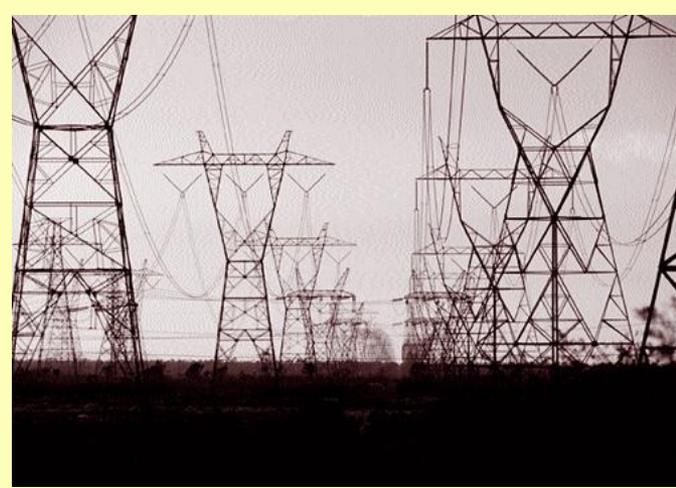


# Module T1 Transmission



Read Module T1.

Work problems 1, 2, 3, 6, 7, 8, 9 at end of Module T1.

Turn in on Tuesday, April 2.



# I will draw 1 quiz question from the following questions.

All questions can be answered based on information contained in Module T1. However, you may have to think.

1. What are the typical voltage levels for bulk transmission lines?
2. What are the typical voltage levels for the subtransmission system?
3. When are underground transmission circuits used?
4. If a transmission line is bundled so that it requires a total of 9 conductors, how many conductors per phase are being used?
5. What is a typical X/R ratio for a transmission line? (X, R are series reactance, resistance, respectively).
6. Which is larger for a transmission circuit: capacitance between the phases or capacitance between phases and ground?
7. Under heavily loaded conditions, is transmission line capacitance a desirable or undesirable influence? (think power factor)
8. Explain why charging capacitance increases with line length.
9. If a transmission line has impedance of  $Z=R+jX=1+j5$ , compute the admittance  $Y=G-jB$ , i.e., find G and B.
10. If a transmission line has impedance of  $Z=R+jX=0+j5$ , compute the admittance  $Y=G-jB$ , i.e., find G and B.
11. Answer the following question using basic knowledge of power calculations in a circuit. A 3 phase transmission line has series impedance of  $Z_{pq}$  ohms. The line has negligible shunt capacitance. The current in the line is 400 amps, when the 3-phase power flowing out of bus 1 into the line is +50 MWs and +20 MVARs. Compute the real & reactive power flowing out of the line into bus 2 if the impedance is: (a)  $Z_{pq}=5+j50$  (b)  $Z_{pq}=0+j50$  (c)  $Z_{pq}=5+j0$

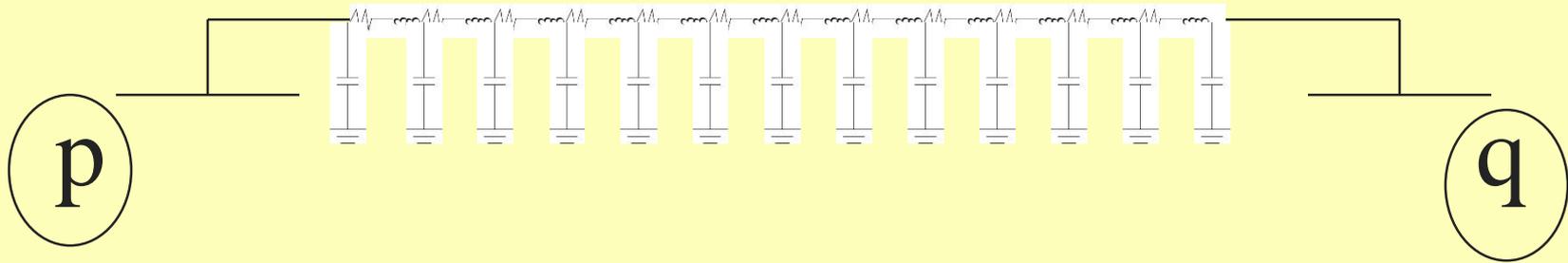
# Electrical characteristics of a transmission line

- When loaded, we observe voltage drop in phase with current; incurs MW losses proportional to current<sup>2</sup>.
  - ➔ **This is modeled with a series resistance,  $R$ .**
- When loaded, we observe voltage drop  $\sim 90^\circ$  ahead of current; incurs MVAR losses proportional to current<sup>2</sup>.
  - ➔ **This is modeled with a series (+) reactance,  $X$ .**
- When unloaded, we observe very small MW losses proportional to voltage<sup>2</sup>.
  - ➔ **This could be modeled with a shunt conductance,  $G$ , but it is negligible, and so we ignore it.**
- When unloaded, we observe fairly large MVAR supply proportional to voltage<sup>2</sup>.
  - ➔ **This is modeled with a shunt susceptance,  $B_C$**

Every inch of the transmission line exhibits the above.

# Electrical characteristics of a transmission line

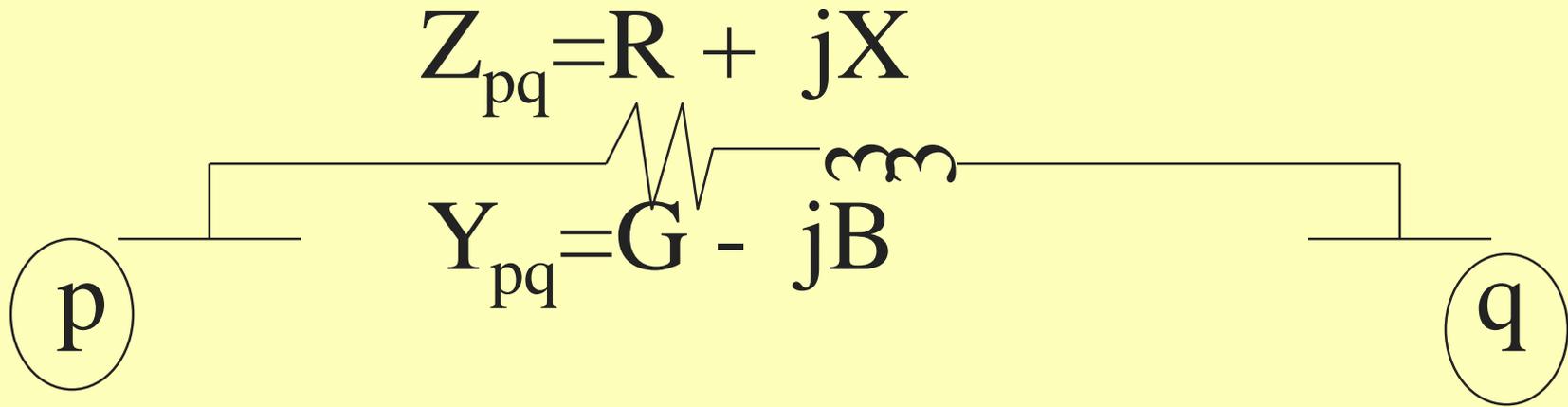
Because every inch of a transmission line exhibits the effects described on the previous slide, a “distributed parameter” model is best.



However, such a model is large and bulky, and experience has it that a “lumped parameter” model works well. The lumped parameter model represents

- the series effect as a single  $R+jX$ , and
- the shunt effect as a single  $jB_C$ .

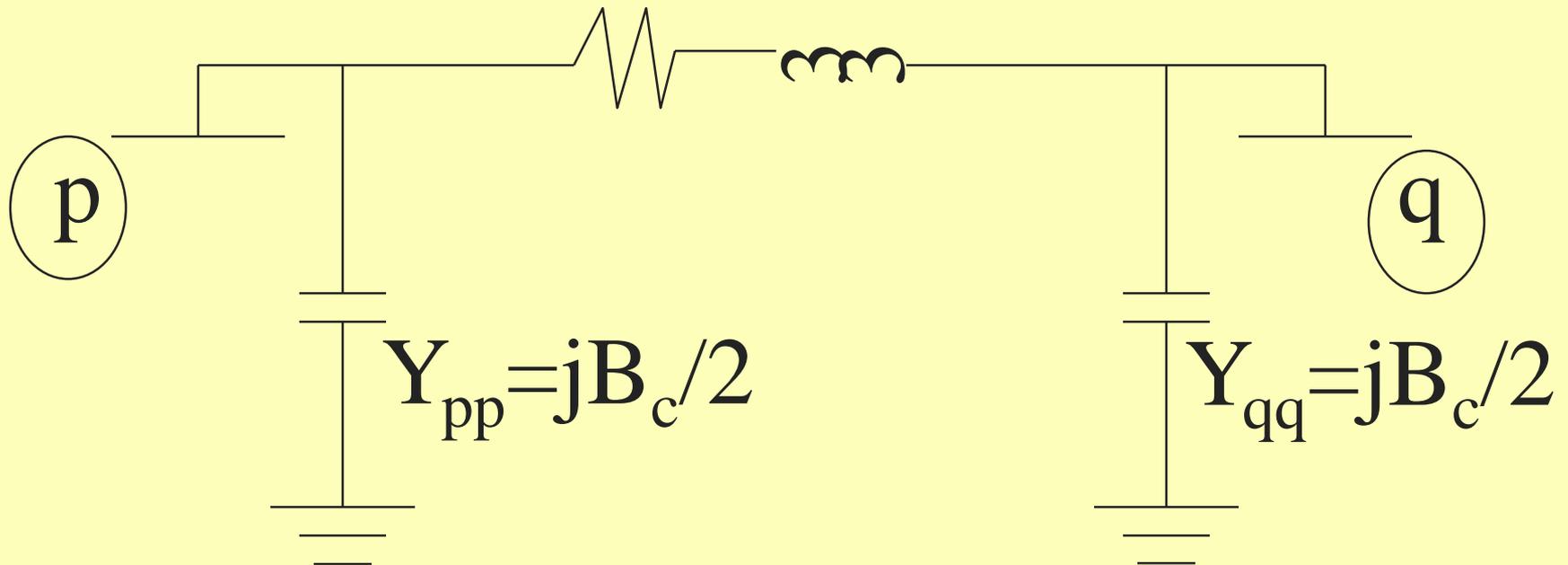
# Series impedance of a 3-phase transmission line



$$Y_{pq} = \frac{1}{Z_{pq}} = \frac{1}{R + jX} \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{R^2 + X^2} - j \frac{X}{R^2 + X^2} = G - jB$$

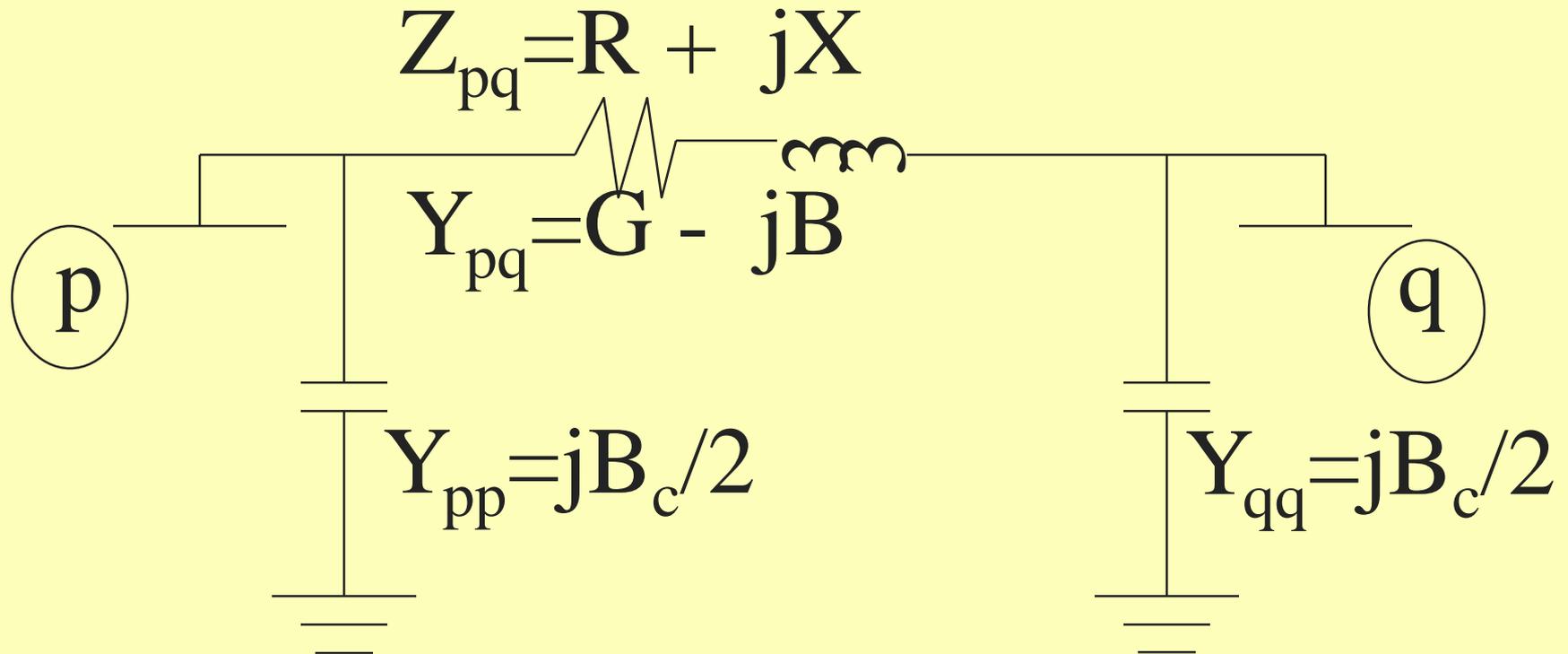
Observe:  $G \neq \frac{1}{R}$      $B \neq \frac{1}{X}$      $G = \frac{R}{R^2 + X^2}$ ;     $B = \frac{X}{R^2 + X^2}$

# Shunt susceptance of a 3-phase transmission line



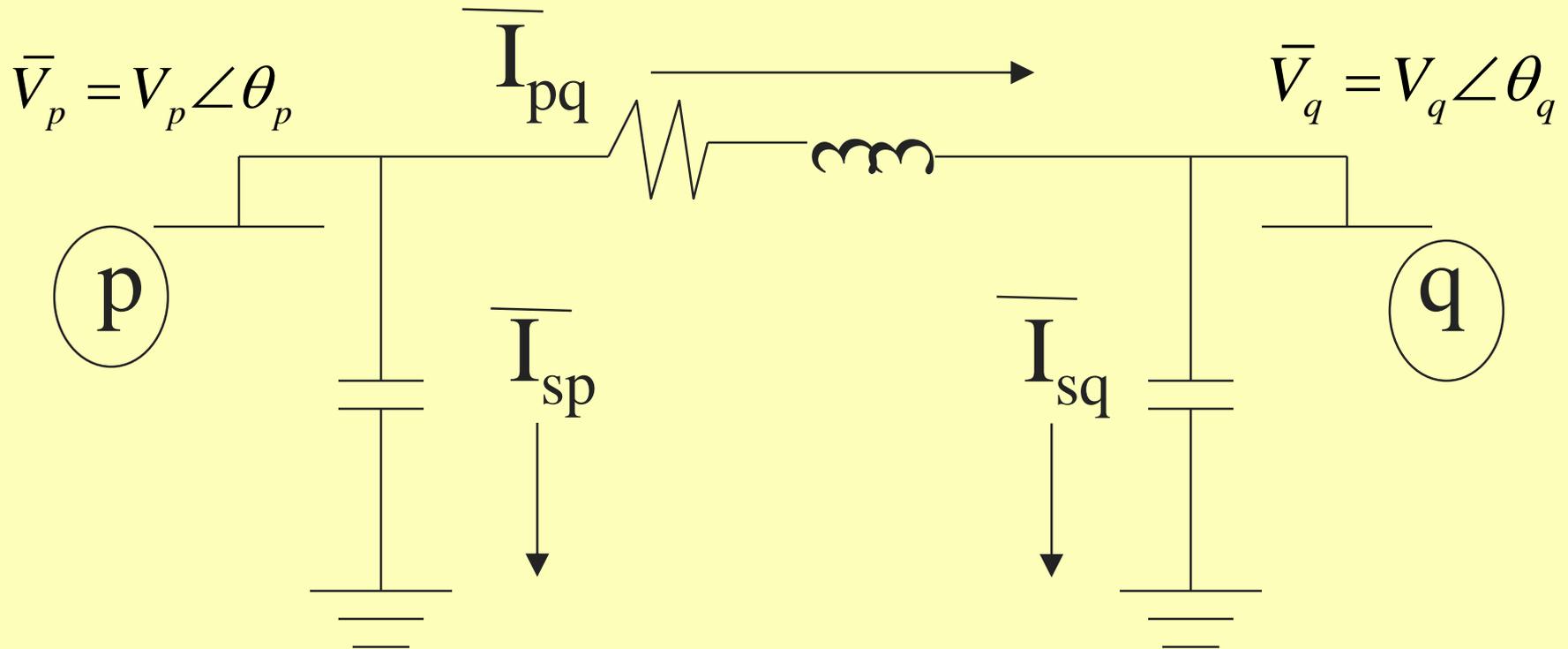
Observe that the total capacitive susceptance,  $jB_c$ , is halved, with  $jB_c/2$  at each end of the transmission line.

# The $\pi$ -equivalent model of a 3-phase transmission line

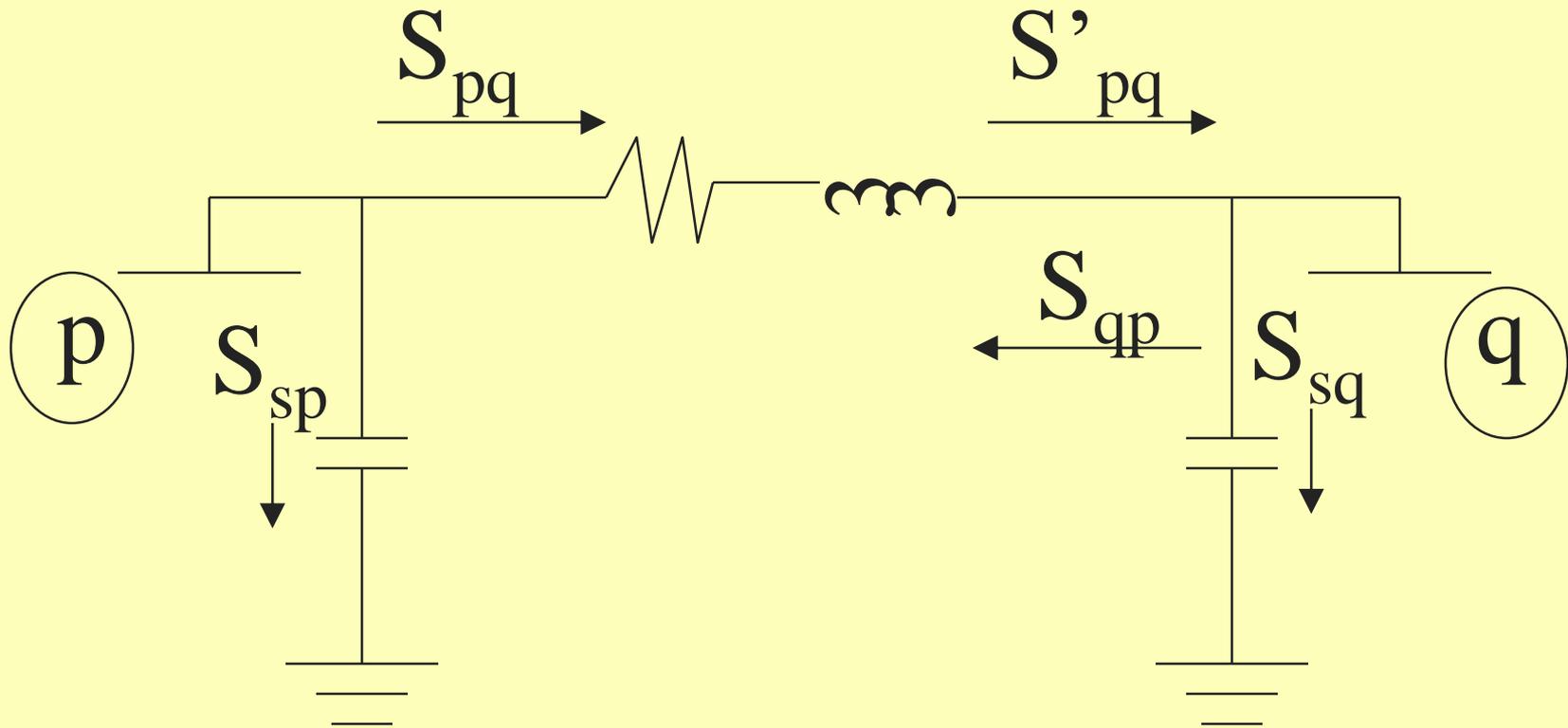


- The “ $\pi$ -equivalent” transmission line model is very standard in power system engineering.
- It is also well-known in other fields as one of several “two-port” networks.

# The $\pi$ -equivalent model of a 3-phase transmission line; nomenclature for voltages and currents



# The $\pi$ -equivalent model of a 3-phase transmission line; nomenclature for apparent power flows



# Power flow into capacitor

Assume all quantities are in per-unit. Then:

$$\bar{I}_{sp} = (\bar{V}_p \angle \theta_p)(Y_{pp}) = (V_p \angle \theta_p)(jB_c / 2)$$

$$S_{sp} = V_p \angle \theta_p [\bar{I}_{sp}]^*$$

$$= V_p \angle \theta_p [(V_p \angle \theta_p)(jB_c / 2)]^*$$

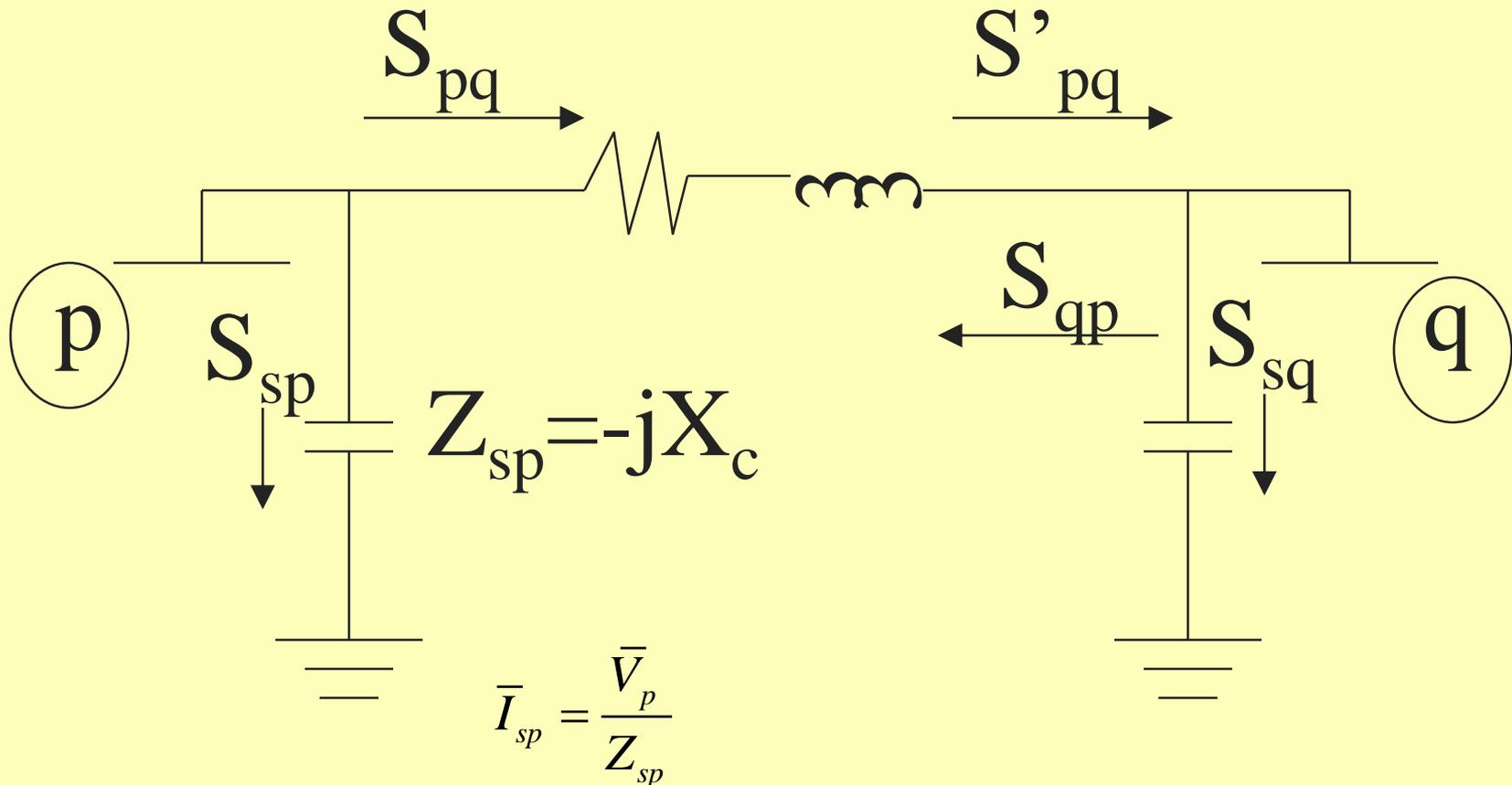
$$= V_p \angle \theta_p (V_p \angle -\theta_p)(-jB_c / 2)$$

$$= (V_p)^2 (-jB_c / 2) = -j(V_p)^2 (B_c / 2)$$

Comments:

- Power into capacitor is purely reactive;
- Neg sign implies  $S_{sp}$  flow direction reversed relative to direction indicated on previous slide; Q supplied to network.
- This result should not be mysterious. Let's look at it from another perspective. Define  $X_c = 1/(B_c/2)$ ...

# Power flow into capacitor



$$S_{sp} = \bar{V}_p \bar{I}_{sp}^* = \bar{V}_p \left( \frac{\bar{V}_p}{Z_{sp}} \right)^* = \bar{V}_p \left( \frac{\bar{V}_p^*}{Z_{sp}^*} \right) = \frac{V_p^2}{(-jX_c)^*} = \frac{V_p^2}{jX_c} = \frac{-jV_p^2}{X_c} = -jV_p^2 B_c / 2$$

# Power flow into series impedance

$$S_{pq} = V_p \angle \theta_p (\bar{I}_{pq})^*$$

$$\bar{I}_{pq} = [V_p \angle \theta_p - V_q \angle \theta_q] Y_{pq}$$

$$Y_{pq} = \frac{1}{R + jX} = G - jB$$

$$\Rightarrow \bar{I}_{pq} = [V_p \angle \theta_p - |V_q| \angle \theta_q] [G - jB]$$

$$(\bar{I}_{pq})^* = [V_p \angle \theta_p - |V_q| \angle \theta_q]^* [G - jB]^*$$

$$= [V_p \angle -\theta_p - V_q \angle -\theta_q] [G + jB]$$

Make this substitution

$$\begin{aligned}
 S_{pq} &= V_p \angle \theta_p [V_p \angle -\theta_p - V_q \angle -\theta_q][G + jB] \\
 &= [V_p^2 - \underbrace{V_p V_q \angle (\theta_p - \theta_q)}] [G + jB]
 \end{aligned}$$

By Euler...

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$$V_p V_q \cos(\theta_p - \theta_q) + jV_p V_q \sin(\theta_p - \theta_q)$$

$$S_{pq} = [V_p^2 - V_p V_q \cos(\theta_p - \theta_q) - jV_p V_q \sin(\theta_p - \theta_q)][G + jB]$$

- Perform complex multiplication
- Collect real and imaginary parts

$$S_{pq} = P_{pq} + jQ_{pq}$$

where.....

$$\Rightarrow P_{pq} = V_p^2 G - V_p V_q G \cos(\theta_p - \theta_q) + V_p V_q B \sin(\theta_p - \theta_q)$$

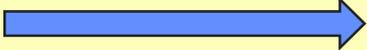
Eq. T1.17

$$\Rightarrow Q_{pq} = V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) - V_p V_q G \sin(\theta_p - \theta_q)$$

Eq. T1.18

These are the “exact”  
power flow equations.

## Some comments:

- $Q_{pq}$  does not include  $S_{sp}$  
- $S_{pq}$  is NOT  $-S_{qp}$  
- May be used in per phase analysis
  - voltages are phase to neutral
  - P and Q are per phase powers
- May be used in per unit analysis

This means reactive equation on previous slide is from bus p into the line but does not include reactive flow into bus p shunt capacitor..

Although bus p shunt capacitor is modeled at the bus, we must remember that it represents capacitance in the line.

This means complex power flowing from bus p into line is not the same as the negative of the complex power flowing from bus q into line..

## A Simplification: Let $R=0$

Normally, for transmission lines,  $X \gg R$ ,  
so that  $Z = R + jX \sim jX$ .

What does setting  $R=0$  do to  $Y=G-jB$ ?

(This is the second time we have mentioned this in these slides – see slide #5)

$$G - jB = \frac{1}{R + jX} = \frac{1}{R + jX} \frac{R - jX}{R - jX} = \frac{R - jX}{R^2 + X^2}$$

So with  $R=0$ ,

$$G - jB = \frac{0 - jX}{0^2 + X^2} = \frac{-jX}{X^2} = \frac{-j}{X} = -jB$$

So  $R=0$  implies  $G=0$ .

Let's see what the approximation  $R=0$  ( $G=0$ ) does to the power flow equations.

$$\begin{aligned} P_{pq} &= V_p^2 G - V_p V_q G \cos(\theta_p - \theta_q) + V_p V_q B \sin(\theta_p - \theta_q) \\ &= |V_p| |V_q| B \sin(\theta_p - \theta_q) \end{aligned}$$

$$\begin{aligned} Q_{pq} &= V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) - V_p V_q G \sin(\theta_p - \theta_q) \\ &= V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) \end{aligned}$$

Are these equations similar to anything you have seen before?

$$P_{pq} = V_p V_q B \sin(\theta_p - \theta_q) \quad \text{Eq. T1.20}$$

$$Q_{pq} = V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) \quad \text{Eq. T1.21}$$

Recall the equations for the synchronous gen

$$P_{out} = \frac{3V_t E_f \sin \delta}{X_s}$$

$$Q_{out} = \frac{3V_t E_f \cos \delta}{X_s} - \frac{3V_t^2}{X_s}$$

Make the following substitutions:

$$X_s = 1 / B$$

$$E_f = V_p$$

$$\delta = \theta_p - \theta_q$$

$$V_t = V_q$$

$$P_{pq} = |V_p| |V_q| B \sin(\theta_p - \theta_q)$$

$$Q_{pq} = |V_p| |V_q| B \cos(\theta_p - \theta_q) - |V_p|^2 B$$

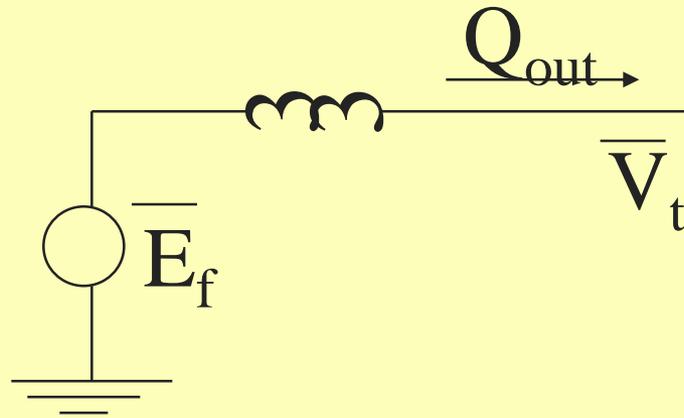
The real power equation is the same as eqt. T1.20.

But note that the reactive power equation is the negative of eqt. T1.21. Why?

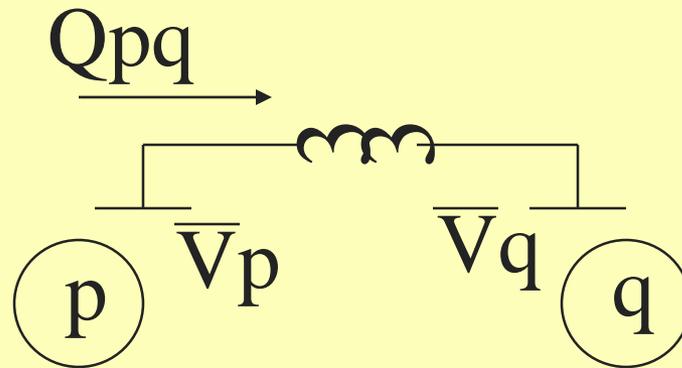
$$P_{pq} = V_p V_q B \sin(\theta_p - \theta_q) \quad \text{Eqt. T1.20}$$

$$Q_{pq} = V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) \quad \text{Eqt. T1.21}$$

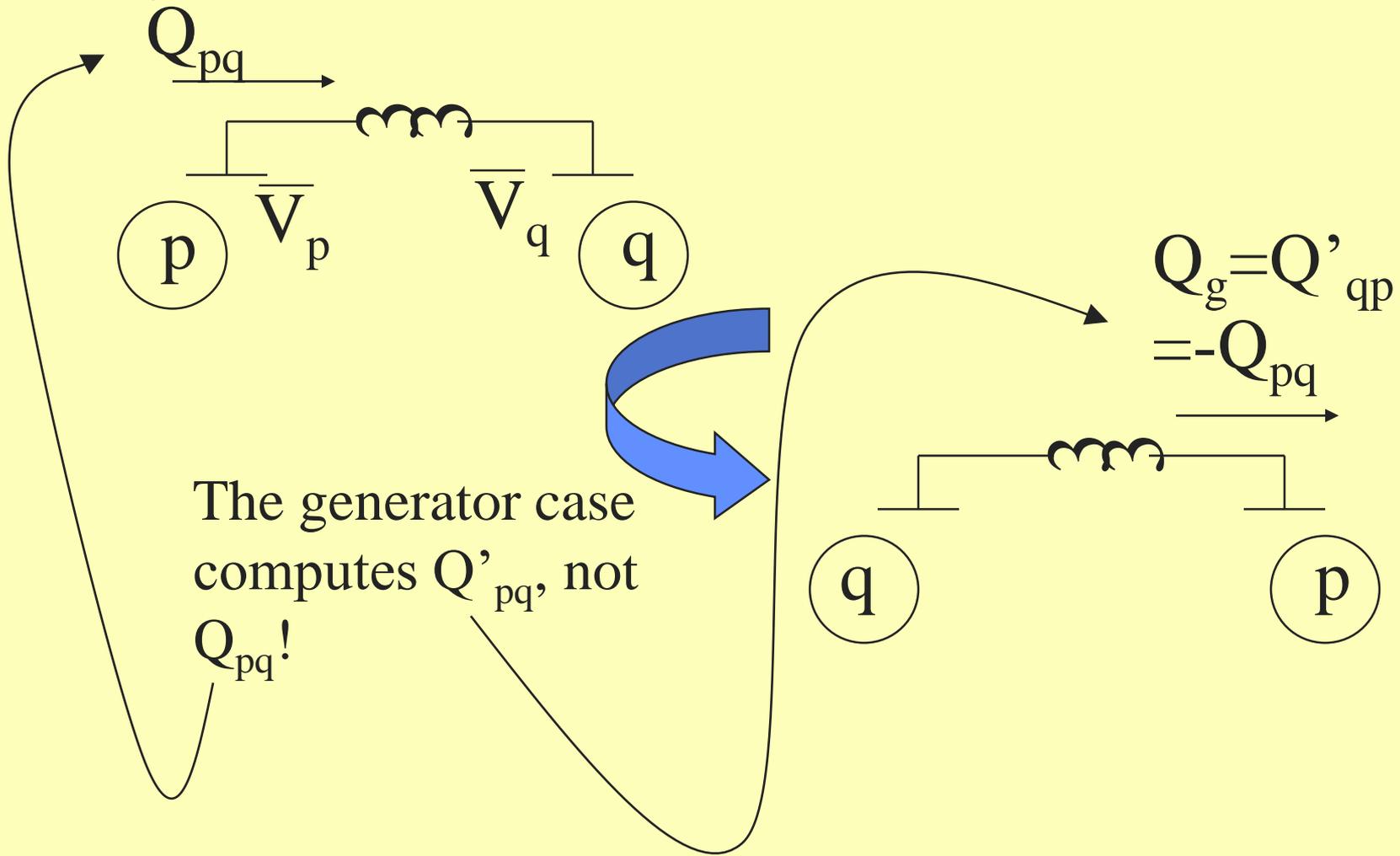
For the generator case, we compute:



For the transmission case, we compute:



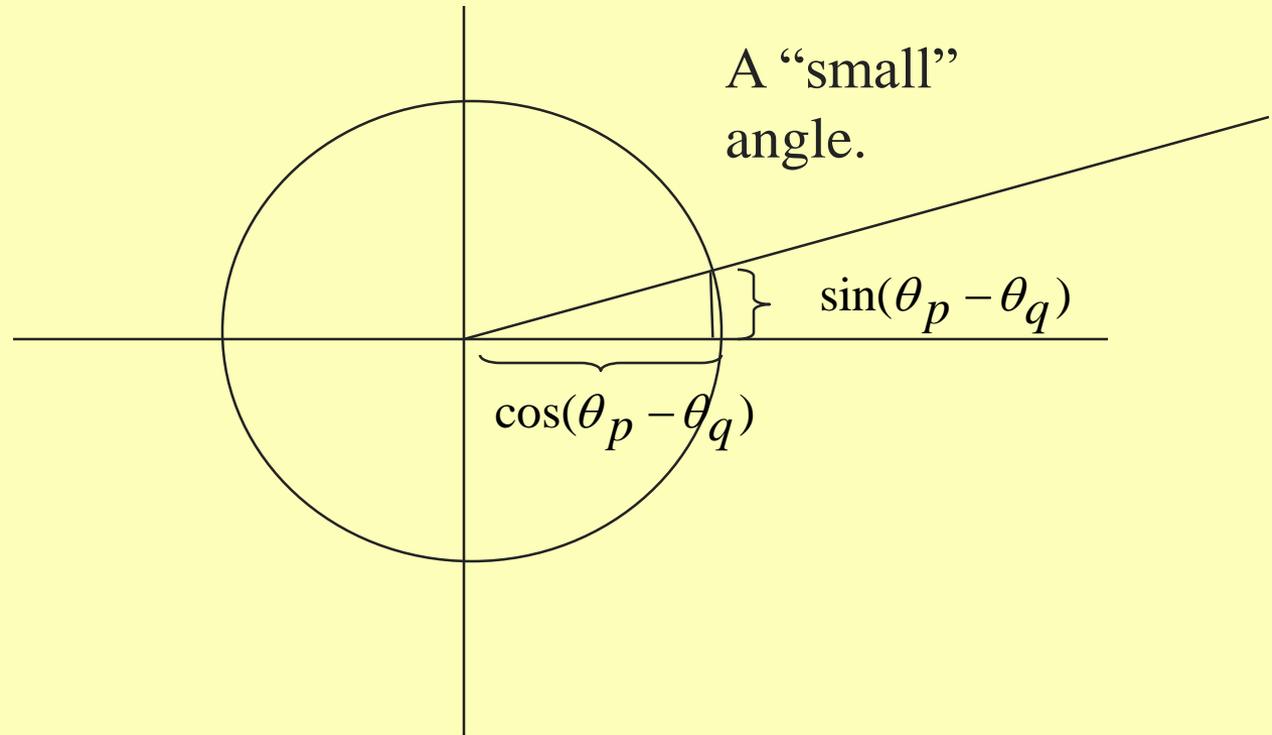
Grab the p-bus, flip it right-hand side, and locate  $Q_g$ . This is the  $Q$  computed for the syn gen.



A second simplification: small angle approximation

Assume that  $\theta_p - \theta_q$  is “small.”

This does tend to be the case for most transmission circuits.



$$\sin(\theta_p - \theta_q) \approx \theta_p - \theta_q \quad \text{Angles in radians!}$$

$$\cos(\theta_p - \theta_q) \approx 1.0$$

Then the two power flow equations become:

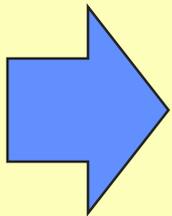
$$P_{pq} = V_p V_q B \sin(\theta_p - \theta_q) \quad \text{Eq. T1.20}$$

$$Q_{pq} = V_p^2 B - V_p V_q B \cos(\theta_p - \theta_q) \quad \text{Eq. T1.21}$$

Apply small angle approximations

$$\sin(\theta_p - \theta_q) \approx \theta_p - \theta_q$$

$$\cos(\theta_p - \theta_q) \approx 1.0$$



$$P_{pq} = V_p V_q B (\theta_p - \theta_q)$$

$$Q_{pq} = V_p^2 B - V_p V_q B = V_p B (V_p - V_q)$$

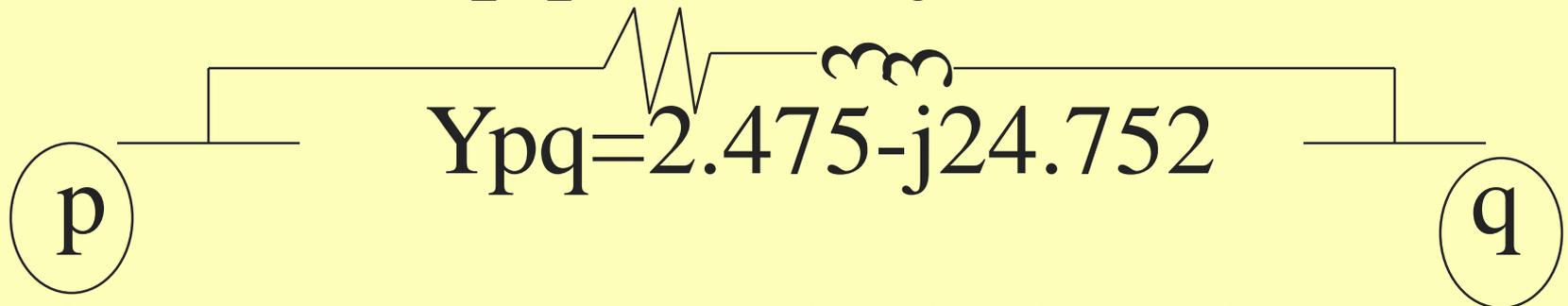
$$P_{pq} = V_p V_q B(\theta_p - \theta_q) \quad \text{Eq. T1.22}$$

$$Q_{pq} = V_p B(V_p - V_q) \quad \text{Eq. T1.23}$$

- $V_p$  and  $V_q$  are usually very close to 1.0
- $B$  is just a constant.
  - $\Rightarrow$  Variations in each of the two power quantities are mainly affected by the difference term on the right hand side.
    - $P_{pq}$  is affected by voltage angle differences
    - $Q_{pq}$  is affected by voltage magnitude differences

	$ V_p $	theta_p (deg)	$ V_q $	theta_q (deg)
Case 1	1.03	30	1.03	10
Case 2	1.06	30	1.03	10
Case 3	1.03	50	1.03	10

$$Z_{pq} = 0.004 + j0.04$$



Compute real  $P_{pq}$  and reactive  $Q_{pq}$  for each case.

Use all three sets of equations:

Eqts. T1.17, T1.18 & T1.20, T1.21 & T1.22, T1.23

Calculations for case 1 are in student text.  
Here are the results for all cases.

	Eqns. T1.17 and T1.18 “Exact” equations		Eqns. T1.20 and T1.21		Eqns. T1.22 and T1.23	
	P_pq	Q_pq	P_pq	Q_pq	P_pq	Q_pq
Case 1	9.139	0.685	8.98	1.58	9.17	0
Case 2	9.48	1.49	9.24	2.42	9.43	0.79
Case 3	17.49	4.46	16.87	6.14	18.33	0

Inspection of the first two columns indicate that

- Cases 1 and 2 have almost the same P\_pq but different Q\_pq. This illustrates the effect of changing voltage magnitude.
- Case 3 has a dramatic change in P\_pq due to the fact that the voltage angle was changed.

	Eqns. T1.17 and T1.18 “Exact” equations		Eqns. T1.20 and T1.21		Eqns. T1.22 and T1.23	
	P_pq	Q_pq	P_pq	Q_pq	P_pq	Q_pq
Case 1	9.139	0.685	8.98	1.58	9.17	0
Case 2	9.48	1.49	9.24	2.42	9.43	0.79
Case 3	17.49	4.46	16.87	6.14	18.33	0

Comparison between the three sets of columns for the three cases indicates that the approximate equations appear fairly accurate for real power flow but not so accurate for reactive power flow.