Module PE1

Introduction to Power Electronics

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Prerequisite Competencies: Basic circuit analysis, found in introductory circuit course

Module Objectives:

1. Gain a broad overview perspective of the power electronic field in terms of the different devices and their characteristics, and in terms of applications of these different devices.
2. Obtain a qualitative understanding of diode V-I switching characteristics and rectifier circuit operation using diodes.
3. Obtain a qualitative understanding of thyristor V-I switching characteristics, and device ratings and corresponding failure conditions.
4. Describe and analyze power semiconductor converters, including AC-DC, DC-AC, AC-AC, and DC-DC, with applications.
5. Obtain an appreciation for the importance of power quality issues in the design of power electronics circuits.

PE1.0 Introduction

Power electronic devices utilize semiconductors. Semiconductors, if operated within their design parameters, are reliable and relatively inexpensive. A semiconductor is a material with an electrical resistance between that of conductors and insulators. The semiconductor controls the flow of electrons by changing the effective resistance of layers of semiconductor material. Power semiconductor materials can handle high voltages and high currents. The field of power electronics began in 1957 when the thyristor was invented by the General Electric Company. The term *power electronics* was coined in the 1960s to distinguish it from classical electronics, which deals with devices and circuits that carry power in the milliwatt to a few watts range at microvolts to low-voltage level. Table PE1.1 illustrates the growth of the industry since that time. Power electronics is a relatively modern subject that emerged in the last two decades due to the continuous evolution of high-power semiconductor devices. There are many applications of power electronic circuits, some of which are summarized in Table PE1.2.

The power portion of any power electronic circuit addresses a specific application as indicated in Table PE1.2. Clearly, the power electronics field requires some background in electric power and energy systems. The solid-state physics aspects of power electronics require some knowledge in semiconductor materials and devices. Perhaps central to the power electronics field is the ability to design the circuitry necessary to perform device switching in a controlled and logical manner to precisely achieve the desired function. The switching control requires both analog and digital circuits, often together with signal processing software logic embedded within a microprocessor, designed based on principles of control theory. Therefore, expertise in the field of power electronics eventually spans the fields of electric power and energy (especially electric machines and electromagnetics), semiconductor materials and devices, digital and analog electronics, microcontrollers, software, signal processing, and control theory. Figure PE1.1 depicts power electronics as an inter- disciplinary subject covering light current, heavy current and systems areas of electrical engineering.

Table PE1.1: Growth of the Field of Power Electronics

1970 – 1980	Second Generation devices PMOSFET, Power Transistor, GTO, IGBT, MCTh, MCT The impact of micro-electronics on PE
1980 – 1990	Third Generation devices High Frequency Power Electronics. Quasi/Multi-resonant converters. (250kHz to 1MHz range).
1990 – 2000	Smart power electronic systems Fuzzy, NN type controls applied to PE Static induction type devices (SiTr, SiTh) Rapid proto-typing Effective simulation tools such as: SABER, DYMOLA-ACSL, MATLAB-SIMULINK for understanding complex systems.

Table PE1.2: Power Electronic Circuit Applications Based On Their Functions

STATIC SWITCHING	THYRISTOR / GTO / IGBT CHOPPER CIRCUITS
Solid state relays	Electric transportation
Logic systems	Generator exciters
Crow-bar protection	High power regulated power supplies
A.C. PHASE CONTROL	D.C. TO A.C. INVERTER
Light dimmers	Aircraft & space power supplies
Voltage controls	UPS systems
Motor speed control	VAR & Harmonic compensators
VAR regulators	Variable – frequency A.C. motor drives
PHASE CONTROLLED RECTIFIERS/INVERTERS	
D.C. Motor drives	
HVDC Transmission System	
Flexible AC Transmission System (FACTS) devices	
Regulated D.C. power supplies	
Wind generator Converters	

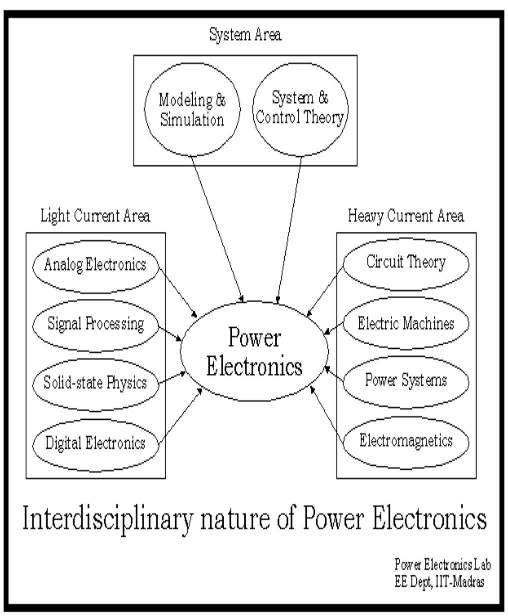
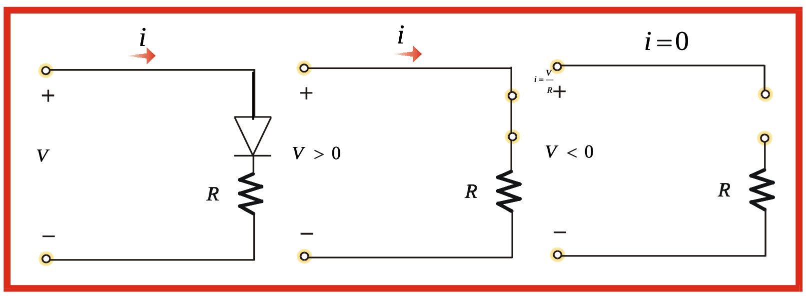


Figure PE1.1: Multi-Disciplinary Aspects of Power Electronics

The purpose of this module is to familiarize the student with the power electronics field and provide material for basic understanding of the subject. We will only briefly mention the solid-state construction of the devices. After a brief introduction to the various power semiconductor devices and their characteristics, commonly used power electronic conversion schemes will be introduced and analyzed. In so doing, a variety of important applications in the energy and power field will be mentioned. Some of these are: ac and dc motor control systems, static frequency conversion, high-voltage dc (HVDC) transmission systems, and high power supply systems. The electronic control portion of the circuit will not be covered here. Such material is usually covered in advanced power electronics courses taught at the senior or graduate level.

PE1.1 The Ideal Diode

A diode is a two-layer p-n semiconductor device. An ideal diode acts like an electrical one-way-valve. The symbol of a diode is shown in Figure PE1.2a. If a positive voltage is applied across the diode, the diode is on and a forward current flows through it. If a negative voltage is applied, the diode is off and no current flows through it. The voltage-current characteristic of an ideal diode is shown in Figure PE1.2b.



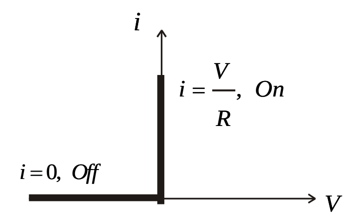
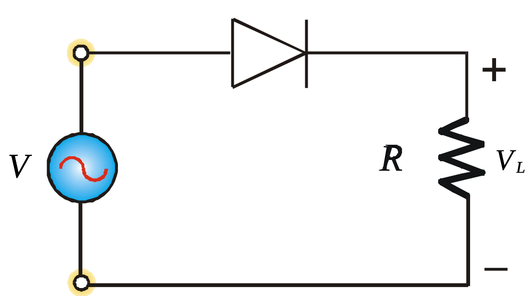


Figure PE1.2a Symbol of Diode **Figure PE1.2b *V-i* Characteristic**

A simple diode half-wave rectifier consists of an alternating voltage source in series with a diode and a resistor as shown in Figure PE1.3a. During the positive half cycle ($0 \le \omega t \le \pi$), the diode conducts and the current flows through the load resistance R. During the negative half-cycle ($\pi \le \omega t \le 2\pi$), the diode is off and acts like an open switch. Figure PE1.3b shows the resulting waveforms for a sinusoidal input voltage source of $v = V_m \sin(\omega t)$.



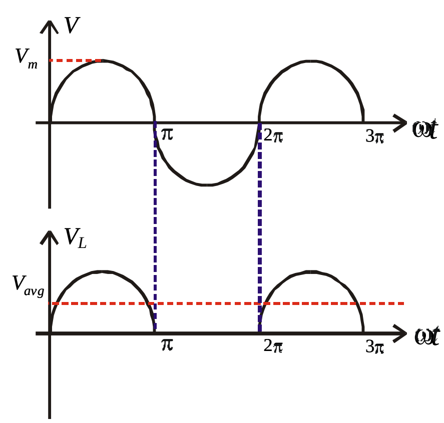


Figure PE1.3a: Diode Rectifier Circuit **Figure PE1.3b: Input & Load Voltage Waveforms**

The average value of a periodic waveform x(t), of period T, is

$$X_{avg} = \frac{1}{T}\int_0^T x(t)dt \qquad \text{or} \qquad \frac{1}{A}\int_0^A x(\alpha)d\alpha \qquad where\ \alpha = \omega t \quad and \quad A = \omega T$$

The average value of the voltage across the load is given by:

$$V_{avg} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin(\omega t)d(\omega t) = \frac{V_m}{\pi}$$

where $A = 2\pi$ but we need only integrate to π because the function is zero between π and 2π .

PE1.2 Controlled electronic switches

The diode is a 2-terminal device and as such, its switching characteristics are fixed, i.e., they cannot be controlled. A number of three-terminal devices are available which provide the ability to achieve such switching control. These include (1) bipolar junction transistors (BJT), (2) metal-oxide semiconducting field-effect transistors (MOSFET), (3) thyristors or silicon-controlled rectifiers (SCR), (4) triacs, and (5) gate turn-off (GTO) thyristors, (6) insulated gate bipolar transistors (IGBT), (7) MOS-controlled transistors (MCT), and (8) MOS-gated transistors (MGT). The MOSFET is a single-carrier device, while the diode, thyristor, GTO, triac, and BJT are two-carrier devices. The IGBT, MGT, and MCT are mixed-carrier devices. Key distinguishing characteristics between these controlled switches are the speed at which they can switch, measured in Hz, and their power handling capabilities, which are a function of their voltage and current ratings. Figure PE1.4 provides an effective way to compare some of these different devices in these terms, and Table PE1.3 provides a convenient summary. Figure PE1.5 compares the circuit symbol and V-I switching characteristic for the diode with a few of these devices. We provide a short introduction to the operation of some of these devices in what follows.

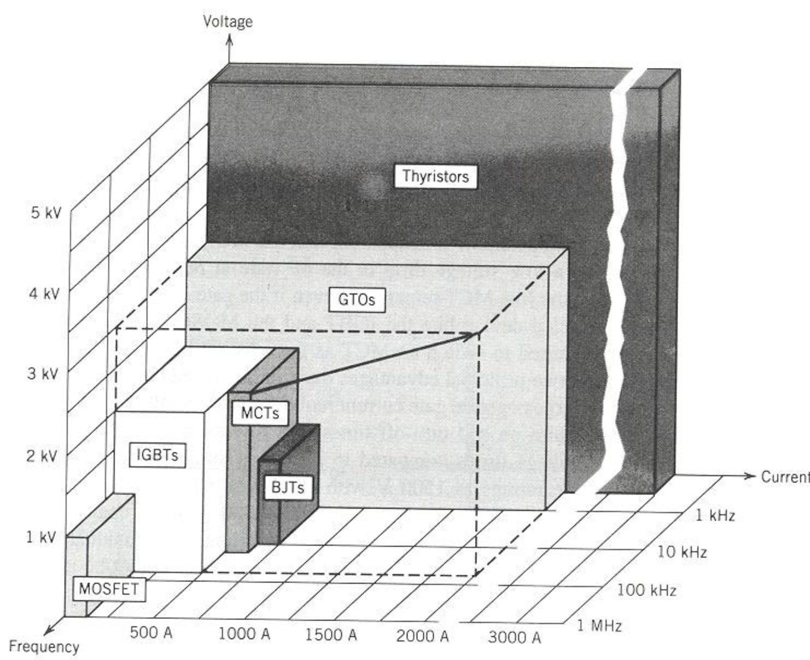


Figure PE1.4: Power Handling and Switching Speeds for Various Controllable Switching Devices

Table PE1.3:
Summary of Power Handling & Switching Speeds for Various Controllable Switching Devices

Device	Power Capability	Switching Speed
BJT/MD	Medium	Medium
MOSFET	Low	Fast
GTO	High	Slow
IGBT	Medium	Medium
MCT	Medium	Medium

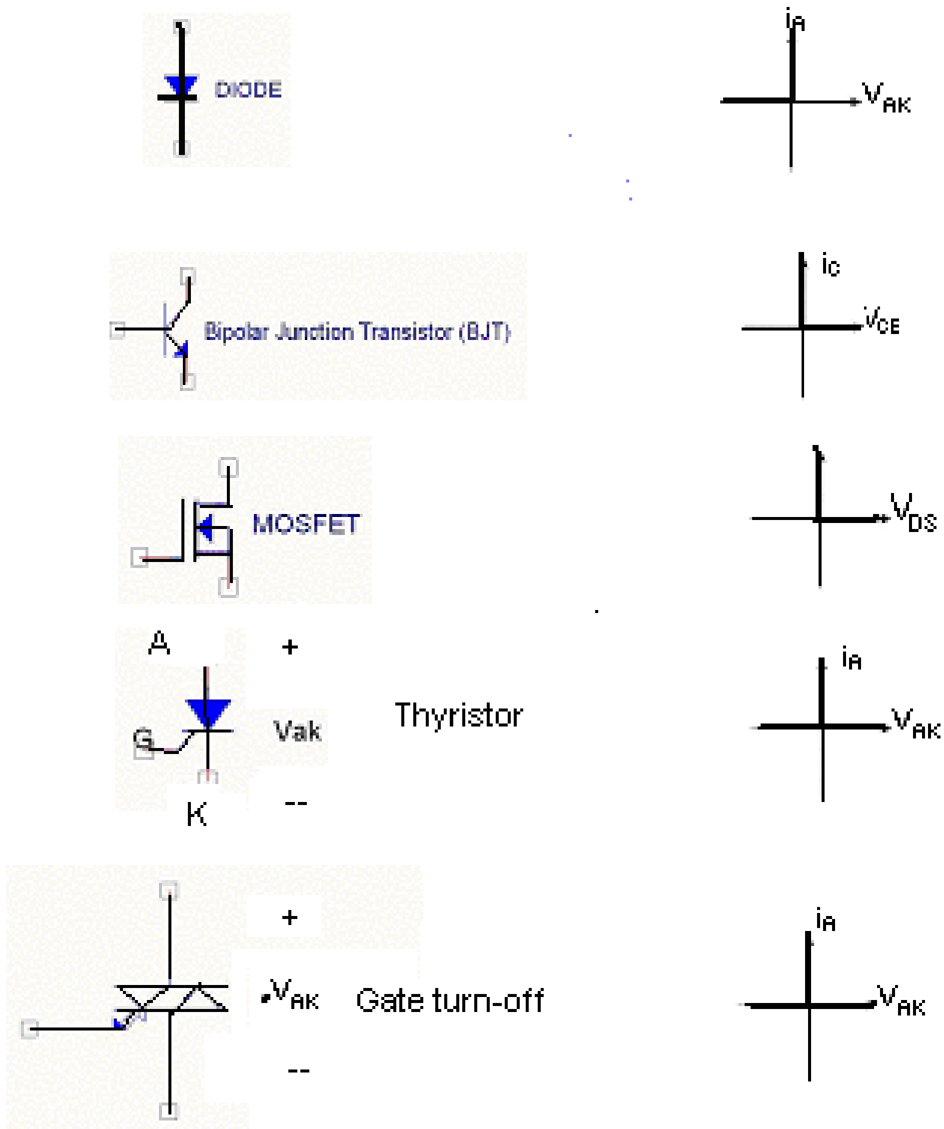


Fig. PE1.5: Power Semiconductor devices and their V-I switching characteristics

PE1.2.1 The thyristor

Solid state devices are created using large pure crystals of silicon, which do not conduct electricity very well. They have to be doped with small amounts of other elements to make it conductive. These elements can either increase or decrease electrons in the crystal lattice. An n-type semiconductor is doped with phosphorus or another element to donate free electrons to the silicon crystal. A p-type semiconductor is doped with boron or some other element to take away electrons from silicon atoms in the crystal, creating positively charged holes that move through the lattice.

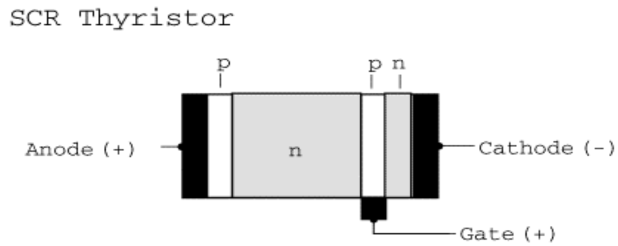


Figure PE1.6: SCR Solid State Construction

When the electrons and holes are brought together from opposite sides, the junction between layers of n-type and p-type materials will conduct electricity. A thyristor is constructed from four layers of doped silicon in a p-n-p-n configuration. By imposing a positive voltage on the embedded p-layer (the gate), an SCR can be turned on at any point in the positive cycle of alternating current. Once the SCR is conducting, it cannot be turned off until the current goes below the holding current, e.g, at the end of each half cycle, at the current zero crossing. Figure PE1.7 shows various forms of the thyristor representation. A more precise version (relative to that of Figure PE1.5) of the static or steady-state *V-I* characteristic curves are given in Figure PE1.8.

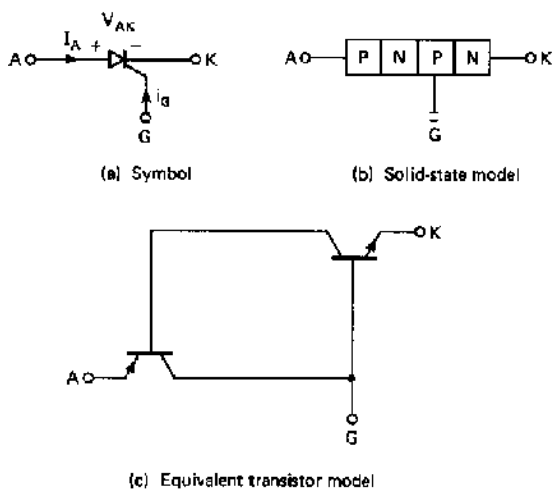


Figure PE1.7: SCR representations

The SCR operates in the following manner. When a positive anode-to-cathode voltage, V_{AK} is applied to the SCR, it will start conducting provided a gate signal pulse, i_G, of sufficient amplitude and duration is also simultaneously applied. The positive-biased voltage at which it conducts depends on the strength of the gate current, as implied by the family of curves shown in Figure PE1.8. The higher the gate current, the less the anode-cathode voltage required to turn-on the SCR. Once the SCR is turned on, the gate signal need not be applied any longer to keep the device conducting. At this point, the gate signal can, therefore, be terminated, and the SCR remains in the conducting state when V_{AK} is very small, as long as sufficient I_A is flowing. The steady-state or equilibrium point at which the SCR conducts, such as *P*, depends upon the resistance (or impedance) connected to it. While it is conducting, the forward anode-to-cathode drop is very small, usually of the order of a few volts, depending on the SCR turn-on speed.

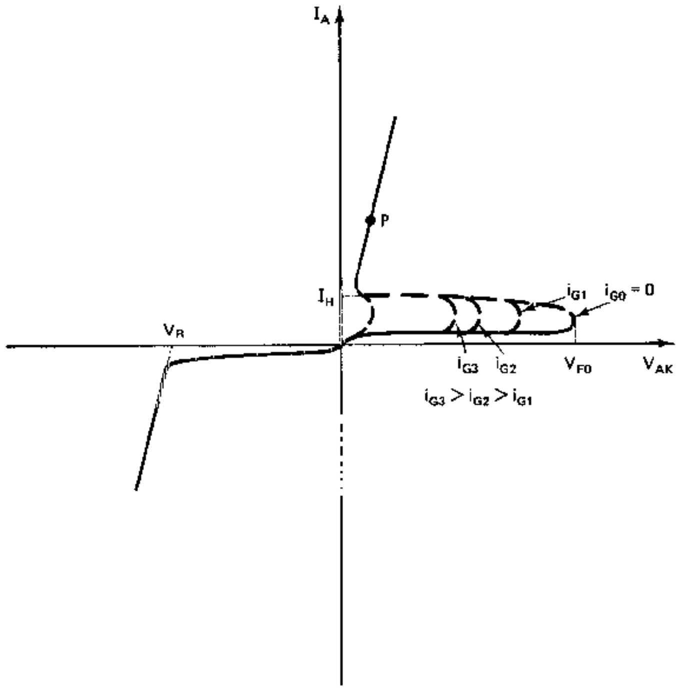


Figure PE1.8: Volt-amp characteristics of SCR

Suppose the thyristor is conducting; it will turn off if the anode-to-cathode voltage is reduced to zero or negative, which is equivalent to applying a reverse-biased voltage. This phenomenon is called *natural commutation,* which is possible only when alternating voltage sources such as sinusoidal voltages are applied to the thyristor. The intent here is to reduce the thyristor current to below the holding current, I_H, to assure commutation. One must make sure not to apply a reverse-biased voltage more than V_R to avoid the avalanche breakdown of the thyristor, a situation which also happens with a diode. If the forward-biased voltage exceeds V_{FO}, then the thyristor continues to conduct even without the presence of a gate signal.

A second method by which a thyristor can be turned off is called *forced commutation,* which implies the need for an external circuit to force the current through it below I_H, or to reduce V_{AK} below zero. A simple form of such a scheme is given in Figure PE1.9. This series—capacitor commutation scheme works in the following manner: When the main SCR is turned-on, it starts charging the capacitor, *C*. When the capacitor voltage reaches the dc supply value, the conditions are ideal to turn off the main SCR; namely, the current through it is below its holding value and the anode-to-cathode potential is about to be reverse-biased. If now a suitable pulse is applied to the commutation SCR, it will turn on, while in this process the main SCR is turned off. The capacitor discharges its voltage through the commutation SCR and the resistor connected in series with it. When the capacitor is completely discharged, conditions are ripe to turn on the main SCR again. In other words, the forced commutation of the main SCR is achieved by employing a second SCR.

Like any other device, thyristors need protection against the following situations:

1. overload
2. overcurrent
3. overvoltage
4. excess *dv/dt*
5. excess *di/dt*

Circuit breakers with thermal protection or expulsion fuses can protect a thyristor against overload conditions. Because of the fast speeds at which this device operates, usually a fast fuse of compatible rating is used to protect it against overcurrents. The SCR may be protected against overvoltages, due to switching or lightning, by connecting a surge arrester such as a metal oxide varistor across it.

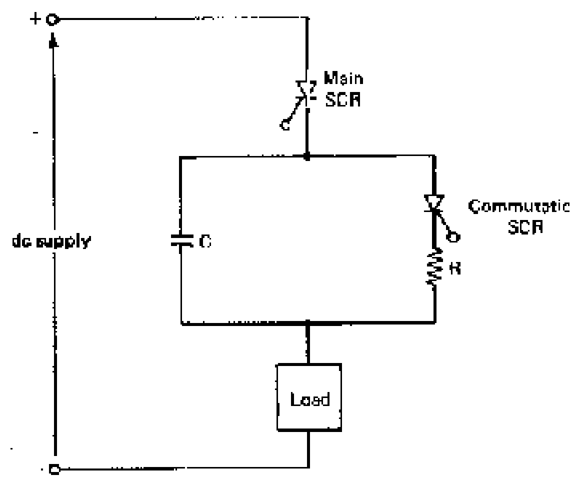


Figure PE1.9: A simple forced-commutation circuit

Because of the SCR's steep dynamic turn-on and turn-off characteristics, it needs both *dv/dt* (dV_{AK}/dt) and *di/dt* (dI_A/dt) protection. While the excessive *dv/dt* is not a serious factor, it can result in premature turn-on of the thyristor even without a gate signal applied to it. The protection against excessive *dv/dt* is typically provided by an RC snubber circuit shown in Figure PE1.10.

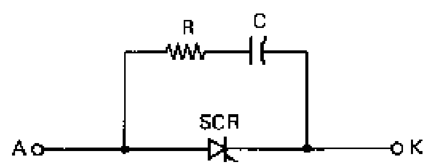


Figure PE1.10: RC snubber circuit

The most popular method to protect an SCR against high *di/dt,* which could create hot spots and burn the junction, is to provide an inductor connected in series with the SCR. The reactor is designed to go into magnetic saturation for currents greater than its rated value so that only a minimal voltage develops across the reactor.

PE1.2.2 The triac

A triac can be conceived as two SCRs connected back-to-back. However, it is manufactured as one integrated device with only one gate terminal. Its schematic symbol is shown in Figure 9-5. The two unmarked terminals play the role of anode and cathode, depending upon the relative polarities of the applied source. A proper (positive or negative) pulse of signal should be applied to the gate at the appropriate time to fire the triac. Figure 9-6 shows the *V-I* characteristic family of the device. As in the case of the SCR (Figure 9-2), the larger the gate signal, the less the forward breakdown voltage required to turn on the triac. In the same vein, it will turn off once the current falls below the holding current, I_H.

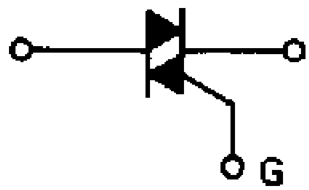


Figure PE1.11: Triac representation

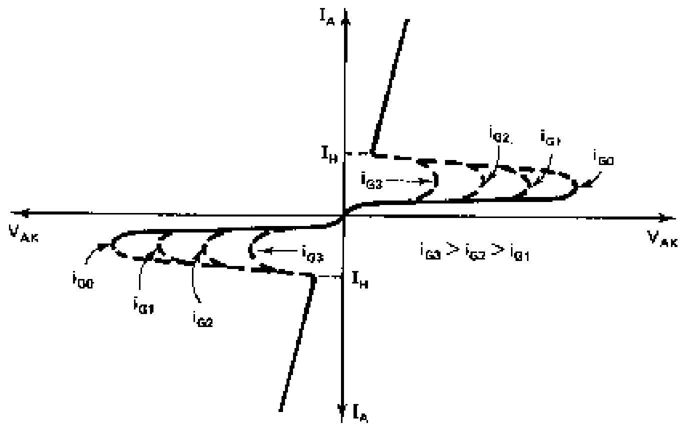


Figure PE1.12: Volt-amp characteristic of triac

In the absence of the gate terminal, the triac becomes a diac, an uncontrolled two-way rectifier, which is equivalent to two diodes connected back-to-back. The *V-I* characteristic of the diac is the envelope characteristic, with $i_{GO} = 0$ in Figure PE1.12.

PE1.2.3 GTO thyristor

A Gate Turn-Off (GTO) SCR is a device that can be turned off under load by applying a relatively large negative current pulse to the gate even if the anode current is more than I_H. Therefore, these devices do not need an external commutation circuit if forced commutation is desired. GTO thyristors are usually represented schematically as in Figure PE1.13.

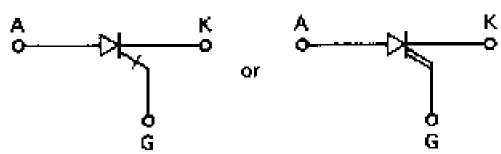


Figure PE1.13: GTO SCR representations

PE1.2.4 Power MOSFET

… to be added later

PE1.2.5 Insulated Gate Bipolar Transistor

… to be added later

PE1.3 AC to DC Converters

Converters with ac input and dc output are classically known as controlled rectifiers, when thyristors or any other controlled devices perform the power switching function to realize variable output dc voltage from constant input ac voltage source. The control is obtained by changing the firing angle α of the thyristors, which as stressed before, act as switches in the conversion scheme to be discussed here. The input could be either a single-phase or a three-phase source. These converters can be configured for half-wave or full-wave rectification, the latter providing better performance, as we shall observe later. Since the input is ac, natural commutation can be achieved. However, forced commutation can also be employed if more versatile control of the output is desired. These converters are very often utilized for dc motor control systems employed in process industries such as steel rolling mills. Other applications include portable tools, standard power supplies, and HVDC transmission systems. In this section, we will explore the basic principles underlying the operation of this type of converter and analyze simple schemes to gain a rudimentary understanding of this important conversion scheme.

PE1.3.1 Single-phase schemes

A simple form of a half-wave rectifier feeding a resistive load, such as an electric heater or incandescent lamp, is shown in Figure PE1.14 (a). If the input source $v_i(\omega t) = V_m \sin(\omega t)$ as drawn in Figure PE1.14 (b), the corresponding output voltage and the resulting current wave are indicated in Figure PE1.14 (c) for a particular triggering angle α. The anode-cathode voltage across the thyristor is depicted in Figure PE1.14 (d). The triggering current pulse of suitable magnitude and duration required to turn on the SCR at $\omega t = \alpha$ is shown in Figure PE1.14 (e). If $\alpha = 0$, we realize an uncontrolled half-wave rectifier employing a diode.

For this scheme, the average value of the output voltage V_{dc} is:

$$\begin{aligned} V_{dc} &\equiv \frac{1}{2\pi}\int_\alpha^\pi V_m \sin(\omega t) d(\omega t) \\ &= \frac{V_m}{2\pi}\left[-\cos\omega t\right]_\alpha^\pi \qquad \text{(PE1.1)} \\ &= \frac{V_m}{2\pi}\left[1+\cos\alpha\right] \end{aligned}$$

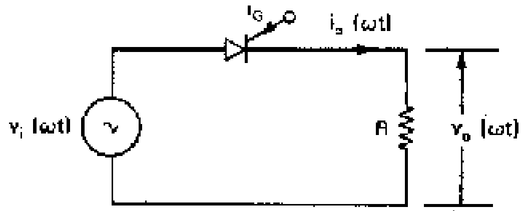
and the corresponding current I_{dc} is given by:

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{2\pi R}\left[1+\cos\alpha\right] \qquad \text{(PE1.2)}$$

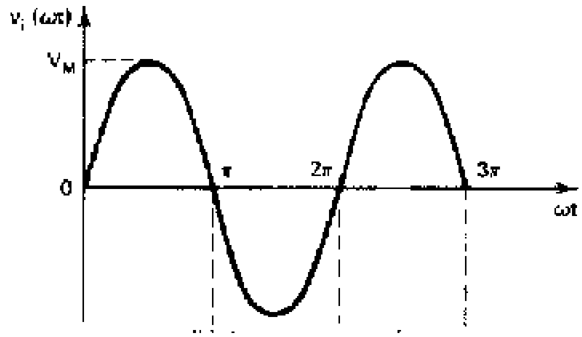
If $\alpha = 0$, then $V_{dc} = V_m/\pi$ and this value goes to zero if $\alpha = \pi$. Therefore, by varying α from 0 to π, one can completely vary the dc output voltage from zero to the maximum value of (V_m/π). The corresponding power dissipated in R will be

$$P_{dc} = \frac{V_{dc}^2}{\mathrm{R}} = I_{dc}^2 R = \frac{V_m^2}{4\pi^2 R}\left[1+\cos\alpha\right]^2 \qquad \text{(PE1.3)}$$

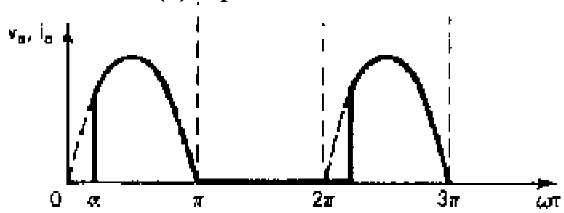
One important observation we can make at this point, with respect to controlled power electronics circuits, is the nonsinusoidal nature of resulting waveforms shown in Figure 9-8(c). The output voltage and current in this circuit contains various harmonics that can be determined by performing a Fourier series analysis (see module PQ1).



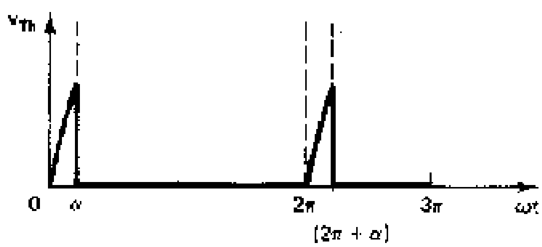
(a) Circuit diagram



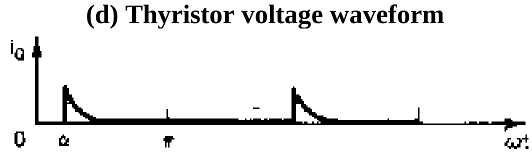
(b) Input source waveform



(c) Output voltage and current waveforms



(d) Thyristor voltage waveform



(e) Gate pulse waveform

Figure PE1.14: Single-phase half-wave controlled rectifier scheme

The rms value of the output voltage, is given by:

$$V_{rms} \equiv \left[\frac{1}{2\pi}\int_{\alpha}^{\pi}\left(V_m \sin(\omega t)\right)^2 d(\omega t)\right]^{1/2}$$
$$= \frac{V_m}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2} \quad \text{(PE1.4)}$$

Consequently,

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{2\sqrt{\pi}R}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2} \quad \text{(PE1.5)}$$

We now define a measure called *Ripple Factor (r)* for determining the smoothness of the output voltage.

$$r(\%) \equiv \frac{\text{rms value of all ac components of } v_0}{V_{dc}} \times 100 \quad \text{(PE1.6)}$$

This can be shown to reduce to:

$$r = \left[\frac{V_{rms}^2}{V_{dc}^2} - 1\right]^{1/2} \times 100 \quad \text{(PE1.7)}$$

One can quickly observe that the output voltage, v_0 resulting from the scheme is far from ideal. It has in fact, no resemblance to the dc output waveform desired. However, the scheme still provides a component of dc output. In the following example, we will gain a better understanding of this issue.

Example PE1.1

We want to vary the intensity of light of an incandescent lamp connected to a 120-V, 60-HZ ac source by means of the scheme discussed above. We want to control the power dissipated in the lamp from 0 to 60 Watts. Investigate P_{dc} as a function of α.. Also draw the variation of V_{dc}, V_{rms} and r as a function of the triggering angle.

Solution From equation (PE1.3), with $\alpha = 0$

$$60 = \frac{\left\{\sqrt{2}(120)\right\}^2}{4\pi^2 R}(1+1)^2$$
$$= \frac{2(120)^2}{\pi^2 R}$$

or,

$$R = \frac{2(120)^2}{60\pi^2} = 48.63\,\Omega$$

Now for any other value of α

$$P_{dc} = 15[1+\cos\alpha]^2 \quad \text{(PE1.8)}$$

From equation (PE1.1)

$$V_{dc} = \frac{\sqrt{2}(120)}{2\pi}[1+\cos\alpha]$$
$$= 27.01[1+\cos\alpha]$$

$$V_{rms} = \frac{\sqrt{2}(120)}{2\sqrt{\pi}}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2} \quad \text{(PE1.9)}$$
$$= 47.87\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2} \quad \text{(PE1.10)}$$

The plots for P_{dc}, V_{dc}, V_{rms} and r are given in Figure PE1.15.

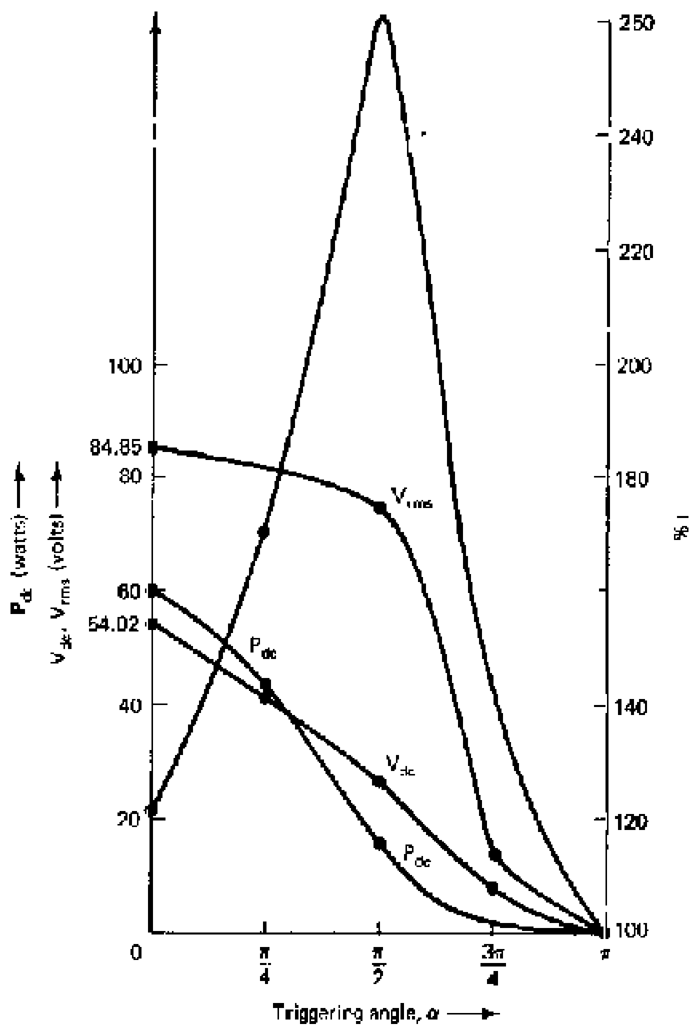


Figure PE1.15: Plots for single-phase controller example

Because of excessive harmonic content in the output waveforms, this is not the best scheme, though it is the simplest. Most of the control is gained in the first half domain of α from 0 to 90°, and the scheme becomes very insensitive in the latter half of the α-domain.

The half-wave or one-pulse scheme discussed above is rendered impractical if the input source has a dc component. A more versatile scheme, which can reduce the ripple is the full-wave scheme employing a bridge circuit shown in Figure PE1.16. It can also be effectively used with nonideal input sources having dc offset.

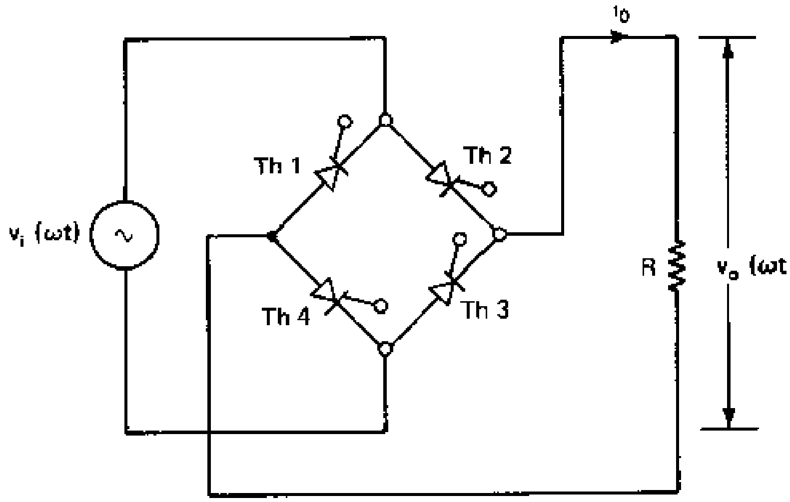


Figure PE1.16: Schematic of a single-phase, full-wave (bridge) controlled rectifier

The full-wave scheme is also known as two-pulse rectifier in the sense that during the negative-half cycle of the source voltage, thyristors l and 3 can be turned on by applying proper pulse signals simultaneously to their gates at any desired triggering angle α. Similarly, a second pulse can be applied 180° later to the gates of the other two thyristors 2 and 4 during the positive-half-cycle of the input voltage. The "two-pulse" terminology results from the fact that two gate pulses are required per cycle. The input and output waveforms are drawn in parts (a) and (b) of Figure PE1.17. At any given time, since two nonconducting SCRs operate in parallel, the entire v_i is applied across them as shown in Figure PE1.17 (c) and (d).

We will now obtain the expressions for the average and the rms values of v_o as in the half-wave case:

$$V_{\text{dc}} = \frac{1}{\pi}\int_{\alpha}^{\pi} v_0(\omega t)d(\omega t)$$

$$= \frac{1}{\pi}\int_{\alpha}^{\pi} V_m \sin(\omega t)d(\omega t)$$

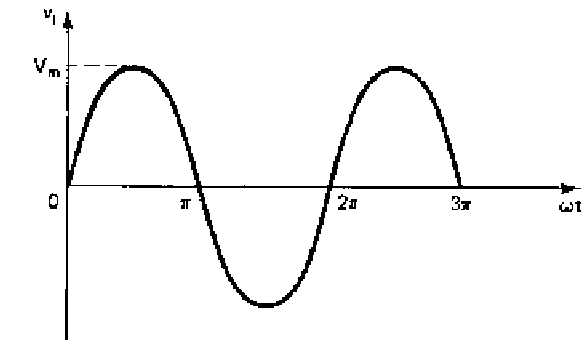
$$V_{\text{dc}} = \frac{V_m}{\pi}[1+\cos\alpha] \tag{PE1.11}$$

and

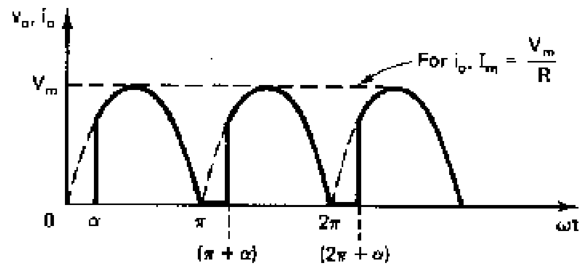
$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_{\alpha}^{\pi} V_m^2 \sin^2(\omega t)d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2} \tag{PE1.12}$$

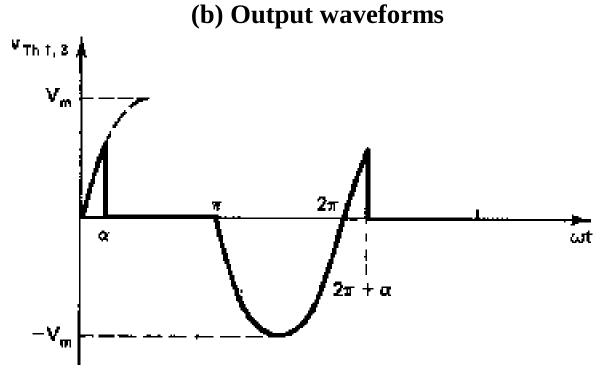
Comparing equations (PE1.1) and (PE1.11) quickly reveals that for a given α, V_{dc} in the full-wave case is twice that in the half-wave case, as might be expected. Also, V_{rms} for the full-wave rectifier is $\sqrt{2}$ times the value for the half-wave case.



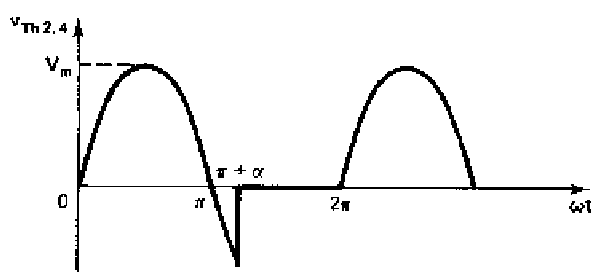
(a) Input source waveform



(b) Output waveforms



(c) Voltage across SCRs 1 and 3



(d) Voltage across SCRs 2 and 4

Figure PE1.17: Waveforms for single-phase, full-wave controlled rectifier scheme supplying resistive load

Example PE1.2

In Figure PE1.16, R = 10 ohms, and $v_i = \sqrt{2}(240)\sin(\omega t)$ volts. For α = 30°, find the following:

a. The average output current, I_{dc}
b. The power dissipated in the load, P_c
c. The rms output current, I_{rms}
d. The average and rms thyristor currents
e. The power factor of the source

Solution For the conditions stipulated, the load is purely passive and resistive.
a. Using equation (PE1.11)

$$I_{dc} = \frac{V_{dc}}{R} = \frac{V_m}{\pi R}[1+\cos\alpha]$$
$$= \frac{\sqrt{2}(240)}{\pi(10)}[1+\cos 30°]$$
$$= 20.16 \text{ A}$$

b. $P_{dc} = I_{dc}^2 R = (20.16)^2(10) = 4{,}064.21 \text{ watts}$

From these numerical values, the reader can realize that the situation described here is akin to controlling an electric range or a dryer at home from a 240-volt, single-phase supply.

c. From equation (PE1.12)

$$I_{rms} = \frac{V_{rms}}{R} = \frac{V_m}{\sqrt{2\pi}R}\left[(\pi-\alpha)+\frac{\sin 2\alpha}{2}\right]^{1/2}$$
$$= \frac{\sqrt{2}(240)}{\sqrt{2\pi}(10)}\left[\left(\pi-\frac{\pi}{6}\right)+\left(\frac{\sin 60°}{2}\right)\right]^{1/2}$$
$$= 23.65\text{A}$$

d. Average thyristor current = $\dfrac{I_{dc}}{2}$ = 10.08 A

rms thyristor current = $\dfrac{I_{rms}}{\sqrt{2}}$ = 16.72 A

e. Power factor of the source ≡

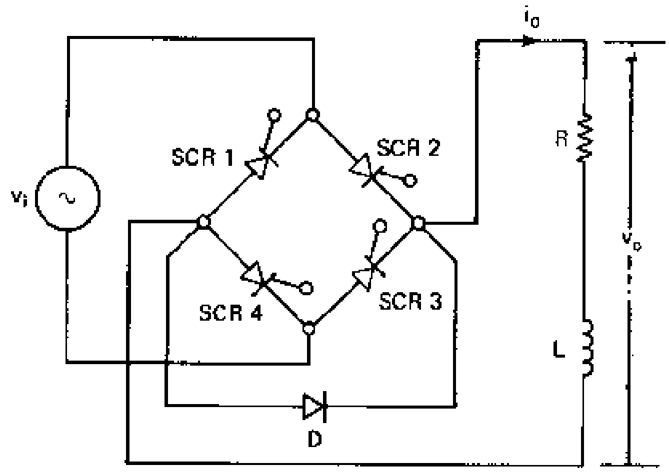
$$\frac{\text{Real power delivered to the load}}{\text{Input apparent power}}$$
$$= \frac{4{,}064.21}{240\times 23.65} = 0.7160 \text{ lagging}$$

(PE1.13)

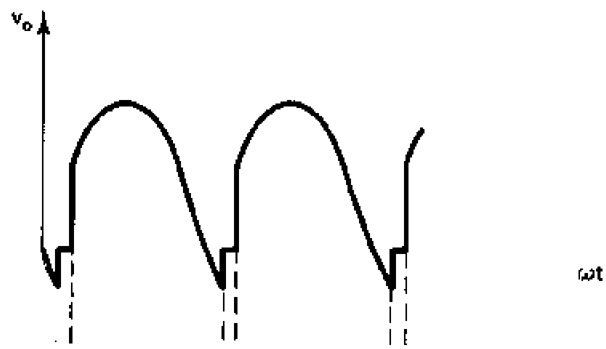
Observation. Even with a purely resistive load, the power factor seen by the source is not unity. In fact, with power electronic circuits, the input power factor can become very small depending upon α and other system parameters.

Inductive Loads

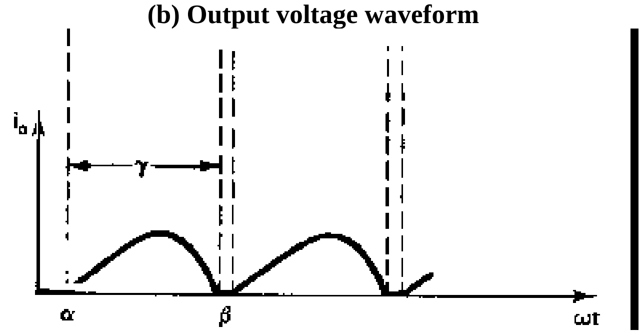
We have thus far considered the load to be purely resistive, which is the easiest type to analyze. Power electronic circuits involving inductive loads and active sources lead to more complex output waveforms and require the numerical solution of nonlinear transcendental equations for output currents. To expose the reader to this aspect, we will now briefly analyze a full-wave controlled rectifier scheme feeding an inductive load. It is illustrated in Figure PE1.18.



(a) Single-phase full-wave controlled rectifier scheme feeding an inductive load



(b) Output voltage waveform



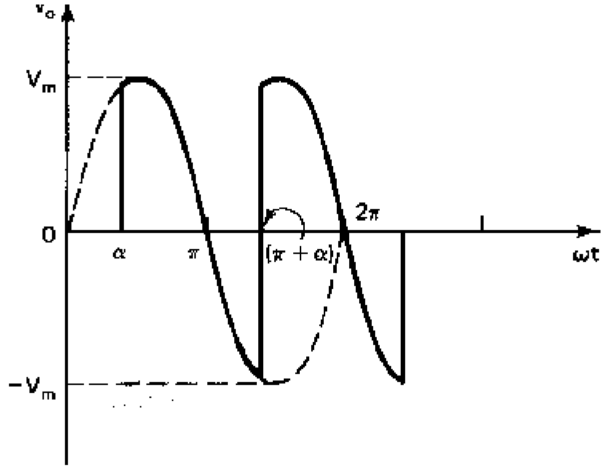
(c) Output current waveform

Figure PE1.18: Single-phase, full-wave controlled rectifier scheme feeding inductive load

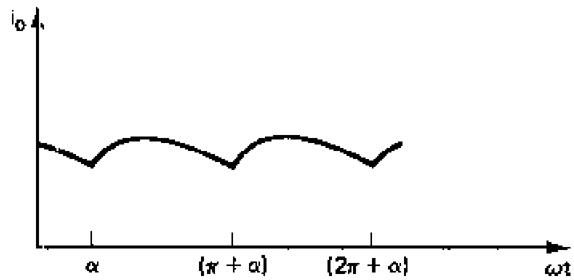
We can draw the following inferences from the output waveforms shown in Figure PE1.18 (b) and (c).

1. The fact that the load is inductive in nature makes i_o lag behind v_o. Therefore, when v_o goes to zero at $(\omega t) = \pi$, i_o is still nonzero, and to facilitate the current flow through the load, a free-wheeling diode (D) is usually connected across the dc circuit as shown in Figure PE1.18 (a).

2. For small L/R ratio, i_o is discontinuous and is obviously nonsinusoidal.



(a) Output voltage waveform



(b) Output current waveform
Figure PE1.19:
Single-phase, full-wave controlled rectifier scheme feeding inductive load (continuous output current case)

3. The output current can be made smoother and continuous by inserting an inductor of suitable value in series with the load circuit. The curves corresponding to this situation are shown in Figure PE1.19. For this case of continuous current operation, we can analyze the harmonic content in the output waveforms by a Fourier series. Let

$$v_o \equiv V_{\mathrm{dc}} + \sum_{n=1}^{\infty}\left(a_n \sin(n\omega t) + b_n \cos(n\omega t)\right)$$
(PE1.14)

$$= V_{\mathrm{dc}} + \sum_{n=1}^{\infty} V_{nm} \cos(n\omega t - \theta_n)$$
(PE1.15)

where V_{nm} = Maximum value of the nth harmonic

$$\text{Component } = \sqrt{a_n^2 + b_n^2}$$
(PE1.16)

$$\theta_n = \text{Arc}\tan(a_n / b_n)$$
(PE1.17)

In equations (PE1.14) to (PE1.17)

$$a_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} v_o \sin(n\omega t)d(\omega t) \tag{PE1.18}$$

In this case

$$\begin{aligned} a_n &= \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\sin(n\omega t)d(\omega t) \\ &= \frac{2V_m}{\pi}\left[\frac{\sin(n+1)\alpha}{(n+1)} - \frac{\sin(n-1)\alpha}{(n-1)}\right] \end{aligned} \tag{PE1.19}$$

Similarly,

$$b_n = \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} v_o \cos(n\omega t)d(\omega t) \tag{PE1.20}$$

$$\begin{aligned} &= \frac{2}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)\cos(n\omega t)d(\omega t) \\ &= \frac{2V_m}{\pi}\left[\frac{\cos(n+1)\alpha}{n+1} - \frac{\cos(n-1)\alpha}{(n-1)}\right] \end{aligned} \tag{PE1.21}$$

The average output voltage (or the dc term in the series) is given by

$$\begin{aligned} V_{\text{dc}} &= \frac{1}{\pi}\int_{\alpha}^{\pi+\alpha} V_m \sin(\omega t)d(\omega t) \\ &= \frac{2V_m}{\pi}\cos\alpha \end{aligned} \tag{PE1.22}$$

Compare this expression with equation (PE1.11) for the resistive load case. For this case, V_{rms}, with n = 1 in equations (PE1.16), (PE1.17), and (PE1.21), can be shown to be equal to $V_m/\sqrt{2}$. The ripple factor r can still be defined by equation (PE1.7).

A quick examination of Figures PE1.17, PE1.18, and PE1.19 reveals that the frequency of output waveforms is twice that of the source frequency. This is to be expected, since the full-wave scheme is a two-pulse converter. The voltage output, therefore, contains only even order harmonics.

A similar series for output current can be expressed as:

$$i_o = I_{\text{dc}} + \sum_{n=1}^{\infty} I_{nm}\cos(n\omega t - \theta_n - \phi_n) \tag{PE1.23}$$

where

$$\begin{aligned} I_{nm} &= \text{Maximum or Peak value of } n\text{th} \\ &\quad \text{harmonic current component} = V_{nm}/Z_n \end{aligned} \tag{PE1.24}$$

$$\begin{aligned} Z_n &= n\text{th harmonic impedance of the load} \\ &= \left(R^2 + (n\omega L)^2\right)^{1/2} \\ \phi_n &= \text{Arc}\tan(n\omega L/R) \end{aligned} \tag{PE1.25}$$

PE1.4.1 Three-phase schemes

The single-phase, full-wave bridge scheme can be extended to accommodate a three-phase input source. Most popular and versatile is the 6-pulse converter, or controlled rectifier. Prior to the invention of thyristors, mercury arc rectifiers were employed to realize controlled rectification. The well-known Pacific ±400 kV, HVDC transmission line linking the northwest region at Celilo, Oregon, to Sylmar, near Los Angeles in the southwest region, adopted the mercury arc rectifiers. We will explore this important application later in this section after gaining a good understanding of this converter scheme.

The circuit of Figure PE1.20 employs a Y-connected, three-phase source $v_i(\omega t)$, delivering dc output $v_o(\omega t)$ to resistive load through a thyristor bridge consisting of six controlled switches; hence it is called a 6-pulse converter.

The operation of the scheme can be described in the following manner: During $\omega t = \pi/3$ to $2\pi/3$, v_{ab} is the most positive voltage relative to other line-to-line voltages and conditions are congenial for thyristors l and 6 to conduct if suitable pulses are applied, or are already applied, to their respective gates, as shown in Figure 9-14(d). The two thyristors will conduct during the following 60° at which time Th 6 will commutate naturally, and if a gate pulse is applied at this moment to the Th 2, output can be delivered to load the next 60° through Th 1 and Th 2. If this process is continued with two thyristors conducting at any given time, we will essentially realize a 6-pulse controlled rectifier. The output voltage v_o and load current i_o are illustrated in Figure 9-14(c) for a typical case. The (d) part of the same figure shows the logical sequence in which the thyristors conduct, and the gate pulses need to be applied to obtain continuous output waveforms.

A close observation of v_o clearly shows that its fundamental frequency of variation is six times that of the input source. Therefore, the harmonic of the output voltage will be multiple orders of 6, a clear advantage with a polyphase source. In addition, the ripple in v_o will also be much lower than in the single-phase scheme we discussed earlier in this chapter. Consequently, we can obtain higher quality (or less ripple) output.

The average value of $v_o = V_{\text{dc}}$ is given by:

$$\begin{aligned} V_{\text{dc}} &= \frac{3}{\pi}\int_{\alpha+(\pi/3)}^{\alpha+(2\pi/3)} v_o(\omega t)\,d(\omega t) \\ &= \frac{3}{\pi}\int_{\alpha+(\pi/3)}^{\alpha+(2\pi/3)} v_{ab}(\omega t)\,d(\omega t) \\ &= \frac{3}{\pi}\int_{\alpha+(\pi/3)}^{\alpha+(2\pi/3)} V_M \sin(\omega t)\,d(\omega t) \\ &= \frac{3V_M}{\pi}\cos\alpha \end{aligned} \tag{PE1.26}$$

where V_M is the maximum line-to-line voltage. In a similar manner, the rms value of the load voltage is:

$$\begin{aligned} V_{\text{rms}} &= \left[\frac{3}{\pi}\int_{\alpha+(\pi/3)}^{\alpha+(2\pi/3)} v_o^2\,d(\omega t)\right]^{1/2} \\ &= V_M\left[\frac{1}{2}+\frac{3\sqrt{3}}{4\pi}\cos(2\alpha)\right]^{1/2} \end{aligned} \tag{PE1.27}$$

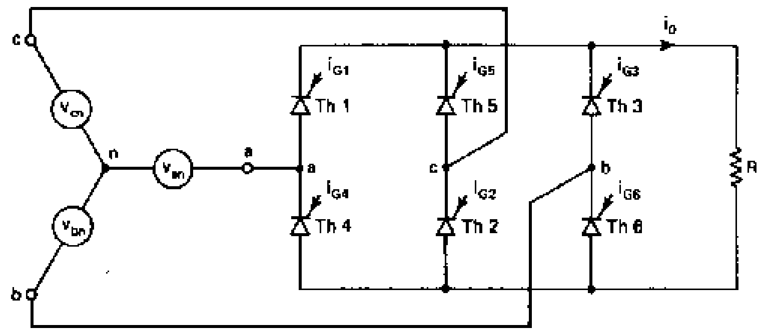
The harmonic coefficients a_n and b_n for this scheme can be obtained by

$$a_n = \frac{6}{\pi}\int_{\alpha-(\pi/3)}^{\alpha} v_o \sin(n\omega t)\,d(\omega t) \tag{PE1.28}$$

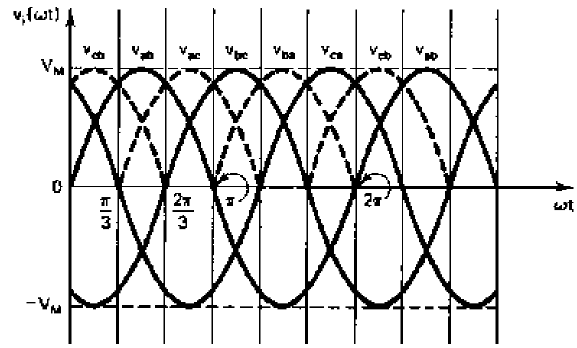
and

$$b_n = \frac{6}{\pi}\int_{\alpha-(\pi/3)}^{\alpha} v_o \cos(n\omega t)\,d(\omega t) \tag{PE1.29}$$

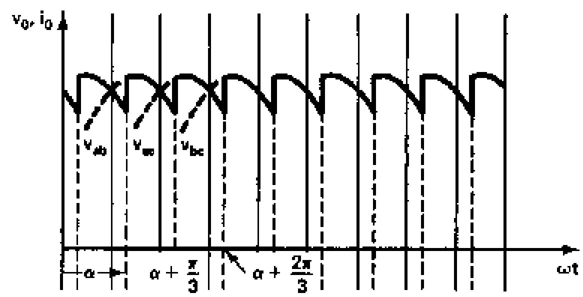
$$n = 6, 12, 18, \ldots$$



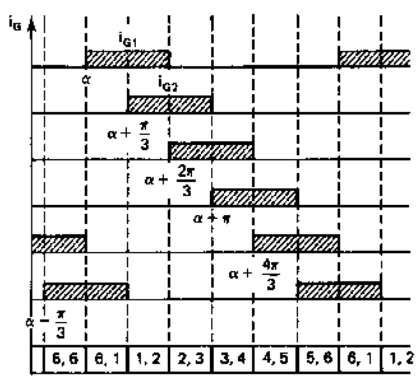
(a) Circuit schematic



(b) Balanced, three-phase input source



(c) Output signal waveforms



(d) Gate pulses and thyristor conduction sequence

Figure PE1.20: Three-phase, full-wave controlled rectifier scheme

A further analysis of V_{dc} from Eq. (PE1.26) reveals that V_{dc} is maximum when α = 0, corresponding to the uncontrolled case. It goes to zero when $\alpha = \pi/2$. Then for $\pi/2 < \alpha \leq \pi$, V_{dc} actually becomes negative. This corresponds to the "inverter" operation, which is possible, provided there is adequate series inductance in the load circuit. Looking from a different point of view, if the dc part of the circuit were a source, one could make the circuit operate as an inverter to realize three-phase, ac output from a dc input source. We will discuss the inverter or, more appropriately, dc-to-ac converter scheme in the following section.

The ripple factor, r, can still be obtained from Eq. (PE1.7), with V_{dc} and V_{rms} defined by Equations (PE1.26) and (PE1.27), respectively.

Before we proceed with a practical application for the three-phase controlled rectifier scheme, we will work an example to gain a better understanding of its basic principle of operation.

Example PE1.3

The three-phase six-pulse controlled rectifier scheme can be adopted to realize a 125-V dc variable supply voltage from a constant voltage, constant frequency ac source such as 480 V or 208 V. However, in order to reduce the ripple, one prefers a 12-pulse, or even a 24-pulse, rectifier scheme, which requires twice, or four times, the number of thyristors we have used in the circuit of Figure PE1.20.

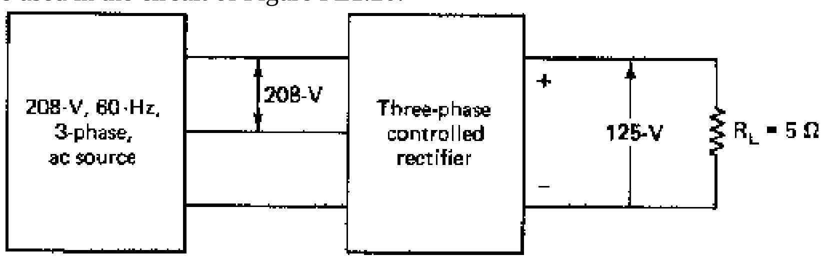


Figure PE1.21: Block diagram for three-phase controlled rectifier scheme delivering resistive load

Perform a detailed analysis of this 6-pulse circuit if it delivers dc power to a 5-ohm resistor at 125—V from a 208-V, 60-Hz three-phase ac supply (Figure PE1.21).

Solution

a. For $V_m = \sqrt{2}(208)$ volts and V_{dc} = 125 V we need to determine α using equation (PE1.26).

$$V_{dc} = 125 = \left\{ \frac{3\left[\sqrt{2}(208)\right]}{\pi} \cos\alpha \right\}$$

or

$$\cos\alpha = \frac{125\pi}{(3)\left(\sqrt{2}\right)(208)} = 0.4450$$

or

$$\alpha = 63.58°$$

b. $I_{dc} = \dfrac{V_{dc}}{R_L} = \dfrac{125}{5} = 25\text{ A}$

c. $P_{dc} = V_{dc} I_{dc} = 125 \times 5 = 3{,}125\,\text{kW}$

Observation 1. Evidently, proper heat sinks will be needed to dissipate the heat in the thyristor.

d. V_{rms} from equation (PE1.27)

$$= \sqrt{2}(208)\left[\frac{1}{2} + \frac{3\sqrt{3}}{4\pi}\cos(127.15°)\right]^{1/2}$$

$$= 147.16\ \text{V}$$

e. Average thyristor current = $\dfrac{I_{\text{dc}}}{3}$

This is true since each thyristor conducts for 1/3 cycle.

$$= 25/3 = 8.34\ \text{A}$$

f. rms value of current in each thyristor

$$= \frac{I_{\text{rms}}}{\sqrt{3}}$$

$$= \frac{V_{\text{rms}}}{\sqrt{3}R_L}$$

$$= \frac{147.16}{(\sqrt{3})(5)} = 16.99\ \text{A}$$

g. Ripple factor, $r = \left[\left(\dfrac{V_{\text{rms}}}{V_{\text{dc}}}\right)^2 - 1\right]^{1/2} \times 100$

$$= \left[\frac{(147.16)^2}{(125)^2} - 1\right]^{1/2} \times 100$$

$$= 42.10\ \%$$

Observation 2. The ripple is very high, thus warranting a 12-pulse or even a 24-pulse scheme for this example system to mitigate the ripple.

h. Input power factor = (P_{input} / S_{input})

$$\cong P_{\text{dc}} / S_{\text{input}}$$

$$\cong \frac{3{,}125}{\sqrt{3}(208)(\sqrt{3})(16.99)}$$

$$= 0.2947\ \text{current lagging}$$

Observation 3. The power factor is extremely low as seen from the source side.

Inductive Loads

Besides supplying dc power to resistive loads, these can be adopted as dc motor drives in process industries, as converters at the terminals of a HVDC transmission line, for portable hand tools and many more applications. In most of these cases the load is no longer fully resistive. A general dc load includes in addition an active source (EMF) and an inductance as we emphasized while discussing single-phase schemes. In these situations, the output voltage and current are no longer in phase. The output current could be continuous or discontinuous, depending upon the value of α and other circuit parameters. Figure PE1.22 illustrates continuous i_o from a passive inductive load.

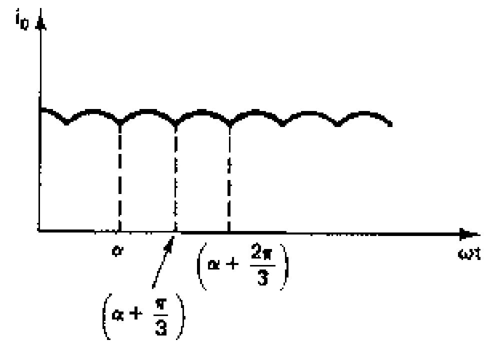


Figure PE1.22: Continuous output waveform for inductive load

HVDC Transmission System Application

High voltage DC (HVDC) energy transmission may be used within an interconnected power grid (which are always AC systems), but they require converters at the terminals. The additional cost of the converters makes HVDC uneconomic, relative to AC transmission, for short distances, but because HVDC lines may be constructed at lower cost per mile than equivalent AC lines, HVDC is quite attractive for long-distance energy transmission. Nonetheless, HVDC is occasionally used in short-distance applications because they serve as an asynchronous and stabilizing link to interconnect two polyphase ac lines of the same frequency, or two different frequencies. A schematic of such a system is given in Figure PE1.23.

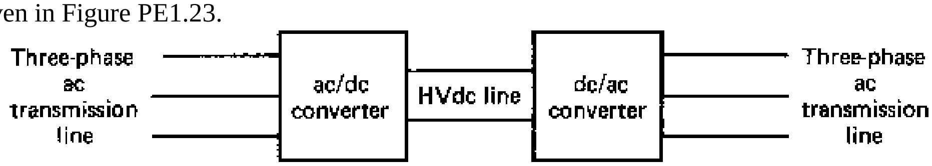


Figure PE1.23: HVDC transmission system schematic

The first solid-state HVDC line is the Eel River Project in New Brunswick, Canada. It started functioning in 1972. It is rated at 320 MW with four three-phase, ac-to-dc bridge converters connected back-to-back on each side of the line. Each of these bridges is rated at 40 kV between the positive pole (or the negative pole) and ground, and is designed to carry 2000 A. In each bridge, there are six SCR valves with each valve having 200 SCRs.

Another such line is the famous Pacific NW/SW Intertie, which is approximately 900 miles (or about 1,450 km.) long, interconnecting two ac high-voltage systems between Celilo, Oregon and Sylmar, California, near Los Angeles. It was originally designed to operate at ±400 kV, utilizing an array of three 133-kV mercury-arc valve groups. This bipolar line with 800 kV between the lines had the capacity to transmit 1,800 amperes or about 1,440 MW of power. By proper control of the valves, the real power could be varied from zero to 1,440 MW in each direction. Putting it differently, the power flow direction implied the converter at one end, serving as a controlled rectifier, and the converter at the other end as an inverter. Their roles could be interchanged to facilitate the flow of energy in the opposite direction.

In 1978, the current capacity of the line was increased from 1,800 amperes to 2,000 amperes, and later, it was further upgraded to ±500 kV with the installation of two 100 kV SCR valve groups on both the positive and negative sides, and at both ends of the line. This latest modification also needed the addition of a new smoothing reactor in series with the line and the replacement of new filters to isolate unwanted harmonics in the dc line.

To understand the principle of the converter operation, or equivalently, the power flow control in the line, let us consider the Pacific Intertie. A simplified circuit diagram of the line is shown in Figure PE1.24. Denote $\alpha_{\text{top}}^{\text{c}}$ and $\alpha_{\text{bottom}}^{\text{c}}$ as the firing angles of the Celilo top and bottom SCRs, respectively, and $\alpha_{\text{top}}^{\text{s}}$ and $\alpha_{\text{bottom}}^{\text{s}}$ as the firing angles of the Sylmar top and bottom SCRs, respectively. If it is desired to keep the voltage at the Celilo converter station at the rated value of 400 kV, the firing angle $\alpha_{\text{top}}^{\text{c}}$ of the top set of the SCR must be controlled so that:

$$V_{dc} = 400 = \left\{ \frac{3\left[\sqrt{2}(500)\right]}{\pi} \cos\left(\alpha^{c}_{top}\right) \right\} \quad \text{by Eq. (PE1.26)}$$

or

$$\alpha^{c}_{top} = \cos^{-1}\left[\frac{400\pi}{(3)\left(\sqrt{2}\right)(500)}\right] = 53.67°$$

The firing angle of the bottom SCR valve should be adjusted so that V_{dc} = +400 kV can be obtained from the ac side.

Or

$$\alpha^{c}_{bottom} = 53.67°$$

which is equal to α^{c}_{top}.

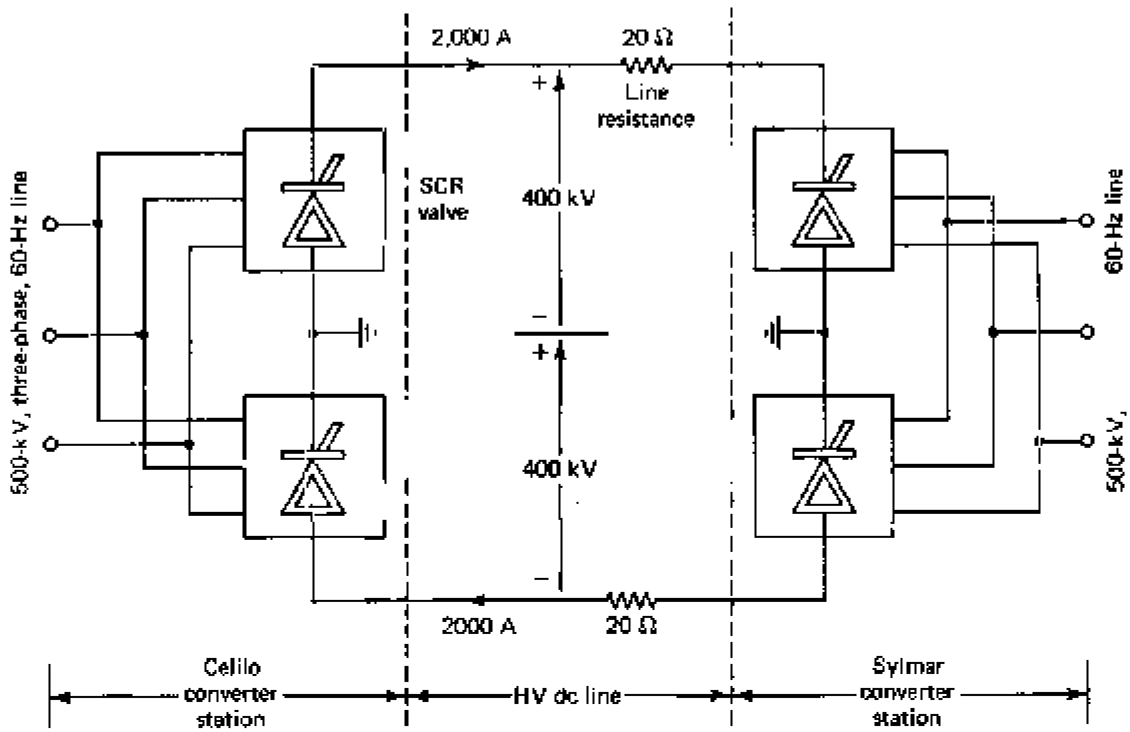


Figure PE1.24: Pacific HVDC transmission intertie representation diagram

If it is desired to send 1,600 MW from the north to the south, which happens in the summer, the dc voltage from one pole to the ground at the Sylmar converter station ought to be at 400 - 2 × 20 = 360 kV. This situation warrants that the converter at the Sylmar station operate as an inverter. The α^{s}_{top} at the Sylmar end must now be adjusted such that:

$$+360 = -\frac{3\left[\sqrt{2}(500)\right]}{\pi} \cos\left(\alpha^{s}_{top}\right)$$

(The negative sign on the right side takes care of the reversal of power flow.)
or

$$\alpha^{s}_{top} = 122.22°$$

We can deduce that $\alpha_{\text{bottom}}^{\text{s}}$ will also have to be =

$$\alpha_{\text{bottom}}^{\text{s}} = 122.22°$$

In summary, the power flow magnitude and direction are essentially controlled by proper adjustment of the firing angles of the valves at both converter stations.

PE1.5 DC to AC Converters

These inverters convert dc power to ac power at desired voltage and frequency. While we have already established that these are employed at the output side of a HVDC transmission line, other applications for these converters include:

1. Uninterruptible power supplies (UPS) for computers, which require high quality power supply,
2. Induction motor drives, which demand variable input ac voltage supply,
3. Space power systems,
4. Induction heating, and others.

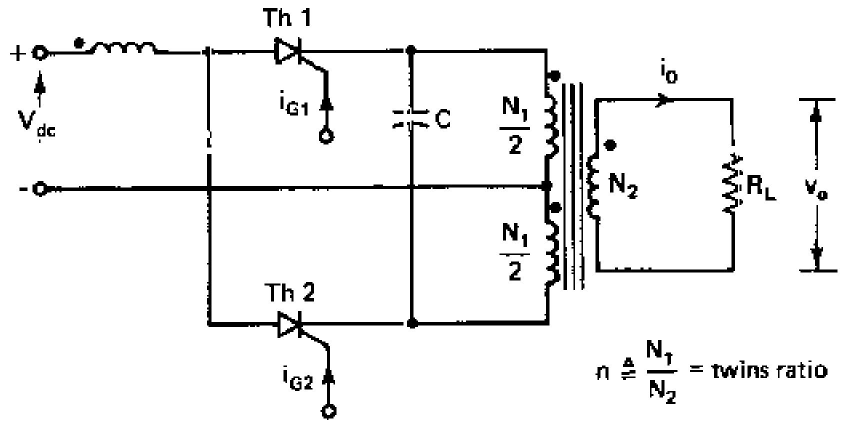
In the following material, we will discuss the basic single-phase and three-phase circuits employed in one or more of the applications mentioned above to understand their principle of operation and the quality of the output waveforms.

PE1.5.1 Single-phase inverter

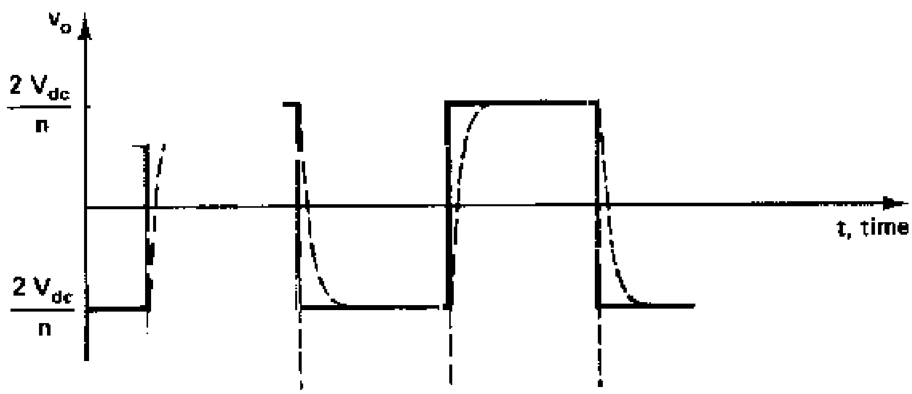
A simple scheme to obtain constant ac output voltage at a desired frequency, or range of frequencies, is illustrated in Figure PE1.25. This circuit employs two thyristors, Th 1 and Th 2, which can be made to conduct alternately by applying suitable gate signals to each of them, a commutating capacitor *C*, and an output transformer with center tap in the primary winding. The circuit essentially operates in the following sequential manner:

Assume Th 1 is turned on by the application of i_{Gl} to its gate, the input voltage V_{dc} will now be applied across the top half of the primary winding of the transformer. Since the bottom half of the primary is magnetically linked to the top winding, V_{dc} will be induced in the lower winding also. This implies a voltage $2V_{\text{dc}}$ will build across the capacitor and will remain at this value until Th 2 is turned on by injecting i_{G2} through its gate. Since the voltage across the capacitor cannot change instantaneously, a reverse-biased voltage will be momentarily applied across Th l, causing it to turn off. At this stage, a negative voltage V_{dc} appears across the bottom half of the transformer primary winding. This induces a negative V_{dc} across the top half of the primary winding, resulting in $-2V_{\text{dc}}$ across the capacitor. This situation will continue until Th 1 is turned on again, completing one cycle of operation of the circuit. The various salient waveforms of this circuit are also shown in Figure PE1.25. We can make the following important observations on the operation of this circuit:

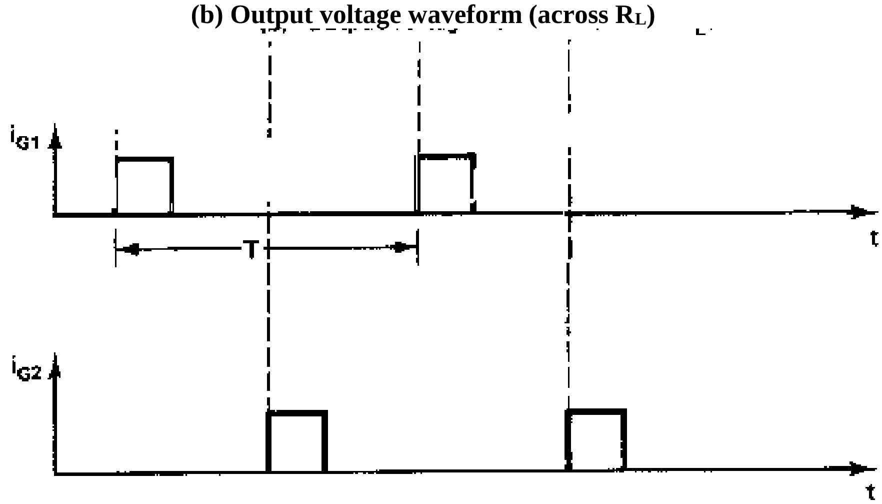
1. The output frequency of variation $f = 1/T$ Hertz; where T is duration between two successive gate pulses applied to each thyristor.
2. The output waveform is far from sinusoidal in shape. Hence, proper filters are required to extract the fundamental component from the almost rectangular output voltage v_o.
3. If the load is an ac induction motor, i_o will be different from v_o (Figure PE1.25 (b)). The reader is encouraged to go through standard power electronics or electronic drives material listed at the end of this chapter to gain an understanding on such circuits involving active sources and inductive parameters.
4. The capacitor C aids in achieving the forced commutation of the two thyristors at proper instants. It should be chosen along with n and R_t such that each thyristor has adequate time to turn off. At this stage, compare the forced commutation process attained here with the simplest circuit described in Figure PE1.9.
5. With the advent of high-power transistors and GTO thyristors, a power electronic designer has better choices of devices along with the standard thyristors for circuits such as the one described here.



(a) Circuit diagram



(b) Output voltage waveform (across R_L)

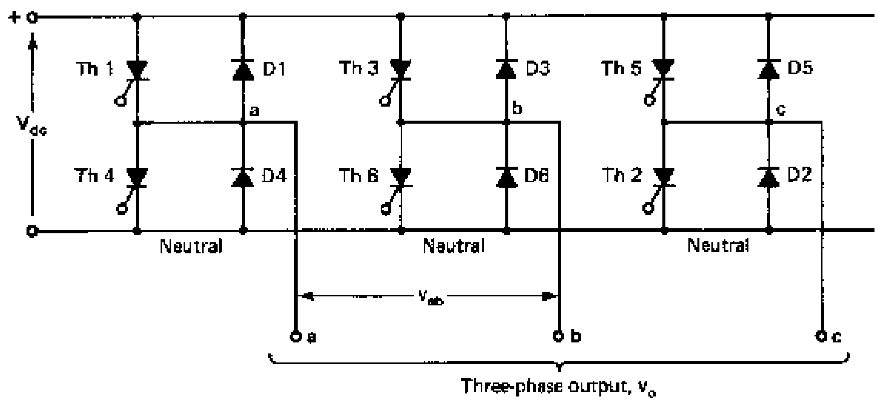


(c) Gate pulse waveforms

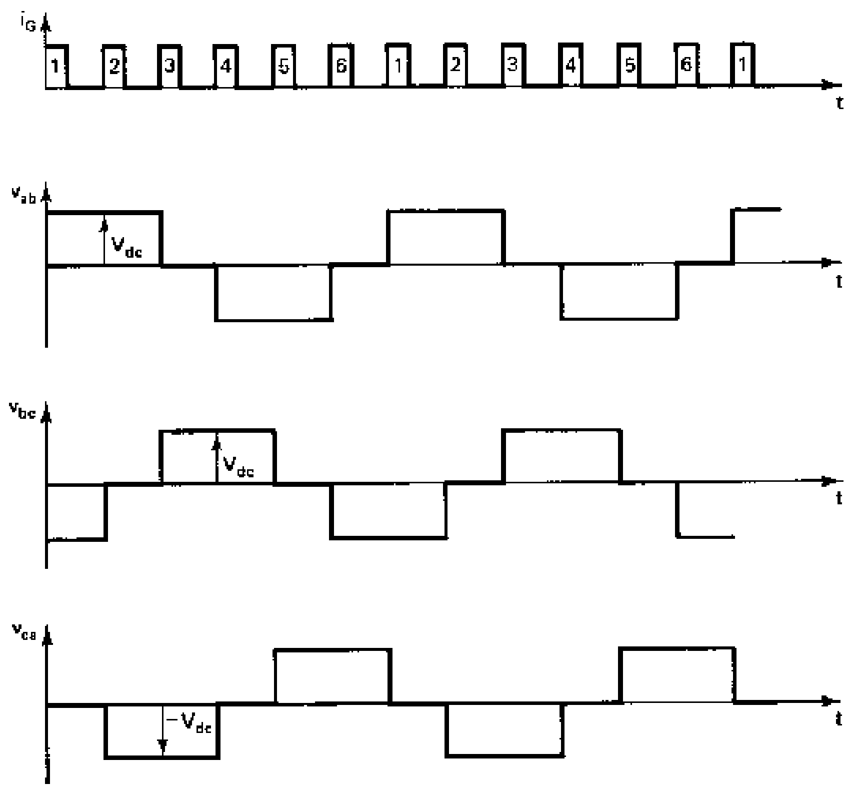
Figure PE1.25: Single-phase dc-to-ac converter

PE1.5.2 Three-phase inverter

Figure PE1.26 illustrates a commonly used scheme employing six electronic switches (Th 1 to Th 6) and 6 free-wheeling diodes (D1 to D6). this figure does not explicitly show the commutation circuits to turn off the switches, or gating circuits.



(a) Circuit schematic



(b) Gate current and output voltage waveforms

Figure PE1.26: Three-phase dc-to-ac converter

The operation of this circuit is similar to the single-phase inverter scheme described earlier in this section, except each thyristor must be turned on at equal intervals of time in the natural sequence of 1 to 6, as shown in Figure PE1.26. The time interval between two successive pulses applied to each thyristor's gate will dictate the frequency of output of the ac waveform. The figure also shows the output waveforms v_{ab}, v_{bc}, and v_{ca}, which have a high harmonic content. We could improve the quality of the output voltage by doubling or even quadrupling the number of switching devices to realize a 12-pulse or a 24-pulse converter. We have to pay a higher price to achieve higher quality of power supply.

There is yet another way to improve the quality of the 6-pulse inverter described above. Instead of a single gate pulse, one could design an efficient control logic for gating circuitry, whereby each gating pulse frequency and width is varied in such a manner as to approach sinusoidal output voltage. Such a scheme is called *pulse width modulation (PWM)* process, which is commonly adopted for modern ac drives. The PWM process is conceptually depicted in Figure PE1.27.

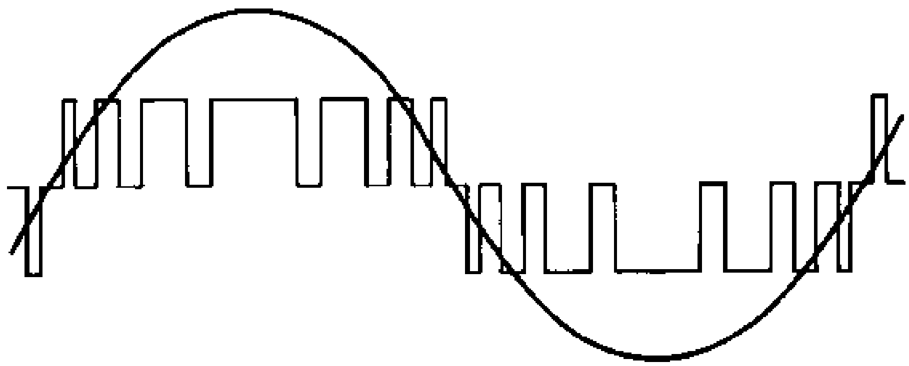


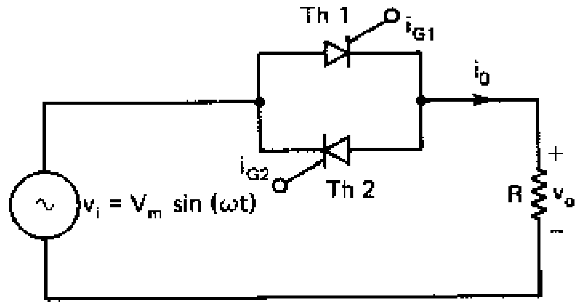
Figure PE1.27: Pulse width modulation process

PE1.6 AC to AC Converters

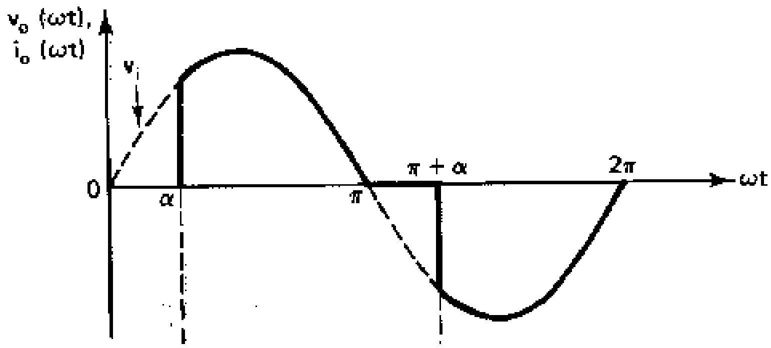
The primary purpose of this type of converter is to vary the rms value of the output voltage applied to an ac load from a constant input source. The control is achieved by means of changing the firing angle of the switching device; hence the name *phase angle* or simply, *phase control.* Since the entire power circuit is ac, the commutation process can be accomplished naturally. In other words, no external commutation circuits are required to deactivate the thyristors. Important applications for this type of controller vary from the simplest such as light dimmers to the sophisticated ac drives for induction motors. Another application can be found in tap changing mechanisms for power transformers. As in the case of the converters discussed in the earlier two sections, we will briefly discuss both single-phase and three-phase schemes.

PE1.6.1 Single-phase full-wave controller

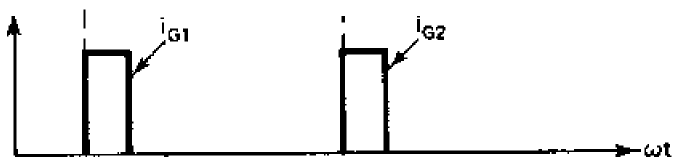
Consider the following full-wave controller feeding a resistive load from a standard single-phase supply. Figure PE1.28 shows such a situation. It also shows the output voltage and current waveforms and the instants at which the gating pulses are applied. One could use a single triac in the place of the two thyristors.



(a) Circuit diagram



(b) Output waveforms



(c) Gating pulse waveforms

Figure PE1.28: Single-phase, full-wave, ac-to-ac converter scheme

The analysis of this scheme is similar to the full wave ac-to-dc converter circuit we discussed in Section PE1.3. The average value of the output voltage is obviously zero. If one of the thyristors is replaced by a diode, V_{dc} across the load is not zero. The effective value of v_o is given by:

$$V_{rms} = \left[\frac{1}{\pi}\int_0^{\pi} v_0^2 d(\omega t)\right]^{1/2}$$

$$= \frac{V_m}{\sqrt{2\pi}}\left[(\pi - \alpha) + \frac{\sin 2\alpha}{2}\right]^{1/2} \qquad \text{(PE1.12)}$$

which is exactly the same as in the case of a full-wave controlled rectifier. The rms value of each thyristor voltage can be shown to be $V_{rms}/\sqrt{2}$. We can observe from Figure PE1.28 that v_o has significant odd harmonic components, which can be evaluated by using standard Fourier series. At this point let us consider an example to gather further knowledge on this scheme.

Example PE1.4

In the circuit of Figure PE1.28, let $v_i = \sqrt{2}(240)\sin(\omega t)$ and R = 24 Ω. For α = 45° or π/4 radians, determine:

a. V_{rms}, I_{rms} and real power absorbed by the load
b. The power factor as seen by the source
c. The rms value of third harmonic component of v_o

Solution

a. By equation (PE1.12)

$$V_{\text{rms}} = \frac{\sqrt{2}(240)}{\sqrt{2\pi}}\left[\left(\pi - \frac{\pi}{4}\right) + \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right]^{1/2}$$

$$= 228.84 \text{ V}$$

Therefore

$$I_{\text{rms}} = \frac{228.84}{24} = 9.535 \text{ A}$$

Real power absorbed, $P_{\text{dc}} = V_{\text{rms}} I_{\text{rms}}$ (why?) $= I_{\text{rms}}^2 R = 2{,}181.97$ watts

b. Input apparent power S = (240) (240/24)
= 2,400 VA

Therefore, input power factor

$$= \frac{P_{\text{dc}}}{\text{S}} = \frac{2{,}181.97}{2{,}400} = 0.9092 \text{ lagging}$$

c. The third harmonic component of v_o can be computed as follows:

$$a_3 = \frac{2}{\pi}\int_{\alpha}^{\pi} v_0 \sin(3\omega t)\, d(\omega t) \tag{PE1.30}$$

which will reduce to:

$$a_3 = \frac{V_m}{\pi}\left[\frac{\sin 4\alpha}{4} - \frac{\sin 2\alpha}{2}\right] \tag{PE1.31}$$

For $\alpha = \pi/4$

$$a_3 = \frac{\sqrt{2}(240)}{\pi}\left[\frac{\sin \pi}{4} - \frac{\sin\left(\frac{\pi}{2}\right)}{2}\right]$$

$$= -54.02 \text{ volts}$$

Similarly,

$$b_3 = \frac{2}{\pi}\int_{\alpha}^{\pi} v_0 \cos(3\omega t) d(\omega t)$$

(PE1.32)

which can be shown to yield:

$$b_3 = \frac{V_m}{\pi}\left[\frac{\cos(4\alpha)-1}{4} - \frac{\cos(2\alpha)-1}{2}\right]$$

(PE1.33)

For $\alpha = \pi/4$, eq. (PE1.33) results in

$$b_3 = 0$$

Therefore,

$$V_{3\text{rms}} = \frac{V_{3m}}{\sqrt{2}} = \frac{\left(a_3^2 + b_3^2\right)^{1/2}}{\sqrt{2}} = \frac{54.02}{\sqrt{2}} = 38.2 \text{ V}$$

or the per unit value of the third harmonic

$$= \frac{V_{3\text{rms}}}{V_{\text{rms}}} = \frac{38.20}{228.84} = 0.1669$$

$$= 16.69\%$$

Viewed differently, the third-harmonic component will be = 38.20/240 = 0.1592 or 15.92 percent of the input rms value. This value is fairly significant, and the designer must devise means of mitigating this particular harmonic content.

PE1.6.2 Single-phase full-wave controller

The schematic diagram of a Y-connected three-phase converter often adopted for induction motor drives is drawn in Figure PE1.29.

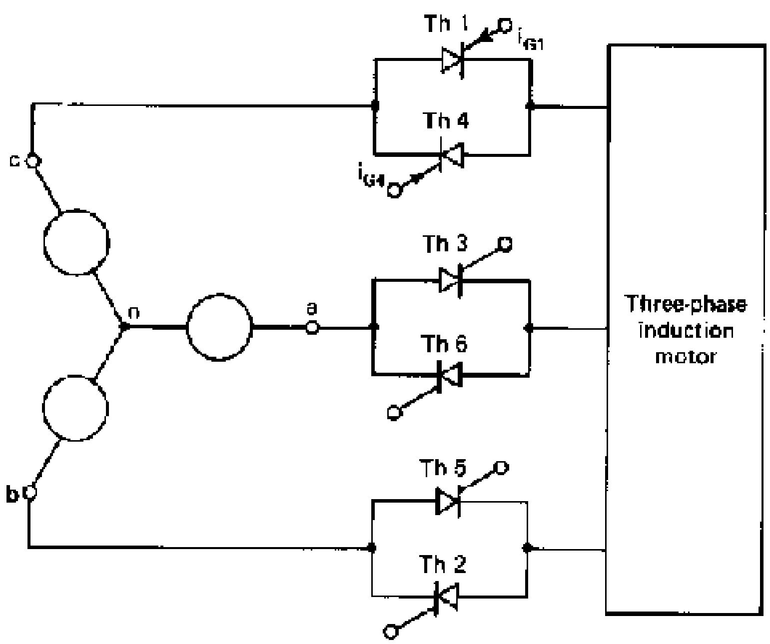
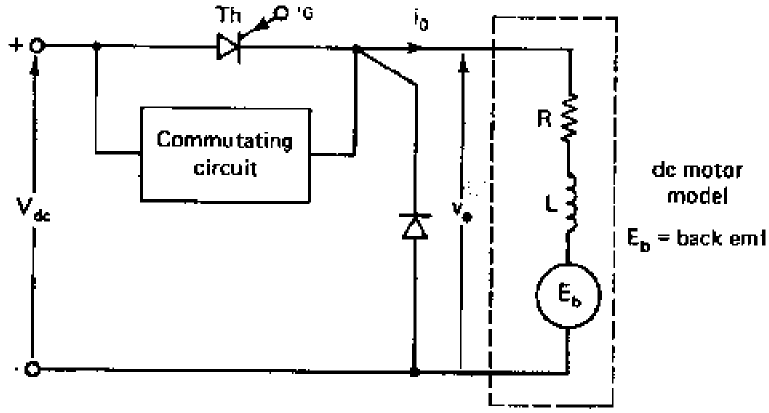


Figure PE1.29: Three-phase, ac-to-ac Converter Scheme (Y-connected)

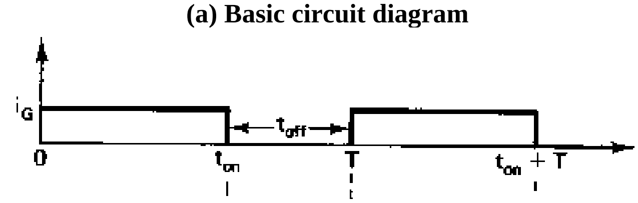
The basic principle of operation is very similar to the single-phase scheme presented in Figure PE1.28. The logical sequence at which the gate signals are applied is exactly the same as in the three-phase, controlled rectifier scheme presented in Figure PE1.20 and the accompanying material explained in Sec. PE1.2.

PE1.7 DC to DC Converters

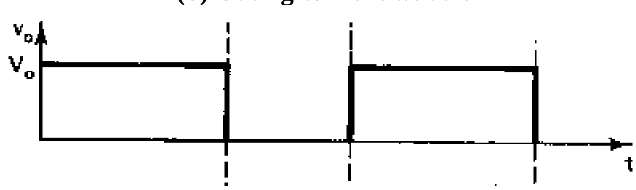
This type of converter is intended for developing variable dc output voltage, usually required for the speed control of dc motors, from a constant dc source, such as a battery. This converter scheme, classically known as a *chopper,* employs one or more electronic switches, devices for commutating the switches at required instants, and, of course, gating circuits. The simple forced commutation scheme presented in Figure PE1.9, though not versatile, can perform the functions of the dc-to-dc converter. The basic principle of operation calls for controlling the ratio of the turn-on-time (t_{on}), of each device to its periodic time, T, as illustrated in Figure PE1.30.



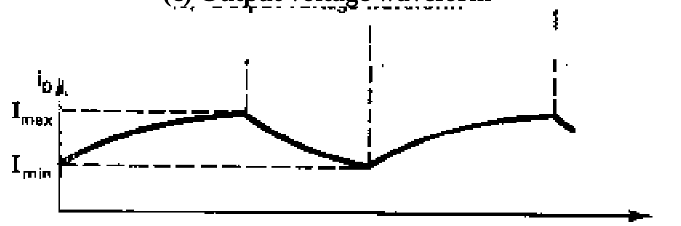
(a) Basic circuit diagram



(b) Gating current waveform



(c) Output voltage waveform



(d) Output current waveform

Figure PE1.30: A simple dc-to-dc converter scheme

We can easily observe from Figure PE1.30 that:

$$V_o = \left(\frac{t_{on}}{T}\right)V_{\mathrm{dc}} = \left(\frac{t_{on}}{t_{on}+t_{off}}\right)V_{\mathrm{dc}}$$

(PE1.34)

The load voltage V_o can be varied in one of the following ways:

1. Varying t_{on} while keeping T constant, which implies adopting the pulse width modulation (PWM) principle introduced in Figure PE1.27 of Section PE1.4,
2. Keeping t_{on} constant, while varying T, which means employing the frequency modulation principle of switching, or
3. A judicious combination of 1 and 2.

We can also observe from Figure PE1.30 (d) that i_o is not an ideal dc waveform, which may require additional conditioning if warranted.

PE1.8 SUMMARY

The primary objective of this chapter is to enable the reader to gain a rudimentary knowledge on the rapidly emerging and expanding field of power electronics. To obtain this objective, the principles of operation of power electronic switching devices and converter schemes have been introduced. The problems demanding a designer's attention have been brought out; some of these problems are gating circuitry design, commutating circuitry design, harmonics generation, and means of reducing their effects with proper filter design. The four commonly used converter schemes, namely, ac-to-dc (controlled rectifier), dc-to-ac (inverter), ac-to-ac (phase controller), and dc-to-dc (choppers) are introduced. Several applications encountered in energy and power system applications have either been briefly described, or introduced. In an introductory level of material, it is very difficult, if not impossible, to offer detailed coverage of the material presented here. The reader is therefore strongly recommended to refer to the following reference books:

ADDITIONAL READING MATERIAL

1. Chapman, S. J., *Electric Machinery Fundamentals,* New York: McGraw-Hill Book Company, 1985.
2. Datta, S. K., *Power Electronics and Controls,* Reston, Virginia: Reston Publishing Company, Inc., 1985.
3. Dewan, S. B., and A. Straughen, *Power Semiconductor Circuits,* New York: John Wiley & Sons, Inc., 1975.
4. Institute of Electronic and Electrical Engineers, IEEE Tutorial Course, *Power Electronics Applications in Power Systems,* 78 EH0135-4-PWR, Piscataway, NJ: IEEE Service Center, 1978.
5. Kusko, A., *Solid-State dc Motor Drives,* Cambridge, Mass.: MIT Press, 1969.
6. Motto, J. W. Jr., ed., *Introduction to Solid-State Power Electronics,* Youngwood, Pennsylvania: Westinghouse Electric Corporation, 1977.
7. Nasar, S. A., *Electric Machines and Electromechanics,* New York: Schaum's Outline Series in Engineering, McGraw-Hill Book Company, 1981.