

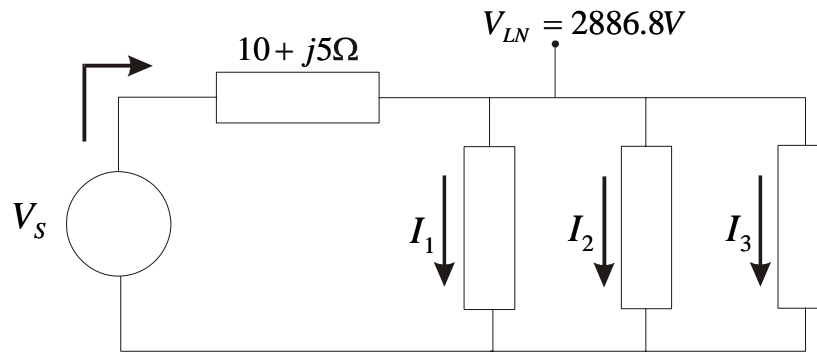
# Module B3

## Problem 1

The 3-phase loads are connected in parallel. One is a purely resistive load connected in wye. It consumes 300kW. The second is a purely inductive 300kVAR load connected in wye. The third is a purely capacitive 300kVAR load connected in wye. The line-to-line voltage at the load is 5kV. A 3-phase distribution line supplying this load has an impedance of  $10+j5$  ohms per phase.

- Calculate the currents drawn by each load (magnitude and phase).
- Indicate the power factor of each load. Remember that non-unity power factors must also include whether they are lagging or leading.
- What is the power factor of the entire load? That is, what is the power factor seen by the transmission line at the load end?
- Calculate the real and reactive power supplied at the sending end of the distribution line.

## Solution to problem 1



$$(a) V_{Load} = \frac{5kV}{\sqrt{3}} = 2886.8\angle 0^\circ$$

Note that:

$$S = V \cdot I^* \Rightarrow I = \left( \frac{S}{V} \right)^*, S = \frac{S_{3\phi}}{3}$$

Therefore:

$$S_1 = 100 \times 10^3 W, S_2 = j100 \times 10^3 VAR, S_3 = -j100 \times 10^3 VAR.$$

$$\text{for purely resistive load: } I_1 = \left( \frac{S_1}{V_{Load}} \right)^* = \frac{100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle 0^\circ A}}$$

$$\text{for purely inductive load: } I_2 = \left( \frac{S_2}{V_{Load}} \right)^* = \frac{-j100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle -90^\circ A}}$$

$$\text{for purely capacitive load: } I_3 = \left( \frac{S_3}{V_{Load}} \right)^* = \frac{j100 \times 10^3}{2886.8} = \underline{\underline{34.64\angle 90^\circ A}}$$

Note in the above that for the resistive load, the current and voltage are in phase, for the inductive, the current lags by 90 deg, and for the capacitive, the current leads by 90 deg.

(b) Load 1 :  $pf_1 = 1.0$

Load 2 :  $pf_2 = 0$  lagging

Load 3 :  $pf_3 = 0$  leading

(c) Need current angle with respect to  $V_{Load} = 2886.98\angle 0^\circ$

$$I_{Load} = I_1 + I_2 + I_3 = 34.64\angle 0^\circ - j34.64 + j34.64 = 34.64\angle 0^\circ$$

so  $V_{Load}$  and  $I_{Load}$  are in phase !

$$\Rightarrow \underline{\underline{pf_{Load} = 1.0}}$$

(d)  $V_s = V_{Load} + I_{Load}(Z_t) = 2886.8 + (34.64\angle 0^\circ) \cdot (10 + j5) = 3237.8\angle 3.07^\circ$

$$\Rightarrow S_{3\phi} = 3V_s I_{Load}^* = 3 \cdot (3237.8\angle 3.07^\circ) \cdot (34.64\angle 0^\circ) = 336,472\angle 3.07^\circ \text{VA}$$

$$\underline{\underline{P_{3\phi} = 335.99 \text{ kW}}}, \underline{\underline{Q_{3\phi} = 18.02 \text{ kVAR}}}$$

Alternatively, one could compute losses and add to load :

$$S_{loss(3\phi)} = 3(|I_{Load}|^2 Z_t) = 3(34.64)^2 \cdot (10 + j5) = 35,999.9 + j17,998.9$$

$$\Rightarrow S_{3\phi} = S_{load(3\phi)} + S_{loss(3\phi)} = 300 \times 10^3 + 35,997.9 + j17,998.9 \text{VA}$$

$$\Rightarrow \underline{\underline{P_{3\phi} = 335.99 \text{ kW}}}$$

$$\Rightarrow \underline{\underline{Q_{3\phi} = 18.0 \text{ kVAR}}}$$

## Problem 2

A three phase load has a per phase impedance, connected in Y, of  $100 + j30\Omega$ . The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of  $10 + j5\Omega/\phi$ .

(a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.

(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

## Solution to problem 2

$$(a) V_{AN} = \frac{1500}{\sqrt{3}} = 866.025 \text{V} \Rightarrow I_A = \frac{V_{AN}}{Z_{LOAD}} = \frac{866.025}{100 + j30} = 7.945 - j2.384 \text{A} = 8.295\angle -16.7^\circ \text{A}$$

$$V_{sending,AN} = V_{AN} + I_A \cdot Z_{Line} = 866.025\angle 0^\circ + (8.295\angle -16.7^\circ) \cdot (10 + j5)$$

$$= 957.39 + j15.89 = 957.53\angle 0.95^\circ \text{V}$$

$$\Rightarrow \underline{\underline{|V_{Sending,AB}| = 957.53 \cdot \sqrt{3} = 1658.5 \text{V}}}}$$

$$(b) S = 3 \cdot V_{sending,AN} \cdot I_a^* = 3 \cdot (957.53\angle 0.95^\circ) \cdot (8.295\angle 16.7^\circ) = 22,706.5 + j7224.7 \text{VA}$$

$$\Rightarrow \underline{\underline{P = 22.706 \text{ kW}}}, \underline{\underline{Q = 7.225 \text{ kVAR}}}$$

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**Problem 3**

A three-phase load consumes 100kVA at 0.7 pf lagging. The line-to-line voltage magnitude at the load is 1500V. The three-phase distribution line supplying this load has an impedance of  $10 + j5\Omega/\phi$

- (a) Calculate the line-to-line voltage magnitude at the sending end of the distribution line.  
(b) Calculate the real and reactive power supplied at the sending end of the distribution line.

**Solution to problem 3**

- (a) Note that  $\theta = \cos^{-1}(0.7) = 45.57^\circ$  (the angle is positive because the pf is lagging), and  $\sin(45.7) = 0.714$ .

$$V_{an} = \frac{1500}{\sqrt{3}} = 866.025 \angle 0^\circ \text{V}, \quad S_{1\phi} = \frac{100 \times 10^3}{3} (0.7 + j0.714) \text{VA}$$

$$I_L = \left( \frac{S_{1\phi}}{V_{an}} \right)^* = \frac{33.3 \times 10^3 \times (0.7 - j0.714)}{866.025} = 38.45 \angle -45.57^\circ \text{A}$$

$$V_{\text{sending},an} = V_{an} + I_L \cdot Z_L = 866.025 \angle 0^\circ + 38.45 \angle -45.57^\circ \cdot (10 + j5) = 1279.2 \angle -6.211^\circ$$

$$\underline{\underline{|V_{\text{sending},AB}| = \sqrt{3} \times 1279.2 = 2215.3 \text{V}}}}$$

- (b)

$$S_{\text{sending}} = 3 \cdot (V_{\text{an},\text{sending}}) \cdot (I_L)^* = 3(1279.2 \angle -6.211^\circ) \cdot (38.45 \angle -45.57^\circ) = 147.556 \angle 39.36^\circ \text{VA}$$

$$\Rightarrow \underline{\underline{P_{\text{sending}} = 114.09 \text{kW}, \quad Q_{\text{sending}} = 93.58 \text{kVAR}}}}$$

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**Problem 4**

The complex power absorbed by a three-phase load is 1500kVA at 0.8 pf lag

$$P_{1\phi} = \underline{\hspace{2cm}} \qquad Q_{1\phi} = \underline{\hspace{2cm}}$$

If the Line voltage at the load in problem 1 is 8660.2540 V, what is the voltage magnitude across each phase of the load, if the load is connected as follows,

$$|V_d| = \underline{\hspace{2cm}} \qquad |V_y| = \underline{\hspace{2cm}}$$

What is the magnitude of line current drawn by this load?

$$|I_L| = \underline{\hspace{2cm}}$$

**Solution to problem 4**

The complex power absorbed by a three-phase load is 1500kVA at 0.8 pf lag

Note that  $\theta = \cos^{-1}(0.8) = 36.87^\circ$  (the angle is positive because the pf is lagging), and  $\sin(36.87) = 0.6$ .  
 Then  $P_{1\phi} = 1500(0.8)/3 = 400 \text{ kW}$ ,  $Q_{1\phi} = 1500(0.6)/3 = 300 \text{ kVAR}$ .

$$P_{1\phi} = \underline{400 \text{ kW}} \quad Q_{1\phi} = \underline{300 \text{ kVAR}}$$

If the Line voltage at the load in problem 1 is 8660.254 V, what is the voltage magnitude across each phase of the load, if the load is connected as follows,

$$|V_A| = \underline{8660.254 \text{ V}} \quad |V_y| = \underline{5000 \text{ V}} = \frac{8660.254}{\sqrt{3}}$$

What is the magnitude of line current drawn by this load?

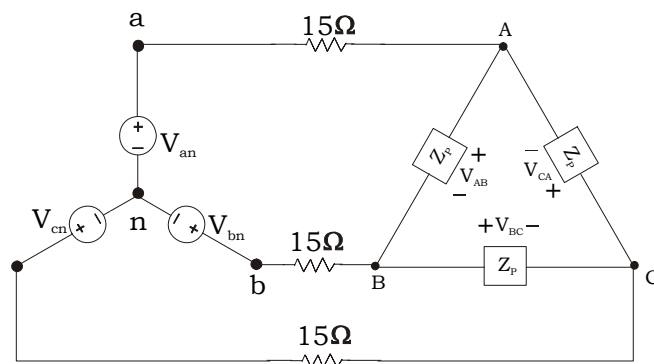
$$|I_L| = \underline{100 \text{ A}}$$

$$|I_L| = \frac{1500 \times 10^3}{\sqrt{3} \times 8660.254}$$

### Problem 5

In the circuit shown below,  $V_{an} = 12,000 + j0 \text{ V (rms)}$ . Assume positive phase sequence. The balanced source supplies 1.5 MW and 0.3 MVAR to the three phase balanced load. Find:

- The rms line current.
- $Z_p$



### Solution to problem 5

$$(a) S_{3\phi} = (1.5 + j0.3) \times 10^6 \text{ VA}$$

$$S_{1\phi} = \frac{1}{3}(1.5 + j0.3) \times 10^6 \text{ VA}$$

$$V_{an} I_{an}^* = S_{1\phi} = 12,000 I_{an}^*$$

$$I_{an}^* = \frac{(1.5 + j0.3) \times 10^6}{3 \times 12,000} = 42.492 \angle 11.31^\circ \text{ A}$$

$$I_{an} = \underline{\underline{42.492 \angle -11.31^\circ \text{ A}}}$$

$$(b) Z_p = 3Z_{py}$$

$$Z_{py} = \frac{V_{an}}{I_{aA}}$$

$$V_{AN} = V_{aN} - I_{aA}^* \times 15 = 12,000 - 42.492 \angle -11.31^\circ \times 15$$

$$= 11,375.68 \angle 0.6296^\circ \text{ V}$$

$$Z_{py} = \frac{11,375.68 \angle 0.6296^\circ}{42.492 \angle -11.31^\circ} = 267.7135 \angle 11.939^\circ$$

$$Z_{py} = 3 \times 267.7135 \angle 11.939^\circ = \underline{\underline{803.1405 \angle 11.939^\circ \Omega}}$$

### Problem 6

A three phase source is supplying a balanced three phase load over a transmission line having impedance of  $Z_L = 2 + j20$  ohms per phase. The voltage at the source end of the transmission line is  $2887 \angle 0$  volts line to neutral. The current supplied through the transmission line is  $I_L = 100 \angle -30^\circ$  amperes.

1. Determine the power factor seen by the source, and specify whether it is leading or lagging.
2. Determine the voltage (line to neutral) at the load.
3. Determine the power factor of the load, and specify whether the load is
  - a. leading or lagging
  - b. inductive or capacitive
4. Determine the real and reactive power consumed by the load.

### Solution to problem 6

1.  $\text{pf} = \cos(30) = 0.8660$ , and it is lagging.
2.  $V_{\text{LOAD}} = 2887 \angle 0 - 100 \angle -30 (2 + j20) = 713.8 - j1632 = 2366.5 \angle -43.6$
3.  $\text{pf angle} = \text{angle at which voltage leads the current} = -43.6 - (-30) = -13.6$ , so  $\text{pf} = \cos(-13.6) = 0.972$ , ...and the current is leading the voltage! This means the power factor is leading (part a) and the load must be capacitive (part b).
4.  $S = 3V_{\text{LOAD}} (I)^* = 3(2366.5 \angle -43.6)(100 \angle +30) = 690044 - j166939 = 709.950 \angle -13.6 \text{ kVA}$