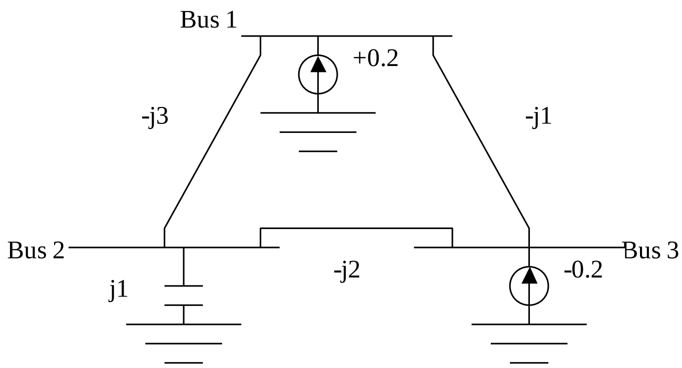
Solutions to Problems

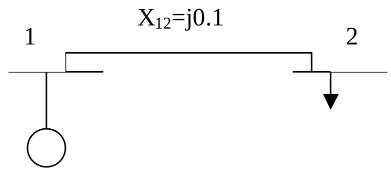
1. Form the matrix equation $\underline{YV}=\underline{I}$ for the network below.



Answer:

$$\begin{bmatrix} -j4 & j3 & j1 \\ j3 & -j4 & j2 \\ j1 & j2 & -j3 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0 \\ -0.2 \end{bmatrix}$$

2. Form the Y-bus for the following two-bus power system, in both rectangular and polar notation. The reactance X_{12} is given in per unit.



Answer:

$$y_{12} = \frac{1}{x_{12}} = \frac{1}{j0.1} = -j10$$

$$Y_{bus} = \begin{bmatrix} Y_{12} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -j10 & j10 \\ j10 & -j10 \end{bmatrix} = \begin{bmatrix} 10\angle -90° & 10\angle 90° \\ 10\angle 90° & 10\angle -90° \end{bmatrix}$$

3. Draw the network, in terms of branches only, for the following admittance matrix

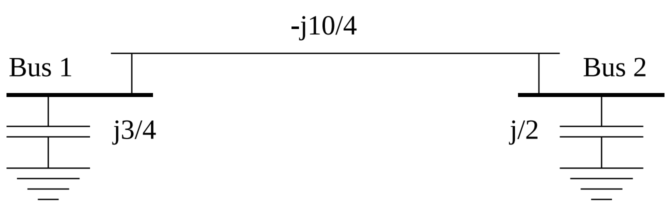
$$\underline{Y} = \begin{bmatrix} -\frac{j7}{4} & \frac{j10}{4} \\ \frac{j10}{4} & -j2 \end{bmatrix}$$

Answer:

$$Y_{11} = y_{12} + y_1 \qquad\qquad Y_{22} = y_{21} + y_2$$

$$\frac{-j7}{4} = \frac{-j10}{4} + y_1 \qquad\qquad -j2 = \frac{-j10}{4} + y_2$$

$$y_1 = \frac{j3}{4} \qquad\qquad y_2 = \frac{j}{2}$$



4. A 2000 bus system has 250 generators, all of which are modeled in a power flow program with constant (known) terminal voltage.
    a. How many type PV buses are there in the power flow model?
    b. How many type PQ buses are there in the power flow model?
    c. What is the minimum number of equations required to solve this problem?
    d. How many bus voltage magnitudes are unknown in this problem?
    e. How many bus voltage angles are unknown in this problem?

Answer:

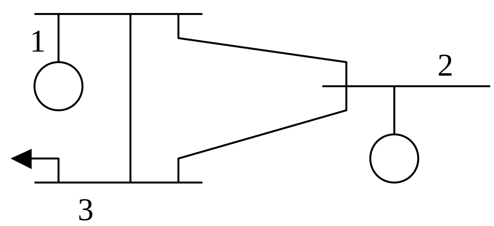
a. 249
b. 1750
c. 3749
d. 1750
e. 1999

5. For a certain bus type, when formulating the power flow problem, we reduce the dimensionality of the problem by removing the reactive power equation from the set of equations to be solved. What bus type is this? Explain why this does not result in an under-constrained problem such that there are more unknowns than equations.

Answer:

The PV Bus. Since the Q_i variable doesn't appear in any other equations, it can be removed to reduce the dimensionality of the equation set and avoid having an under-defined problem where the number of unknowns is greater than the number of equations.

6. Consider the system illustrated below.
   a. Identify the unknowns comprising the solution vector in the power flow problem.
   b. Identify the minimum number of real power equations and the minimum number of reactive power equations to be solved in the power flow problem.

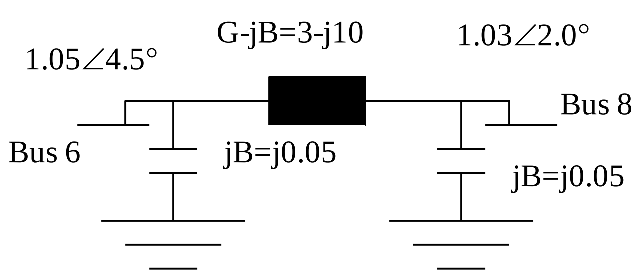


Answer:
   a. The three unknowns for this system are θ_2, θ_3, and $|V_3|$.
   b. One reactive power equation is required for bus 3 and two real power equations, one each for bus 1 and 2, are required to solve the power flow problem for this system.

7. George and Clara read the output from an industrial-grade power flow program; part of it is as follows:

From bus no.	To bus no.	Pflow (MW)	Qflow (MVAR)
6	8	53.8	2.4

The voltages and transmission line admittance parameters are given on the diagram below. All parameters are on a 100 MVA base.



They thought the power flow program had a "bug" because they did the following per unit calculations and compared the result to the Q flow of the power flow program output.

$$I_{6,8} = (V_6 - V_8)(G + jB) = (1.05\angle 4.5^\circ - 1.03\angle 2.0^\circ)(3 - j10) = 0.518\angle -3.83^\circ$$

$$S_{6,8} = V_6 I_{6,8}^{\,*} = (1.05\angle 4.5^\circ)(0.518\angle 3.83^\circ) = 0.544\angle 8.33^\circ = 0.538 + j0.079$$

$$\Rightarrow P_{6,8} = 53.8\,\text{MW}, \qquad Q_{6,8} = 7.9\,\text{MVARS}$$

Were George and Clara correct? Explain.

Answer:

George and Clara are computing S = P + jQ without including the effects of the reactive power injection from the charging capacitance. $C_{charge} = (100)(0.05)(1.05)^2 = 5.5125$

8. A two-bus power system is interconnected by a transmission line having series admittance of Y_{12}=0.304-j1.88 pu and *total* line charging of $Y_{charging}$=j0.064 pu. Bus 2 is a load bus with specified demand as P_D pu and Q_D pu. Bus 1 is a generator bus with specified terminal voltage magnitude of $|V_1|$ pu. We desire to solve the power flow problem for this system.
    a. Form the Y-bus for this system with all elements given in polar form.
    b. Identify the variables in the solution vector.
    c. It is possible to write a power flow equation for real and reactive injections at buses 1 and 2 (giving a total of four equations). From these four equations, write down the mismatch equation(s) that are required in the solutions procedure. Express each equation symbolically (no numbers). Denote each equation by g_i.
    d. Write down the Jacobian matrix $\underline{J}$ to be used in the solution procedure. Indicate the elements in the matrix using partial derivative notation; you do not need to differentiate anything or provide any numerical values.
    e. Write down the update formula for this system in terms of $\underline{J}$ and the solution vector.
    f. A generator is now brought on-line at bus 2 so that it becomes a type PV bus. Repeat part d.

Answer:

a.
$$Y_{bus} = \begin{bmatrix} .304 - j1.848 & -.304 + j1.88 \\ -.304 + j1.88 & .304 - j1.88 \end{bmatrix} = \begin{bmatrix} 1.873\angle -80.66^\circ & 1.9\angle 99.18^\circ \\ 1.9\angle 99.18^\circ & 1.873\angle -80.81^\circ \end{bmatrix}$$

b.
$$solution = \begin{bmatrix} \theta_2 \\ |V_2| \end{bmatrix}$$

c.
$$g_1 = P_{2,sch} - P_2 = P_{2,sch} - \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\,cos(\theta_k - \theta_j) + B_{kj}\,sin(\theta_k - \theta_j)\right)$$

$$g_2 = Q_{2,sch} - Q_2 = Q_{2,sch} - \sum_{j=1}^{N} |V_k||V_j|\left(G_{kj}\,sin(\theta_k - \theta_j) - B_{kj}\,cos(\theta_k - \theta_j)\right)$$

d.

$$\vec{J} = \begin{bmatrix} \frac{\partial g_1}{\partial \theta_2} & \frac{\partial g_1}{\partial |V_2|} \\ \frac{\partial g_2}{\partial \theta_2} & \frac{\partial g_2}{\partial |V_2|} \end{bmatrix}$$

e. $$\vec{J}\Delta\vec{X} = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \vec{J} \begin{bmatrix} \Delta\theta_2 \\ \Delta|V_2| \end{bmatrix}$$

f. $$J = \begin{bmatrix} \frac{\partial g_1}{\partial \theta_2} \end{bmatrix}$$