## Problem 11

A balanced three phase industrial facility consists of two parallel loads, as follows:

- 1200 kVA at 0.5 power factor lagging
- 200 kW (entirely resistive load)

The two loads are supplied by a three phase, distribution feeder circuit having impedance of 4+j2 ohms per phase. The load voltage is 13.2 kV line-to-line.

a. Find the magnitude and angle of the feeder current
b. Find the magnitude of the line-to-line voltage at the sending end of the distribution feeder circuit.
c. Determine the capacitive VARS required to correct the power factor of the entire load to 0.80 lagging.
d. Determine the susceptance of the capacitor necessary to supply these vars at the stated load voltage.

## Solution to problem 11

$a)$

$$pf = 0.5 \quad \rightarrow \quad 0.5 = \cos(\theta) \quad \rightarrow \quad \theta = 60°$$

$$S_1 = P_1 + jQ_1 = 1200(0.5 + j0.866) = 600 + j1039.2$$

$$S_2 = P_2 + jQ_2 = 200 + j0$$

$$S_T = S_1 + S_2 = 600 + j1039.2 + 200 + j0 = 800 + j1039.2$$

$$|S_T| = \sqrt{P_T{}^2 + Q_T{}^2} = \sqrt{(800)^2 + (1039.2)^2} = 1311.5\ kVA$$

$$I = \frac{1}{\sqrt{3}} \cdot \frac{S}{V} = \frac{(1311.5)\cdot 10^3}{(\sqrt{3})(13.2)\cdot 10^3} = \underline{\underline{57.4\ A}}$$

$$\angle I = \tan^{-1}(Q_T / P_T) = \tan^{-1}(1039.2/800) = 52.41\text{deg}$$

$b)$

$$V_{LN,Load} = (13.2)\cdot 10^3 / \sqrt{3} = 7621$$

$$V_{LN,src} = 7621 + 57.4\angle -52.4(4 + j2) = 7852 - \text{j}111.9 = 7853\angle -0.82° Volts$$

$$|V_{LL,src}| = 7853\sqrt{3} = 13.602 kV$$

$c)$

$$pf = 0.8 \quad \rightarrow \quad \theta = 36.9°$$

$$Q'_T = P_T \tan(\theta) = 800 \tan(36.9°) = 600\ kVARs$$

$$Q_C = Q_T - Q'_T = 1039.2 - 600 = 439.2$$

$$Q_C = \underline{\underline{439.2}} kVARS$$

$d)$

$$B = Q_C / (V_{LL})^2 = 439200 / (13{,}200)^2 = 0.00252 mhos$$

## Problem 12

Show that ABCD parameters for the following circuits are given as indicated.

a. Shunt or "I" circuit as given in Figure D1.14.
b. T-circuit as given in Figure D1.15.
c. Pi-circuit as given in Figure D1.16. Note the circuit configuration and matrix when Y1=Y2=0

## Solution to Problem 12

**a)** $V_1 = AV_2 + BI_2$ ; $A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1$

; $B = \frac{V_1}{I_2}\Big|_{V_2=0} = 0$

$I_1 = CV_2 + DI_2$ ; $C = \frac{I_1}{V_2}\Big|_{I_2=0} = Y$

; $D = \frac{I_1}{I_2}\Big|_{V_2=0} = 1$

**b)** $A == \frac{V_1}{Y_2}\Big|_{I_2=0}$

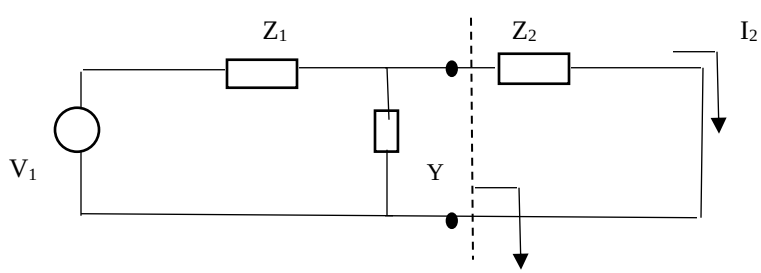
Use voltage division:

$V_2 = V_1 \left[\frac{1/Y}{Z_1 + 1/Y}\right]$

⇨ $\frac{V_1}{V_2} = \left[\frac{Z_1 + 1/Y}{1/Y}\right]$

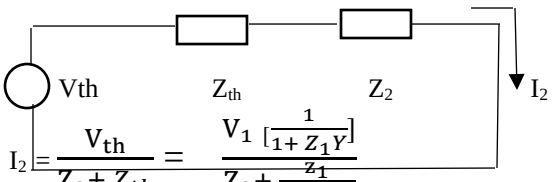
⇨ $Z_1 Y + 1 = A$

$B == \frac{V_1}{I_2}\Big|_{V_2=0}$



Use Thevenin $Zth = \frac{Z_1 1/Y}{Z_1 + 1/Y} = \frac{Z_1}{1 + Z_1 Y}$

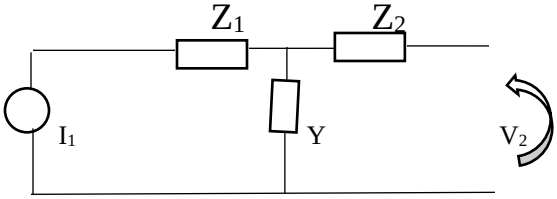
$$Vth = V_1 \left[\frac{\frac{1}{Y}}{Z_1 + \frac{1}{Y}}\right] = V_1 \left[\frac{1}{1 + Z_1 Y}\right]$$




$$I_2 = \frac{V_{th}}{Z_2 + Z_{th}} = \frac{V_1 \left[\frac{1}{1 + Z_1 Y}\right]}{Z_2 + \frac{Z_1}{1 + Z_1 Y}}$$

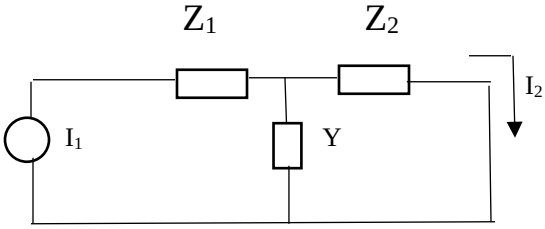
$$\frac{V_1}{I_2} = \frac{Z_2 \left[\frac{Z_1}{1 + Z_1 Y}\right]}{\frac{1}{1 + Z_1 Y}} = Z_1 + Z_2 (1 + Z_1 Y) = Z_1 + Z_2 + Z_1 Z_2 Y = B$$

$$C = \frac{I_1}{V_2}\Big|_{I2=0}$$




$$V_2 = \frac{I_1}{Y} = \frac{I_1}{V_2} = Y = C$$

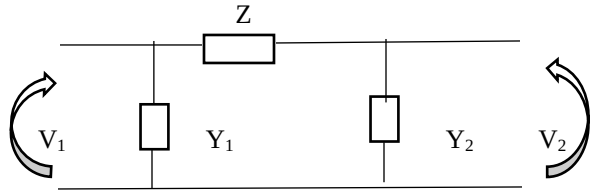
$$D = \frac{I_1}{I_2}\Big|_{V2=0}$$




Use current division: $I_2 = I_2 \left[\frac{1/Y}{1/Y + Z_2}\right]$

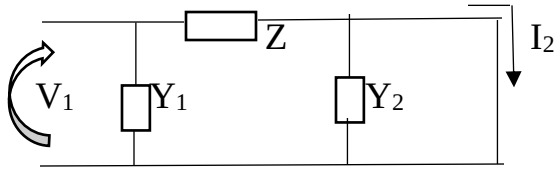
$$\frac{I_1}{I_2} = 1 + Y Z_2 = D$$

**c)** $A == \frac{V_1}{V_2}\Big|_{I_2=0}$

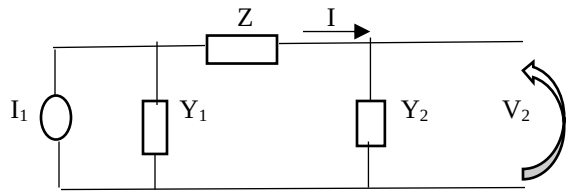


Use Voltage division: $V_2 = V_1 \left[\frac{1/Y_2}{Z + 1/Y_2}\right] \Rightarrow \frac{V_1}{V_2} = 1 + Y_2 Z = A$

$B == \frac{V_1}{I_2} \Big|_{V_2=0}$




$I_2 = \frac{V_1}{Z} \Rightarrow \frac{V_1}{I_2} = Z$

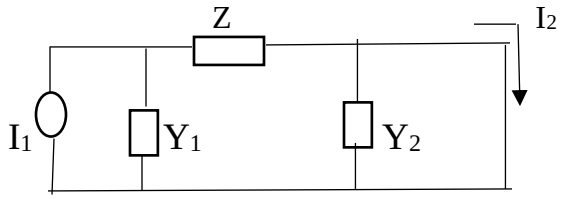



$C = \frac{I_1}{V_2} \Big|_{I2=0}$

Use current division: $V_2 = \frac{I}{Y_2} = \frac{I_1 \left[\frac{1/Y_1}{\frac{1}{Y_!} + Z + 1/Y_2}\right]}{Y_2}$

$\frac{I_1}{V_2} = Y_1 + Y_2 + Y_1 Y_2 Z = C$

$D = \frac{I_1}{I_2} \Big|_{V2=0}$




Use current division: $I_2 = I_1 \left[\frac{\frac{1}{Y_1}}{\frac{1}{Y_!} + Z}\right] = I_1 = \frac{1}{1 + Y_1 Z}$

$\frac{I_1}{I_2} = 1 + Y_1 Z$

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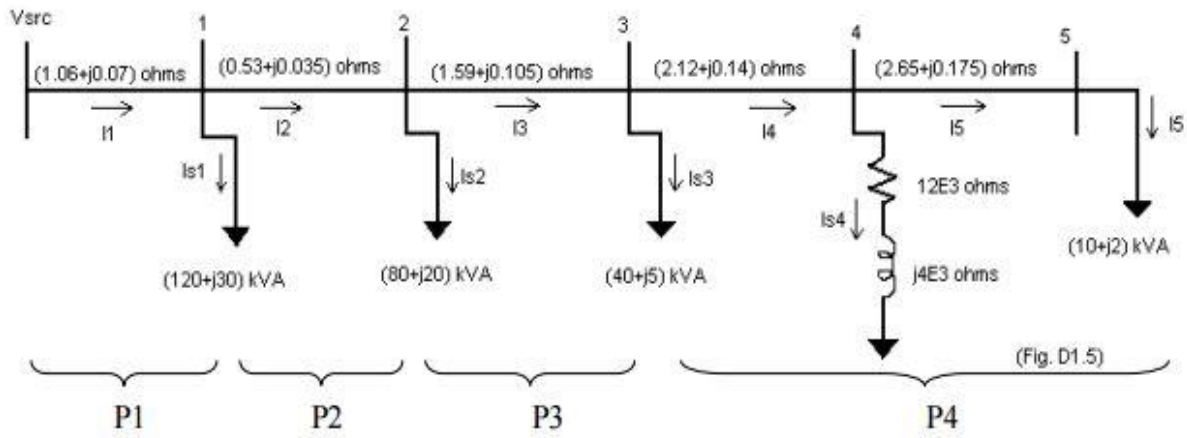
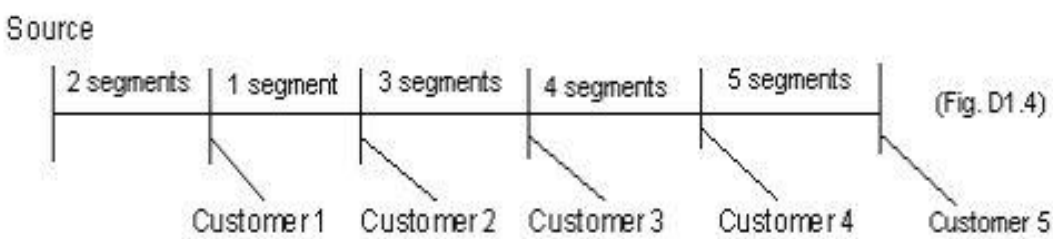
## Problem 13

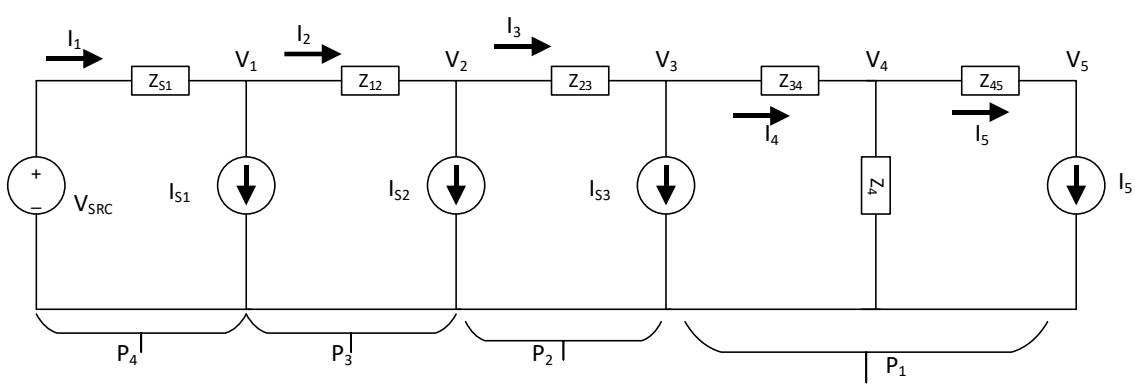
Consider the following circuit partitioned according to P1…,P4, where a 3-phase radial feeder is supplying 5 customer loads. The line-to-line voltage of the last bus (#5) is 11kV. Line impedances and load values are given in the figure. This problem is solved in Example D1.1 using KCL and KVL. In the exercise below, the goal is to use ABCD parameters to solve it, i.e., obtain $V_{src}$ given V5=11,000/sqrt(3).




a. P1: Compute $I_5$. Get ABCD parameters. Compute $[V_3 \;\; I_4]^T$.
b. P2: Compute $I_{s3}$ and then $I_3$. Get ABCD parameters. Compute $[V_2 \;\; I_3]^T$. (Note that you will already have $I_3$ so really this calculation just gives you $V_2$).
c. P3: Compute $I_{s2}$ and then $I_2$. Get ABCD parameters. Compute $[V_1 \;\; I_2]^T$. (Note that you will already have $I_2$ so really this calculation just gives you $V_1$).
d. P4: Compute $I_{s1}$ and then $I_1$. Get ABCD parameters. Compute $[V_{src} \;\; I_1]^T$. (Note that you will already have $I_1$ so really this calculation just gives you $V_{src}$)

## Solution to Problem 13




$Z_S = 0.53 + j0.035$
$Z_4 = 12*10^3 + j\,4*10^3$
$Z_{S1} = 2\,Z_S$
$Z_{12} = Z_S$

$S_1 = (120 + j30) * 10^3$
$S_2 = (80 + j20) * 10^3$
$S_3 = (40 + j5) * 10^3$
$S_5 = (10 + j2) * 10^3$

$Z_{23}$= 3$Z_S$
$Z_{34}$= 4 $Z_S$
$Z_{4S}$= 5$Z_S$

a) **Compute $I_5$. Get ABCD parameters. Compute $[V_3 \;\; I_4]^T$.**

$$\overline{V_5} = \frac{11000}{\sqrt{3}} \llcorner -30 \qquad S_5 = \overline{V_5}\;\overline{I_5^*} \Rightarrow \overline{I_5^*} = \left(\frac{S_5}{\overline{V_5}}\right) = \frac{(10+j2)^* * 10^3/3}{\frac{11000}{\sqrt{3}} \llcorner -30}$$

$$\overline{I_5^*} = 0.040206 - j0.353\ A$$

$$\begin{pmatrix} \overline{V_3} \\ \overline{I_4} \end{pmatrix} = \begin{pmatrix} 1 + \frac{Z_{34}}{Z_4} & Z_{34} + Z_{45} + \frac{Z_{45}Z_{34}}{Z_4} \\ \frac{1}{Z_4} & 1 + \frac{Z_{45}}{Z_4} \end{pmatrix} \begin{pmatrix} \overline{V_5} \\ \overline{I_5} \end{pmatrix}$$

$$= \begin{pmatrix} 1 + \frac{4(0.53 + j0.035)}{12 * 10^3 + j\,4 * 10^3} & 9(0.53 + j0.035) + \frac{20((0.53 + j0.035)^2}{12 * 10^3 + j\,4 * 10^3} \\ \frac{1}{12 * 10^3 + j\,4 * 10^3} & 1 + \frac{5(0.53 + j0.035)}{12 * 10^3 + j\,4 * 10^3} \end{pmatrix} \begin{pmatrix} \frac{11000}{\sqrt{3}} \llcorner -30 \\ 0.040206 - j0.353 \end{pmatrix}$$

$$= \begin{pmatrix} 6.35442 * 10^3 \llcorner -30.0055 \\ 1.03544 \llcorner -44.76 \end{pmatrix}$$

**b) Compute $I_{s3}$ and then $I_3$. Get ABCD parameters. Compute $[V_2 \;\; I_3]^T$. (Note that you will already have $I_3$ so really this calculation just gives you $V_2$)**

$V_1 = AV_2 + BI_2$ ; $A = \frac{V_1}{V_2}\Big|_{I_2=0} = 1$

$$\begin{pmatrix} \overline{V_1} \\ \overline{I_1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \overline{V_2} \\ \overline{I_2} \end{pmatrix}$$

; $B = \frac{V_1}{I_2}\Big|_{V_2=0} = 0$

$I_1 = CV_2 + DI_2$ ; $C = \frac{I_1}{V_2}\Big|_{I_2=0} = Z$

$$\begin{pmatrix} \overline{V_1} \\ \overline{I_1} \end{pmatrix} = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \overline{V_2} \\ \overline{I_2} \end{pmatrix}$$

; $D = \frac{I_1}{I_2}\Big|_{V_2=0} = 1$

$$\overline{I_3} = \overline{I_{S3}} + \overline{I_4}$$

$$\overline{I_{S3}} = V_3\,\overline{I_{S3}} = \left(\frac{S_3}{V_3}\right)^*$$

$$= \left(\frac{(40+\mathrm{j}5)*10^3/3}{6.35442*10^3 \angle -30.0055}\right)^* = 2.1146 \angle -37.13\ A$$

$$\begin{pmatrix}\overline{V_3}\\ \overline{I_3}\end{pmatrix} = \begin{pmatrix}6.35442*10^3 \angle -30.0055\\ 3.14389 \angle -39.637\end{pmatrix}$$

$$\begin{pmatrix}\overline{V_2}\\ \overline{I_3}\end{pmatrix} = \begin{pmatrix}1 & 3(0.53+\mathrm{j}0.035)\\ 0 & 1\end{pmatrix}\begin{pmatrix}6.35442*10^3 \angle -30.0055\\ 3.14389 \angle -39.637\end{pmatrix}$$

$$= \begin{pmatrix}6359.41 \angle -30.0101\\ 3.14389 \angle -39.637\end{pmatrix}$$

**c) Compute $I_{s2}$ and then $I_2$. Get ABCD parameters. Compute $[V_1\ \ I_2]^T$. (Note that you will already have $I_2$ so really this calculation just gives you $V_1$).**

$$\overline{I_2} = \overline{I_{S2}} + \overline{I_3}$$

$$\overline{I_{S2}} = V_2\,\overline{I_{S2}} = \left(\frac{S_2}{V_2}\right)^*$$

$$= \left(\frac{(80+\mathrm{j}20)*10^3/3}{6359.41 \angle -30.0101}\right)^* = 4.322 \angle -44.046\ A$$

$$\begin{pmatrix}\overline{V_2}\\ \overline{I_2}\end{pmatrix} = \begin{pmatrix}6359.41 \angle -30.0101\\ 7.4608 \angle -42.19\end{pmatrix}$$

$$\begin{pmatrix}\overline{V_1}\\ \overline{I_2}\end{pmatrix} = \begin{pmatrix}1 & 0.53+\mathrm{j}0.035\\ 0 & 1\end{pmatrix}\begin{pmatrix}6359.41 \angle -30.0101\\ 7.4608 \angle -42.19\end{pmatrix}$$

$$= \begin{pmatrix}6363.3 \angle -30.015\\ 7.4608 \angle -42.19\end{pmatrix}$$

**d) Compute $I_{s1}$ and then $I_1$. Get ABCD parameters. Compute $[V_{src}\ \ I_1]^T$. (Note that you will already have $I_1$ so really this calculation just gives you $V_{src}$)**

$$\overline{I_1} = \overline{I_{S1}} + \overline{I_2}$$

$$S_1 = \overline{V_1}\ \overline{I_5^*} \Rightarrow \overline{I_{S1}^*} = \left(\frac{S_1}{\overline{V_1}}\right)^*$$

$$= \left(\frac{(120+\mathrm{j}30) * 10^3/3}{6363.3 \angle -30.015}\right)^* = 6.479 \angle -44.06\ A$$

$$\begin{pmatrix} \overline{V_1} \\ \overline{I_1} \end{pmatrix} = \begin{pmatrix} 6363.3 \angle -30.015 \\ 13.9385 \angle -43.055 \end{pmatrix}$$

$$\begin{pmatrix} \overline{V_{src}} \\ \overline{I_1} \end{pmatrix} = \begin{pmatrix} 1 & 2(0.53 + \mathrm{j}0.035) \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 6363.3 \angle -30.015 \\ 13.9385 \angle -43.055 \end{pmatrix}$$

$$= \begin{pmatrix} 6377.94 \angle -30.037 \\ 13.9385 \angle -43.055 \end{pmatrix}$$