 + j8\bar{I}_1 + 12\bar{I}_2 = 0 \qquad (12+j12)\bar{I}_2 + j8\bar{I}_1 = 0$ (2)

Solving the second equation for $\bar{I}_1 \Rightarrow \bar{I}_1 = \frac{-12}{j8}[1+j]\bar{I}_2 = \frac{j3}{2}[1+j]\bar{I}_2$

$\Rightarrow \bar{I}_1 = \frac{3}{2}[-1+j]\bar{I}_2$. Now substitute into (1): $(8+j16)(\frac{3}{2})[-1+j]\bar{I}_2 + j8\bar{I}_2 = 5\angle 45°$

$\Rightarrow [(12+j24)[-1+j] + j8]\bar{I}_2 = 5\angle 45° \Rightarrow \bar{I}_2 = 5\angle 45° / [-4(9+j)] = .138\angle 141.35°$

Now get $\bar{V}_2 = \bar{I}_2 R = (.138\angle 141.35°)(12) = 1.656\angle 141.35°$

Note the above calculation has two errors in the last two lines. These errors are:

a. next to last line: should be I_2=.138 /_ -141.35°. b. last line: should be V_2=-I_2R=1.656/_38.65°.

7. The ohmic values of circuit parameters of a transformer, having turns ration of N_1/N_2=5, are R_1=0.5Ω, R_2=0.021Ω, X_1=3.2Ω, X_2=0.12Ω, R_c=350Ω, and X_m=98Ω. (R_1, X_1, R_c, and X_m are given referred to the primary side; R_2 and X_2 are given referred to the secondary). Draw the approximate equivalent circuit of the transformer, with all quantities referred to (a) the primary and (b) the secondary. Show the numerical values of the circuit parameters.

The circuits are respectively shown in Fig. 2-10(*a*) and Fig. 2-10(*b*). The calculations are as follows:

(*a*)

$$R' \equiv R_1 + a^2R_2 = 0.5 + (5)^2(0.021) = 1.025\ \Omega$$
$$X' \equiv X_1 + a^2X_2 = 3.2 + (5)^2(0.12) = 6.2\ \Omega$$
$$R'_c = 350\ \Omega$$
$$X'_m = 98\ \Omega$$

(*b*)

$$R'' \equiv \frac{R_1}{a^2} + R_2 = \frac{0.5}{25} + 0.021 = 0.041\ \Omega$$
$$X'' \equiv \frac{X_1}{a^2} + X_2 = \frac{3.2}{25} + 0.12 = 0.248\ \Omega$$
$$R''_c = \frac{350}{25} = 14\ \Omega$$
$$X''_m = \frac{98}{25} = 3.92\ \Omega$$

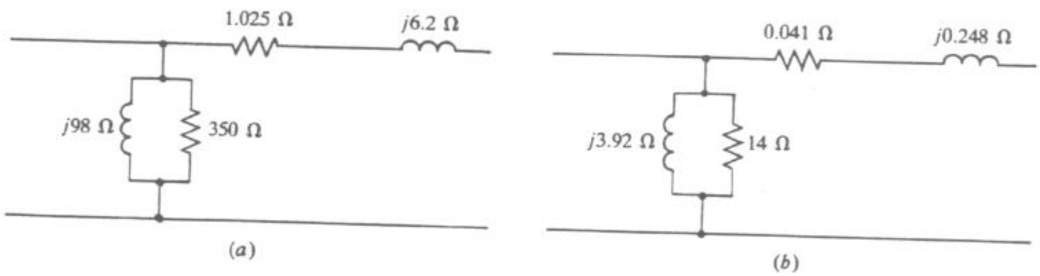


Fig. 2-10

8. The "exact" equivalent circuit parameters of a 150-kVA, 2400volt/240volt transformer are R_1=0.2 Ω, R_2=2mΩ, X_1=0.45Ω, X_2=4.5mΩ, R_c=10kΩ, and X_m=1.55kΩ. (R_1, X_1, R_c, and X_m are given referred to the primary side; R_2 and X_2 are given referred to the secondary). Using the circuit referred to the primary, determine the (a) percent voltage regulation and (b) efficiency of the transformer operating at rated load (150 kVA) with 0.8 lagging power factor. Assume that V_2=240 volts and note that percent voltage regulation is given by ($V_{no\text{-}load}$-V_{load})/V_{load} (where these are voltage magnitudes).

   Note that in the below solution, "a" refers to the turns ratio N_1/N_2.

See Figs. 2-3(a) and 2-4. Given $V_2 = 240$ V, $a = 10$, and $\theta_2 = \cos^{-1} 0.8 = -36.8°$,

$$a\mathbf{V}_2 = 2400\underline{/0°} \quad \text{V}$$

$$I_2 = \frac{150 \times 10^3}{240} = 625 \text{ A} \qquad \text{and} \qquad \frac{\mathbf{I}_2}{a} = 62.5\underline{/-36.8°} = 50 - j37.5 \quad \text{A}$$

Also, $a^2R_2 = 0.2\ \Omega$ and $a^2X_2 = 0.45\ \Omega$, so that

$$\begin{aligned}
\mathbf{E}_1 &= (2400 + j0) + (50 - j37.5)(0.2 + j0.45) \\
&= 2427 + j15 = 2427\underline{/0.35°} \quad \text{V} \\
\mathbf{I}_m &= \frac{2427\underline{/0.35°}}{1550\underline{/90°}} = 1.56\underline{/-89.65°} = 0.0095 - j1.56 \quad \text{A} \\
\mathbf{I}_c &= \frac{2427 + j15}{10 \times 10^3} \approx 0.2427 + j0 \quad \text{A}
\end{aligned}$$

Therefore

$$\begin{aligned}
\mathbf{I}_0 &= \mathbf{I}_c + \mathbf{I}_m = 0.25 - j1.56 \quad \text{A} \\
\mathbf{I}_1 &= \mathbf{I}_0 + (\mathbf{I}_2/a) = 50.25 - j39.06 = 63.65\underline{/-37.85°} \quad \text{A} \\
\mathbf{V}_1 &= (2427 + j15) + (50.25 - j39.06)(0.2 + j0.45) \\
&= 2455 + j30 = 2455\underline{/0.7°} \quad \text{V}
\end{aligned}$$

(a)

$$\text{percent regulation} \equiv \frac{V_{\text{no-load}} - V_{\text{load}}}{V_{\text{load}}} \times 100$$

$$= \frac{V_1 - aV_2}{aV_2} \times 100 = \frac{2455 - 2400}{2400} \times 100 = 2.3\%$$

(b)

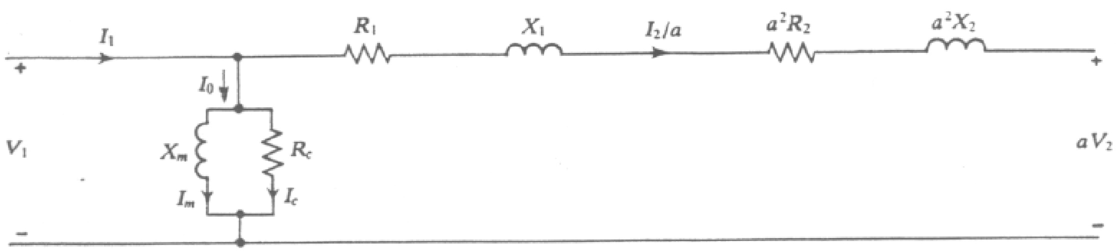
$$\text{efficiency} \equiv \frac{\text{output}}{\text{input}} = \frac{\text{output}}{\text{output} + \text{losses}}$$

$$\begin{aligned}
\text{output} &= (150 \times 10^3)(0.8) = 120 \text{ kW} \\
\text{losses} &= I_1^2R_1 + I_c^2R_c + I_2^2R_2 \\
&= (63.65)^2(0.2) + (0.2427)^2(10 \times 10^3) + (625)^2(2 \times 10^{-3}) = 2.18 \text{ kW}
\end{aligned}$$

Hence

$$\text{efficiency} = \frac{120}{122.18} = 0.982 = 98.2\%$$

9. Using the "approximate equivalent circuit #1", shown below, repeat the calculations of problem 8 and compare the results (note again that "a" is the turns ration N_1/N_2).



$$a\mathbf{V}_2 = 2400\underline{/0^\circ} \quad \text{V}$$

$$\frac{\mathbf{I}_2}{a} = 50 - j37.5 \quad \text{A}$$

$$R_1 + a^2R_2 = 0.4\ \Omega$$

$$X_1 + a^2X_2 = 0.9\ \Omega$$

Hence,

$$\mathbf{V}_1 = (2400 + j0) + (50 - j37.5)(0.4 + j0.9)$$

$$= 2453 + j30 = 2453\underline{/0.7^\circ} \quad \text{V}$$

$$\mathbf{I}_c = \frac{2453\underline{/0.7^\circ}}{10 \times 10^3} = 0.2453\underline{/0.7} \quad \text{A}$$

$$\mathbf{I}_m = \frac{2453\underline{/0.7^\circ}}{1550\underline{/90^\circ}} = 1.58\underline{/-89.3^\circ} \quad \text{A}$$

$$\mathbf{I}_0 = 0.2453 - j1.58 \quad \text{A}$$

$$\mathbf{I}_1 = 50.25 - j39.08 = 63.66\underline{/-37.9^\circ} \quad \text{A}$$

The phasor diagram is shown in Fig. 2-9.

(*a*) $$\text{percent regulation} = \frac{2453 - 2400}{2400} \times 100 = 2.2\%$$

(*b*) $$\text{efficiency} = \frac{120 \times 10^3}{120 \times 10^3 + (63.66)^2(0.4) + (0.2453)^2(10 \times 10^3)} = 0.982 = 98.2\%$$

Notice that the approximate circuit yields results that are sufficiently accurate.

10. The coefficient of coupling for coupled coils is defined as

$$k = \frac{M}{\sqrt{L_{11}L_{22}}}$$

(a) Show that the idea case of no-leakage flux results in k=1.
(b) Determine for the actual case, where leakage flux exists, whether k>1 or k<1.
(c) The coupled circuit below has a coefficient of coupling of 1, Determine the energy stored in the mutually coupled inductors at time t=5 msec, where L_{11}=2.653 mH and L_{22}=10.61 mH.