Module E3

Problem 1

A three-unit system is given by the following data

$$C_1(P_{g1}) = 0.015 \cdot (P_{g1})^2 + 2 \cdot (P_{g1}) + 6$$

$$C_2(P_{g2}) = 0.025 \cdot (P_{g2})^2 + 7 \cdot (P_{g2}) + 3$$

The total system demand is 500MW. The lower and upper limits for each generator unit are 20 and 300MW, respectively.

(a) Determine the optimal dispatch ignoring inequality constraints
(b) And identify whether it is a feasible dispatch or not (support your answer)

$$F(P_{g1}, P_{g2}, \lambda) = C_1(P_{g1}) + C_2(P_{g2}) - \lambda(P_{g1} + P_{g2}) - 500$$

Solution to problem 1

$$\frac{\partial F}{\partial P_{g1}} = 0 \Rightarrow \frac{\partial C_1}{\partial P_{g1}} - \lambda = 0 \qquad \frac{\partial F}{\partial P_{g2}} = 0 \Rightarrow \frac{\partial C_2}{\partial P_{g2}} - \lambda = 0$$

$$\frac{\partial F}{\partial \lambda} = 0 \Rightarrow P_{g1} + P_{g2} - 500 = 0$$

The above equation result in

(a)

$$\begin{bmatrix} 0.03 & 0 & -1 \\ 0 & 0.05 & -1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -7 \\ 500 \end{bmatrix} \Rightarrow \begin{bmatrix} P_{g1} \\ P_{g2} \\ \lambda \end{bmatrix} = \begin{bmatrix} 375 \\ 125 \\ 13.25 \end{bmatrix}$$

(b) And we note that it is not feasible because $\underline{\underline{P_{g1} > P_{g1MAX} = 300\,MW}}$

Problem 2

Generator cost rate functions, in \$/hr, for a three unit system are given as

$$C_1(P_1) = 0.004 P_1^2 + 5.3 P_1 + 500$$

$$C_2(P_2) = 0.006 P_2^2 + 5.5 P_2 + 400$$

$$C_3(P_3) = 0.009 P_3^2 + 5.8 P_3 + 200$$

Limits on the generation levels are $200 \le P_1 \le 450,\ 150 \le P_2 \le 350,\ 100 \le P_3 \le 225.$ These three generators must supply a total demand of 975 MW.

(a) Form the linear matrix equation necessary to solve the unconstrained optimization problem.
(b) The solution to the unconstrained optimization problem is $P_1 = 482.9MW,\ P_2 = 305.3MW,$ $P_3 = 186.5MW.$ For this solution (i.e., ignoring limits)
    (i) Compute lambda
    (ii) Determine the total cost rate

(iii) How much would the total cost rate change if the total load increased from 975 to 976 MW? (Indicate whether the total cost rate increases or decreases).

(c) Form the linear matrix equation necessary to solve the next iteration of getting the solution to this problem.

$$\frac{\partial C_1}{\partial P_1} = 0.008\,P_1 + 5.3 \Rightarrow 0.008\,P_1 - \lambda = -5.3$$

Solution to problem 2

$$\frac{\partial C_2}{\partial P_2} = 0.012\,P_2 \Rightarrow 0.012\,P_2 - \lambda = -5.5$$

$$\frac{\partial C_3}{\partial P_3} = 0.018\,P_3 + 5.8 \Rightarrow 0.018\,P_3 - \lambda = -5.8$$

(a)
$$\begin{bmatrix} 0.008 & 0 & 0 & -1 \\ 0 & 0.012 & 0 & -1 \\ 0 & 0 & 0.018 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} -5.3 \\ -5.5 \\ -5.8 \\ 975 \end{bmatrix}$$

(c) (i) We can use any of the $\dfrac{\partial C_i}{\partial P_i}$

$$\lambda = 0.008 \cdot (482.9) + 5.3 = 0.012 \cdot (305.3) + 5.5 = 0.018 \cdot (186.8) + 5.8$$

$$C_{Total} = 9.163\,\$/MW = C_1(P_1) + C_2(P_2) + C_3(P_3)$$

$$= 0.004(482.9)^2 + 5.3(482.9) + 500$$

(ii)
$$+ 0.006(305.3)^2 + 5.5(305.3) + 400$$

$$+ 0.009(186.8)^2 + 5.8(186.8) + 200$$

$$= 3992.1 + 2638.4 + 1597.5$$

$$= \underline{\underline{8228\$/hr}}$$

(iii) A 1 KW change in load would <u>increase</u> the total cost by $\lambda = \$9.163/hr$

(c) Now since P_1 exceeds its limit, we need to bring in the corresponding constraint with its Lagrange Multiplier.

$$F = C_1 + C_2 + C_3 - \lambda(P_1 + P_2 + P_3 - 975) - \mu(P_1 - 450)$$

And when we apply KKT, we will get

$$\begin{bmatrix} -0.008 & 0 & 0 & -1 & -1 \\ 0 & 0.012 & 0 & -1 & 0 \\ 0 & 0 & 0.018 & -1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \lambda \\ \mu \end{bmatrix} = \begin{bmatrix} -5.3 \\ -5.5 \\ -5.8 \\ 975 \\ 450 \end{bmatrix}$$

Problem 3

A three-unit system is given by the following data. The total system demand is 1100MW. Generator constraints are $0 < P_{g1} < 550$, $0 < P_{g2} < 300$, $0 \le P_{g3} < 300$

$$C_1(P_{g1}) = 0.010 \cdot (P_{g1})^2 + 0.3 \cdot (P_{g1}) + 1$$
$$C_2(P_{g2}) = 0.030 \cdot (P_{g2})^2 + 0.2 \cdot (P_{g2}) + 3$$
$$C_3(P_{g3}) = 0.020 \cdot (P_{g3})^2 + 0.9 \cdot (P_{g3}) + 5$$

(a) Identify the objective function for this optimization problem.
(b) Identify the Lagrangian function assuming no constraints are binding.
(c) Identify the KKT conditions assuming no constraints are binding.
(d) Find the solution to the problem assuming no constraints are binding.
(e) Find the solution to the problem accounting for any binding constraints.
(f) Find the total cost of supplying the 1100MW using the solution found in part (e)
(g) Approximately the total cost of supplying the 1100MW change if the upper limit on generator 1 was increased from 550MW to 560MW.

Solution to problem 3

(a) $C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3})$

(b) $\dfrac{\partial L}{\partial P_{g1}}\left(\bar{P_g}\right) = f\left(P_g\right) - \lambda\left[P_{g1} + P_{g2} + P_{g3} - 1100\right]$

$\dfrac{\partial L}{\partial P_{g1}} = 0.02 P_1 + 0.3 - \lambda = 0$

(c)

$\dfrac{\partial L}{\partial P_{g2}} = 0.06 P_2 + 0.2 - \lambda = 0$

$\dfrac{\partial L}{\partial P_{g3}} = 0.04 P_3 + 0.9 - \lambda = 0$

$\dfrac{\partial L}{\partial \lambda} = P_1 + P_2 + P_3 - 1100 = 0$

(d) $\begin{bmatrix} 0.02 & 0 & 0 & -1 \\ 0 & 0.06 & 0 & -1 \\ 0 & 0 & 0.04 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.2 \\ -0.9 \\ 1100 \end{bmatrix}$

$$\Rightarrow P_{g1} = 607.3 MW$$

$$\Rightarrow P_{g2} = 204.1 MW$$

$$\Rightarrow P_{g3} = 288.6 MW$$

$$\Rightarrow \lambda = 12.45\$ / MW - hr$$

(d) We note that P_{g1} violates its upper limit. Therefore we add in the equation $P_{g1} = 550$ and also argument the KKT condition for the first equation to be $0.02 P_{g1} + 0.3 - \lambda - \mu_1 = 0$. So our equation become

$$\begin{bmatrix} 0.02 & 0 & 0 & -1 & 0 \\ 0 & 0.06 & 0 & -1 & 0 \\ 0 & 0 & 0.04 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_1 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.2 \\ -0.9 \\ 550 \\ 1100 \end{bmatrix}$$

$$\Rightarrow P_{g1} = 550 MW$$
$$\Rightarrow P_{g2} = 227 MW$$
$$\Rightarrow P_{g3} = 323 MW$$

$$\Rightarrow \lambda = 13.82\$ / MW - hr$$
$$\Rightarrow \mu_1 = 2.52\$ / MW$$

But P_{g3} violates its upper limit so we must reformulate again:

$$\begin{bmatrix} 0.02 & 0 & 0 & -1 & -1 & 0 \\ 0 & 0.06 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0.04 & -1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_1 \\ \mu_3 \end{bmatrix} = \begin{bmatrix} -0.3 \\ -0.2 \\ -0.9 \\ 550 \\ 300 \\ 1100 \end{bmatrix}$$

$$\Rightarrow P_{g1} = 550 MW$$

$$\Rightarrow P_{g2} = 250 MW$$

$$\Rightarrow P_{g3} = 300 MW$$

$$\Rightarrow \lambda = 15.2\$ / MW - hr$$
$$\Rightarrow \mu_1 = -3.9\$ / MW$$
$$\Rightarrow \mu_3 = -2.3\$ / MW$$

(f) $C_1(550) + C_2(250) + C_3(300) = 3191 + 1928 + 2075 = \underline{\underline{7194}}$

(g) $7194 - (3.9) \cdot (10) = \underline{\underline{7155}}$

Problem 4

A three-unit system is given by the following data. The total system demand is 1100MW. Generator constraints are $0 < P_{g1} < 700$, $0 < P_{g2} < 200$, $0 < P_{g3} < 252.3$.

$$C_1(P_{g1}) = 0.008 \cdot (P_{g1})^2 + 0.5 \cdot (P_{g1}) + 5$$
$$C_2(P_{g2}) = 0.030 \cdot (P_{g2})^2 + 0.2 \cdot (P_{g2}) + 3$$
$$C_3(P_{g3}) = 0.020 \cdot (P_{g3})^2 + (P_{g3}) + 5$$

(a) Set up the linear matrix equation to solve the economic dispatch problem, assuming all constraints are satisfied (i.e., ignore constraints. DO NOT solve the equation.

(b) The solution to the problem in (a) is $P_{g1} = 664.5MW$, $P_{g2} = 182.2MW$, and $P_{g3} = 253.3MW$. Reformulate this linear matrix equation to solve the economic dispatch problem for this system, accounting for any violated constraints. Again, you DO NOT need to actually solve the equation, just set it up.

(c) Using only the cost function for generator 1, $C_1(P_{g1})$, together with information given in the part b problem statement, determine the system lambda for the solution to the unconstrained problem.

Solution to problem 4

(a)
$$\begin{bmatrix} 0.016 & 0 & 0 & -1 \\ 0 & 0.06 & 0 & -1 \\ 0 & 0 & 0.04 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \end{bmatrix} = \begin{bmatrix} -0.5 \\ -0.2 \\ 1 \\ 1100 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0.016 & 0 & 0 & -1 & 0 \\ 0 & 0.06 & 0 & -1 & 0 \\ 0 & 0 & 0.04 & -1 & -1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} P_{g1} \\ P_{g2} \\ P_{g3} \\ \lambda \\ \mu_1 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -0.2 \\ -1 \\ 1100 \\ 252.3 \end{bmatrix}$$

(c)
$$\lambda = \frac{\partial c_1}{\partial P_{g1}} = 0.016 P_{g1} + 0.5 = 0.016 \cdot (664.5) + 0.5 = \underline{\underline{11.132 \$ / MW - hr}}$$

Problem 5

Recall that the "system lambda" is the cost to the system owner of producing the next MW over the next hour; it is equal to the incremental cost of an individual unit when the system is economically dispatched for minimum cost and the unit is not at an upper or lower generation limit. A two-unit system is given by the following data.

$$C_1(P_{g1}) = 0.015 \cdot (P_{g1})^2 + 2 \cdot (P_{g1}) + 6$$
$$C_2(P_{g2}) = 0.020 \cdot (P_{g2})^2 + 6 \cdot (P_{g1}) + 4$$

The demand is 300MW

1. Write the KKT conditions that must be satisfied at the optimal solution to this problem, assuming that both units are operating between their respective upper and lower limits.
2. Set up the linear matrix equation to solve the economic dispatch problem for this system, assuming that both units are operating between their respective upper and lower limits. Do NOT solve the system of equations.
3. The solution to the problem in (2) is $P_{g1} = 228.57\,MW$, $P_{g2} = 71.43MW$. Assuming that each unit has a minimum generation capability of 80 MW.
   (a) Indicate why the given solution is not feasible.
   (b) Identify the optimal feasible solution
   (c) Identify the incremental costs of each unit at the optimal feasible solution
   (d) Identify the system lambda at the optimal feasible solution
   (e) Would the total cost of supplying the 300MW increase or decrease (relative to the total cost corresponding to the optimal feasible solution) if the minimum generation capabilities on both units were changed to 79MW ?

Solution to problem 5

1. $L[P_{g1}, P_{g2}, \lambda] = C_1(P_{g1}) + C_2(P_{g2}) - \lambda[P_{g1} + P_{g2} - 300]$

$$\frac{\partial L}{\partial P_{g1}} = 0.03 P_1 + 2 - \lambda = 0$$

$$\frac{\partial L}{\partial P_{g2}} = 0.04 P_2 + 6 - \lambda = 0$$

$$\frac{\partial L}{\partial \lambda} = P_1 + P_2 - 300 = 0$$

2. $$\begin{bmatrix} 0.03 & 0 & -1 \\ 0 & 0.04 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ \lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix}$$ or $$\begin{bmatrix} 0.03 & 0 & 1 \\ 0 & 0.04 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} P_{g1} \\ P_{g2} \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -6 \\ 300 \end{bmatrix}$$

3. (a) Because Generator 2 is below its minimum capability

   (b) $P_{g2} = 80MW, P_{g1} = 300 - 80 \Rightarrow 220\,MW - P_{g1}$

   (c) $$\frac{\partial C_1}{\partial P_{g1}} = 0.03 P_{g1} + 2 = 0.03 \cdot (220) + 2 = 8.6\$/MW - hr = IC_1$$

   $$\frac{\partial C_2}{\partial P_{g2}} = 0.04 P_{g2} + 6 = 0.04 \cdot (80) + 6 = 9.2\$/MW - hr = IC_2$$

   (d) lambda = 8.6$/MW-hr (Since unit 1 would supply the next MW-hr)

   (e) Total cost would decrease

Problem 6

The 'system λ' is the cost to the system owner of producing the next MW over the next hour. It is equal to the incremental cost of an individual unit when the system is economically dispatched for minimum cost and the unit is not at an upper or lower generation limit. A three-unit system is given by the following data. Total system demand is 1000 MW.

$$C_1(P_{g1}) = 0.008 \cdot (P_{g1})^2 + 0.5 \cdot (P_{g1}) + 5$$
$$C_2(P_{g2}) = 0.015 \cdot (P_{g2})^2 + 2 \cdot (P_{g2}) + 6$$
$$C_3(P_{g3}) = 0.020 \cdot (P_{g3})^2 + P_{g3} + 5$$

a) Set up the linear matrix equation to solve the economic dispatch problem for this system. DO NOT solve the equation.

b) The solution to the problem in (a) is P_{g1} = 549.6 MW, P_{g2} = 243.1 MW, and P_{g3} = 207.3 MW. Assume that each unit has a maximum generation capability of 350 MW. Reformulate the linear matrix equation to solve the economic dispatch problem for this system. Again, DO NOT solve the system.

c) What is the incremental cost for unit 1under the condition specified in part (b)? Do you think the system λ is greater than or less than this value?

Solution to problem 6

a)

$$\frac{\partial C_1}{\partial P_{g1}} = 0.016 \cdot (P_{g1}) + 0.5 \quad \Rightarrow \quad 0.01(P_{g1}) - \lambda = -0.5$$

$$\frac{\partial C_2}{\partial P_{g2}} = 0.03 \cdot (P_{g2}) + 2 \quad \Rightarrow \quad 0.03(P_{g2}) - \lambda = -2$$

$$\frac{\partial C_3}{\partial P_{g3}} = 0.04 \cdot (P_{g3}) + 1 \quad \Rightarrow \quad 0.04(P_{g3}) - \lambda = -1$$

$$P_{g1} + P_{g2} + P_{g3} = 1000$$

$$
\begin{bmatrix}
0.016 & 0 & 0 & 1 \\
0 & 0.03 & 0 & 1 \\
0 & 0 & 0.04 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}
\times
\begin{bmatrix}
P_{g1} \\
P_{g2} \\
P_{g3} \\
-\lambda
\end{bmatrix}
=
\begin{bmatrix}
-0.5 \\
-2 \\
-1 \\
1000
\end{bmatrix}
$$

b) Our Lagrangian becomes

$$L(Pg1, Pg2, Pg3, \lambda, \mu) = C_1(P_{g1}) + C_2(P_{g2}) + C_3(P_{g3}) - \lambda(P_{g1} + P_{g2} + P_{g3} - 1000) - \mu(P_{g1} - p_{g1\,mx})$$

and application of KKT results in

$$
\begin{bmatrix}
0.016 & 0 & 0 & 1 & 1 \\
0 & 0.03 & 0 & 1 & 0 \\
0 & 0 & 0.04 & 1 & 0 \\
1 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}
\times
\begin{bmatrix}
Pg1 \\
Pg2 \\
Pg3 \\
-\lambda \\
-\mu
\end{bmatrix}
=
\begin{bmatrix}
-0.5 \\
-2 \\
-1 \\
1000 \\
350
\end{bmatrix}
$$

c)

$$IC1 = (0.016)(350) + 0.5 = \$6.10 / MWhr$$

The system lambda must be greater than 6.1 $/MWhr because ???????

Problem 7

Generator 1 has an incremental cost curve of:
$$IC_1(P_{g1}) = 0.05(P_{g1}) + 2.0$$

and limits of:
$$10\ MW \leq P_{g1} \leq 100\ MW.$$

The generator operates in an economically dispatched system. In this system, it is found that supplying an additional 5 MW costs an additional \$50/hr. Determine P_{g1}.

Solution to problem 7

$$\lambda = \frac{\Delta\ \$/hr}{\Delta P} = \frac{50}{5} = 10\ \$/MWhr$$

$$IC(P_{g1}) = 0.05(P_{g1}) + 2.0 = 10 \quad \Rightarrow \quad P_{g1} = 160\ MW$$

But this result is outside of the power generation bounds. Therefore, $P_{g1} = 100$.

Problem 8

A system consists of two generators supplying a load. Generators 1 and 2 have incremental cost curves as indicated below:
$$IC_1(P_{g1}) = 0.04(P_{g1}) + 2.0$$
$$IC_2(P_{g2}) = 0.06(P_{g2}) + 1.0.$$

and limits of:
$$10\ MW \leq P_{g1} \leq 100\ MW$$
$$30\ MW \leq P_{g2} \leq 100\ MW$$

a)   In this system, when the load is 140 MW, what is the dispatch of these two units?
b)   In this system, when the load is 190 MW, what is the dispatch of these two units?
c)   In this system, under a certain economically dispatched scenario (a scenario different than in part (a) and (b)), it is found that supplying an additional 1 MW costs an additional \$5.68/hr. Determine P_{g1} and P_{g2}.

Solution to problem 8

a)

$$\begin{bmatrix} 0.04 & 0 & 1 \\ 0 & 0.06 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} Pg1 \\ Pg2 \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 140 \end{bmatrix} \qquad P_{g1} = 74.0 \quad P_{g2} = 66.0 \quad \lambda = 4.96$$

b)

$$\begin{bmatrix} 0.04 & 0 & 1 \\ 0 & 0.06 & 1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} Pg1 \\ Pg2 \\ -\lambda \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 190 \end{bmatrix} \qquad P_{g1} = 104.0 \quad P_{g2} = 86.0 \quad \lambda = 6.16$$

But in this case Pg1 is above the acceptable limits. We must re-optimize with the

Generator 1 limit taken into consideration:

$$\begin{bmatrix} 0.04 & 0 & 1 & 1 \\ 0 & 0.06 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} P_{g1} \\ P_{g2} \\ -\lambda \\ -\mu \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \\ 190 \\ 100 \end{bmatrix} \qquad \begin{array}{cc} P_{g1} = 100.0 & P_{g2} = 90.0 \\ \lambda = 6.4 & \mu = -0.4 \end{array}$$

c)

$$IC_1(P_{g1}) = 0.04(P_{g1}) + 2.0 = 5.68 \quad \Rightarrow \quad P_{g1} = 92 \ MW$$

$$IC_2(P_{g2}) = 0.06(P_{g2}) + 1.0 = 5.68 \quad \Rightarrow \quad P_{g2} = 78 \ MW$$

Problem 9

A two unit system has incremental cost curves (the derivatives of the cost curves) of $IC_1=0.01P_1+5$, and $IC_2=0.02P_2+4$, where P_1 and P_2 are given in MW. The demand is 300 MW. Ignoring limits on the generators, determine the values of P_1 and P_2 that minimize the cost of supplying the 300 MW.

Solution to problem 9

$$\begin{bmatrix} 0.01 & 0 & -1 \\ 0 & 0.02 & -1 \\ 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ 300 \end{bmatrix} \Rightarrow \begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 166.67 \\ 133.33 \\ 6.67 \end{bmatrix}$$

Problem 10

A two-generator system is operating on economic dispatch and supplying 420 Mw of load. The total cost of supply is computed from the final EDC solution (i.e., all constraints are satisfied) and found to be $3000/hr. From this same final solution, the LaGrange multipliers are found to be:

Equality constraint	$\lambda=\$15$/Mw-hr
$Pg_1 \geq 20$ Mw	$\mu_{1,L}=0$
$Pg_1 \leq 300$ Mw	$\mu_{1,H}=0$
$Pg_2 \geq 10$ Mw	$\mu_{2,L}=0$
$Pg_2 \leq 200$ Mw	$\mu_{2,H}= -\$4.00$/Mw-hr

Here the subscripts "L" and "H" indicate "<u>L</u>ow limit" and "<u>H</u>igh limit," respectively, and refer to the corresponding inequality constraint. For each question below, you must provide some basis or reasoning for your response.

(a) What would be the (approximate) total cost of supply if the total demand was increased to 421 Mw?

(b) What would be the total cost of supply if the lower limit for generator 2 was increased from 10 Mw to 11 Mw?

(c) What would be the total cost of supply if the upper limit for generator 2 was increased from 200 Mw to 201 Mw?

(d) What are the generation levels in Mw of generators 1 and 2?

(e) What is the incremental cost for generator 1?

Solution to problem 10

   (a) 3000+15=<u>3015</u>

   (b) 3000+0=<u>3000</u>

   (c) 3000-4=<u>2996</u>

   (d) Pg2=200 MW, Pg1=420-200=<u>220 MW</u>

   (e) IC(Pg1)= λ=$15/Mw-hr

Problem 11

A two generator system has cost curves ($/hr) of $C_1(P_1)=0.006P_1^2 +5P_1+3$, and $C_2(P_2)=0.01P_2^2+4P_2+2$, where P_1 and P_2 are given in MW. The total demand is $P_T=500$ MW. The limits on these generators are $0 \le P_1 \le 300$ and $0 \le P_2 \le 300$.

   a. Determine the unconstrained values of P_1 and P_2 that minimize the cost of supplying the 500 MW, and indicate whether this solution is feasible or not.

   b. For the solution found in (a), how much would the total cost of supply change if the total demand increased to 501 MW for one hour ?

   c. Use the complementary condition (the third condition in the KKT conditions), to identify the values of each Lagrange multiplier associated with the inequality constraints.

Solution to problem 11

   a. The uncontrained solution is found from applying the first and second KKT conditions. This results in:

$$
\begin{bmatrix} 0.012 & 0 & -1 \\ 0 & 0.02 & -1 \\ 1 & 1 & 0 \end{bmatrix}
\begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix}
= \begin{bmatrix} -5 \\ -4 \\ 500 \end{bmatrix}
\quad \Rightarrow \quad
\begin{bmatrix} P_1 \\ P_2 \\ \lambda \end{bmatrix}
= \begin{bmatrix} 281.25 \\ 218.75 \\ 8.375 \end{bmatrix}
$$

Both generation values are within their constraints; therefore the solution is feasible.

   b. The LaGrange multiplier λ provides the change in the objective function (which is the total cost of supply) for a 1 MW increase in demand over the next hour, which is exactly what this question is asking, therefore the answer is $8.375.

   a. Because the unconstrained solution is feasible, there are no inequality constraints that are binding. Therefore, by the complementary condition, it must be the case that all Lagrange multipliers on the inequality constraints must be zero, i.e., $\mu_i = 0 \; \forall \; i$.